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JEL classification codes
C72, D72, D74

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Linking Individual and Collective Contests through Noise Level and Sharing Rules*

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We provide a theoretical link between the two most prominent ways of modeling individual and collective contests as proposed by Tullock (1980) and Nitzan (1991) respectively. By introducing Nitzan's sharing rule as a way of modeling individual contests we obtain a contest success function nesting a standard Tullock contest and a fair lottery. We first provide an equivalence result between the proposed contest and Tulllock's contest for the two-player set-up. We then employ this nested contest as a way of introducing noise in multi-player contests when in the Tullock contest a closed form solution for the equilibrium in pure strategies does not exist. We conclude by comparing the proposed contest with the existing ones in the literature.

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1 Introduction

In a contest individuals or groups exert costly effort in order to increase their chances of winning a prize. One of the most famous and prolific ways of modeling individual contests was introduced by Tullock (1980). Some years later, Nitzan (1991) focused on collective action and significantly contributed to the literature by modeling contests among groups. His particular interest was in alternative ways of allocating the contested prize among the members of the winning group. While these seminal studies proposed alternative contest designs that significantly contributed in the evolution of the individual and collective contests literature, a theoretical link between these two strands is missing.

In this study we link these two models of individual and collective contests by introducing Nitzan's sharing rule as a way of modeling individual contests with noise. This approach leads to a contest nesting a standard Tullock contest and a fair lottery. Once this nested contest is further transformed to a Tullock contest with transfers, we use the results of Hillman and Riley (1989) and Stein (2002) to provide an equivalence result for Tullock's and Nitzan's methods in a two-player contest. Moreover, and in contrast to the standard Tullock contest, this nested contest guarantees a closed form solution in pure strategies for the asymmetric N players contest that may be used in several applications. As we show, this solution significantly differs to the ones described in the literature (Amegashie, 2006; Dasgupta and Nti, 1998). While the current mechanism is closer to the one proposed by Tullock under different criteria of equivalence, the existing ones are closer to the Tullock contest in terms of axiomatic properties.

Providing the theoretical link to the studies of Tullock (1980) and Nitzan (1991) and proposing the use of a collective contest mechanism in individual contests is important for a number of reasons. First, and most importantly, it is well known that under certain conditions a pure strategy equilibrium does not exist for the Tullock (1980) contest. We show that in such cases one can employ our results to consider a dual problem in the Nitzan (1991)-equivalent collective contest in which an equilibrium in pure strategies always exists. Second, our results will be of use for contest designers who look for optimum mechanisms under constraints and can now implement a particular type of mechanism that works the best. Third, Baye and Hoppe (2003) show that a Tullock contest can represent various general situations other than only rent-seeking (as initially proposed by Tullock, 1980) and provide relevant equivalence conditions. Hence, the link we establish allows the application of our results in various areas of contests.

The rest of the paper progresses as follows. In Section 2 we provide the structure of the analysis. We establish the link between the two types of contests in Section 3, starting with the two-player case in Section 3.1 and extending our set-up to N-players in

Section 3.2. Section 4 concludes.

2 Model set-up

2.1 Individual contests

Let N players compete for a prize of common value V by exerting non-negative levels of effort. A contest success function (CSF), f_i, maps the vector of efforts to the probability that player $i \in N$ wins the prize (i.e., $f_i : R_+^N \to [0,1]$ such that $\sum_{i \in N} f_i(.) = 1$). Arguably, the most popular CSF is the one proposed by Tullock (1980), in which the probability of player i winning the prize when exerting effort $e_i \geq 0$ is

$$f_i^r(e_1, ..., e_N) = \frac{e_i^r}{\sum_{j=1}^N e_j^r} \text{ if } \sum_{j=1}^N e_j^r > 0 \text{ and } 1/N \text{ otherwise} \qquad (\textbf{r-CSF})$$

where e_j denotes the effort exerted by player j and $r \geq 0$ determines the noise level in the contest. If $r = 0$ then the noise is maximum and players face a fair lottery. If $r \to \infty$ then there is no noise and players compete under an all-pay auction in which the highest effort wins with certainty. Let the cost functions be linear and $c_i > 0$ denote the marginal cost of player i. Without loss of generality, let c_i be increasing in i, i.e., $c_1 \leq c_2 \leq ... \leq c_N$. Player's i payoff is then given by[1]

$$\pi_i^r = \frac{e_i^r}{\sum_{j=1}^N e_j^r} V - c_i e_i \qquad (1)$$

2.2 Collective contests

In a collective contest several groups compete for a prize that has to be further allocated to the N group members. If the prize is non-divisible, then group members also compete in an intragroup contest (see, for example, Choi et al., 2015 and the references therein). If the prize is divisible, then it is distributed among the group members following a predetermined sharing rule. Arguably, the most famous sharing rule is the one proposed

[1]For the reasons of interpretability when we use Nitzan's sharing rule in an individual contest, players' heterogeneity is introduced through cost asymmetries. This is equivalent to asymmetries in terms of valuations or in the effort impact (Gradstein, 1995; Corchón, 2007).

by Nitzan (1991) where the share of the prize allocated to player i is given by[2]

$$f_i^\lambda(e_1, ..., e_N) = \lambda \frac{e_i}{\sum_{j=1}^N e_j} + (1-\lambda)\frac{1}{N}$$

If $\lambda \in [0,1]$, as in most of the literature on collective contests, then a fraction λ of the prize is allocated proportional to players' effort. The remaining fraction of the prize is allocated in an egalitarian manner across the N group members. Given that $\lambda \in [0,1]$, it holds that $f_i^\lambda(e_1, ..., e_N) \in [0,1]$ and the λ-CSF can be interpreted as a nested contest that is a convex combination of the most common version of a Tullock CSF where $r = 1$ and of a fair lottery (i.e., $r = 0$).[3] Note that λ can be interpreted as the degree of noise (or meritocracy) in the competition and clearly resembles to the effect of r in the r-CSF.

In an N-player λ-contest, the payoff of player i is given by

$$\pi_i^\lambda = [\lambda \frac{e_i}{\sum_{j=1}^N e_j} + (1-\lambda)\frac{1}{N}]V - c_i e_i \tag{2}$$

Note λ need not be restricted in the $[0,1]$ interval. However, when $\lambda > 1$ the proposed function f_i^λ may take values outside $[0,1]$ and therefore it can not be interpreted as a CSF representing probabilities. If $\lambda > 1$ then the proposed sharing rule allows for transfers among group members.[4] For individual contests, this is similar to the idea proposed by Appelbaum and Katz (1986) and Hillman and Riley (1989).

3 Link between the two contests

The payoff of player i in the proposed nested contest (2) can be rewritten as:

$$\pi_i^\lambda = \frac{e_i}{\sum_{j=1}^N e_j}\tilde{V} - c_i e_i + (1-\lambda)\frac{V}{N} \tag{3}$$

where $\tilde{V} = \lambda V$. Hence the proposed nested contest is now transformed to a proportional Tullock contest (i.e., $r = 1$) with an additional exogenous parameter $(1-\lambda)V/N$.

The exogenous parameter clearly does not affect the solution, and resembles a Tullock contest with transfers (Hillman and Riley, 1989). Therefore, as long as $\lambda > 0$, solving the

[2]Nitzan (1991) was the first to use this sharing rule in modelling collective contests. This sharing rule was previously introduced in the cooperative production literature by Sen (1966). For a survey on sharing rules in collective rent-seeking see Flamand and Troumpounis (2015).

[3]Amegashie (2012) proposes a similar nested two-player contest that ranges from a Tullock to an all-pay auction.

[4]For group contests allowing this possibility see, for example, Baik and Shogren (1995); Baik and Lee (1997, 2001); Lee and Kang (1998); Gürtler (2005); Balart et al. (2015).

proposed nested contest is equivalent to solving the most tractable and therefore most frequently implemented version of a Tullock contest (i.e., $r = 1$). Consequently, one can follow Stein (2002) to solve the λ-contest and obtain the unique equilibrium (as shown by Matros, 2006). Denoting individual prize valuations by $V_i = \frac{\tilde{V}}{c_i}$, the equilibrium effort in the unique equilibrium of the λ-contest is:

$$e_i = \left(1 - \frac{1}{V_i} \frac{(M-1)}{\sum_{j=1}^{M} \frac{1}{V_j}}\right) \frac{(M-1)}{\sum_{j=1}^{M} \frac{1}{V_j}} \tag{4}$$

where M is the number of active players. Player M is the highest marginal cost player for whom the condition $V_M > \frac{(M-2)}{\sum_{i \leq M-1} \frac{1}{V_j}}$ is satisfied.

Given this link, in order to analyze the relationship between the λ and r-contests, we first consider a two-player case in the next sub-section and then extend the analysis to N players in Section 3.2. In the continuation, we make use of the definitions coined by Chowdhury and Sheremeta (????) regarding equivalence of contests.

Definition 1.

- *Contests are effort equivalent if they result in the same equilibrium efforts.*
- *Contests are strategically equivalent if they result in the same best responses.*
- *Contests are payoff equivalent if in equilibrium they result in the same payoffs.*

3.1 The two-player case

The two-player case is the most common in the literature since it allows one to provide with closed form solutions, clear comparative statics and a graphical representation of the results. Baik (1994) and Nti (1999) solved the two-player r-contest when an equilibrium in pure strategies exists (Baye et al., 1994; Alcalde and Dahm, 2010; Ewerhart, 2014) for the equilibrium in mixed strategies). As shown by Nti (1999), as long as $V_1^r + V_2^r > rV_2^r$ (letting $V_1 \leq V_2$ then $r < 1$ is a sufficient condition, while $r < 2$ is a necessary condition), the two-player r-contest has a unique equilibrium in pure strategies where both players are active (i.e., they exert strictly positive effort) with:

$$e_i = \frac{rV_i^{r+1}V_j^r}{(V_1^r + V_2^r)^2} \tag{5}$$

with $i \neq j$, $i = 1, 2$ and $V_i = \frac{\tilde{V}}{c_i}$.

By comparing the solutions of the λ and r-contest presented in equations (4) and (5) as well as the best responses and the equilibrium payoffs in the two contests, the following equivalence results arise.

Proposition 1. *For any two-player r-contest with an equilibrium in pure strategies (i.e., r such that $V_1^r + V_2^r > rV_2^r$):*

1. *There exists an effort equivalent λ-contest with $\lambda = \frac{r(V_1V_2)^r(V_1+V_2)^2}{V_1V_2(V_1^r+V_2^r)^2}$.*
2. *There exists no strategically equivalent λ-contest (except for $r = \lambda = 0$ and $r = \lambda = 1$ when the two contests coincide).*
3. *There exists no payoff equivalent λ-contest (except for $r = \lambda = 0$, $r = \lambda = 1$ when the two contests coincide and the symmetric case, $c_1 = c_2$).*

Proof: *See the appendix.*

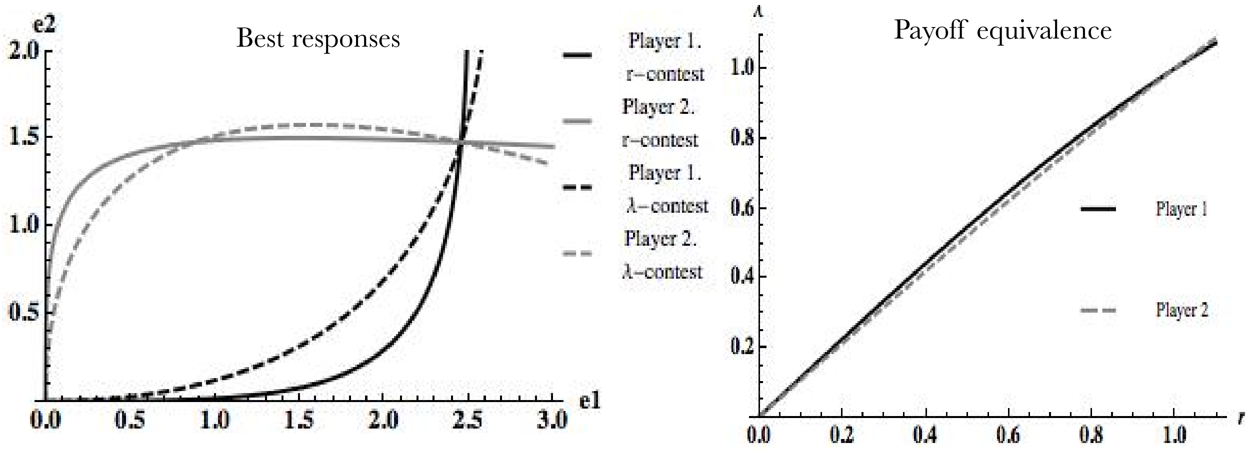

Figure 1: Best response functions and effort equivalence on the left ($r = 0.5$ and $\lambda = 0.524729$) and payoff equivalence on the right. For both graphs $V_1 = 20$ and $V_2 = 12$.

Figure 1 illustrates the result for asymmetric players. On the left, best responses are different for the λ and r-contests but they intersect at the same effort equivalence point. On the right panel, it is clear that the value of λ that guarantees payoff equivalence for player 1 only coincides with that providing payoff equivalence for player 2 when the two contests coincide (i.e., $r = \lambda = 1$ and $r = \lambda = 0$).

For the symmetric case, effort equivalence is obtained when $\lambda = r$. This also translates into payoff equivalence.

By means of comparative statics we can describe several interesting equilibrium properties of the two-player λ-contest and compare them with those of the r-contest:

Proposition 2. *In the two-player λ-contest equilibrium:*

1. *A unique equilibrium in pure strategies exists for any λ.*
2. *Aggregate equilibrium effort is increasing in λ.*
3. *Aggregate equilibrium effort is decreasing in the players' asymmetry.*

Proof: *See the appendix.*

The first two properties substantially differ between the two contests. First, an equilibrium in pure-strategies always exists in the λ-contest, while this is only true for certain parametric restrictions in the r-contest. This difference is attributed to the fact that while zero effort guarantees a zero payoff in the r-contest, this is not true in the λ-contest. In the λ-contest, zero effort may result in negative payoffs since losers have to make a transfer to the winners (Hillman and Riley, 1989). These transfers in the λ-contest make the condition $e_i \geq 0$ non-binding. This guarantees an interior solution, and consequently an equilibrium in pure strategies always exists.

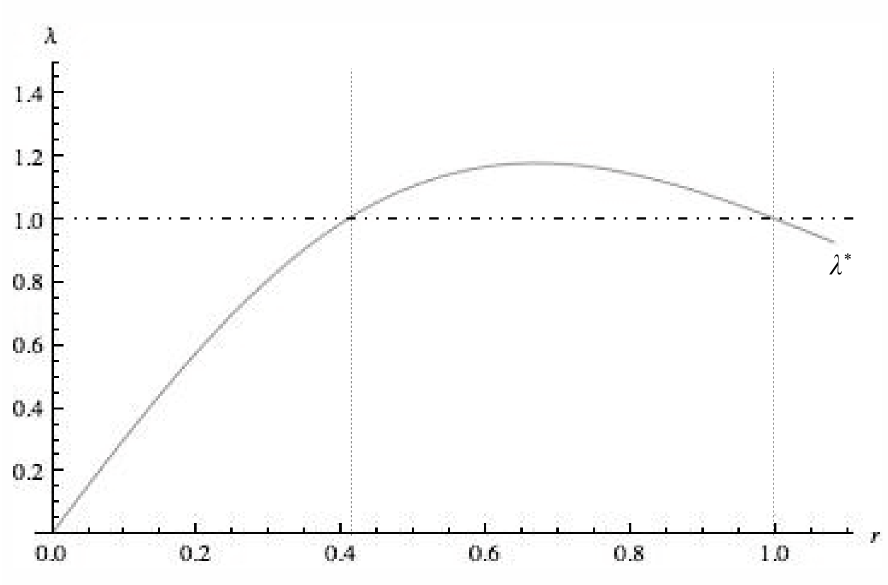

Figure 2: Effort equivalence value of λ, given any r such that an equilibrium in pure strategies exists $V_1^r + V_2^r > rV_2^r$ (with $V_1/V_2 = 10$).

Second, while in the r-contest the comparative statics of aggregate effort with respect to r depend on the degree of asymmetry among the players, in the λ-contest the level of aggregate effort is strictly increasing in λ. As a consequence, the value of λ that guarantees an effort equivalent λ-contest, is not monotonic in r (Figure 2).[5] Finally, the third property linking the asymmetry with aggregate equilibrium effort is in line with the standard result of the r-contest (Nti, 1999).

Before proceeding further, let us focus on an important difference between the λ and r-contests. A well known feature of the r-contest is that it satisfies participation constraint, namely players having a non-negative expected utility in equilibrium. Recall that in the effort equivalent λ-contest the presence of transfers as in Hillman and Riley (1989) may be required (i.e., $\lambda > 1$, as in Figure 2 for r belonging to $[0.41, 1]$). These transfers in turn may violate individuals' participation constraint, as it always happens in their setup. While any competition involving a policy change with "winners" and "losers" does not entail voluntarily participation and hence the participation constraint does not apply, the latter is crucial in situations where agents compete for preexisting rents (Hillman and Riley, 1989). Overall, while the existence of a pure strategy equilibrium for any level of noise is a desirable characteristic of the λ-contest compared to the r-contest, the possible violation of the participation constraint by the λ-contest may challenge its implementability under some particular settings.

Remark 1. *In any λ-contest the participation constraint is satisfied if and only if*

$$\lambda \leq \frac{(V_1 + V_2)^2}{V_1^2 + 2V_1V_2 - V_2^2}$$

Hence, for any r, although a λ-equivalent contest always exists, the latter fails to satisfy participation constraint if

$$\frac{r(V_1V_2)^r(V_1 + V_2)^2}{V_1V_2(V_1^r + V_2^r)^2} > \frac{(V_1 + V_2)^2}{V_1^2 + 2V_1V_2 - V_2^2}$$

As the first inequality describes, as long as λ is low, meaning that either no transfers are involved (i.e., $\lambda \leq 1$) or transfers are present but are not too punishing for low contributors, all individuals will obtain a non-negative payoff in equilibrium and hence the participation constraint is satisfied. Once the transfers become high enough (i.e., $\lambda > \frac{(V_1+V_2)^2}{V_1^2+2V_1V_2-V_2^2}$), then low contributors are severely punished and therefore are better off not participating in the contest.[6] The conditions under which the λ-equivalent contest

[5] Note that, even for $r < 1$, the level of λ that ensures effort equivalence between the two contests might involve transfers as in Hillman and Riley (1989). This depends on the exact level of asymmetry.

[6] This remark follows immediately when guaranteeing non-negative expected utility for player 2 and isolating λ. Since $V_1 > V_2$ the same condition also guarantees that the expected utility of the highest valuation player is also non-negative. Notice that participation constraint means not participating in the contest with transfers whatsoever, and is different to an individual being inactive and exerting zero effort. Zero effort is allowed in the solution of the λ-contest as presented in (4), but the presence of inactive players violates the participation constraint.

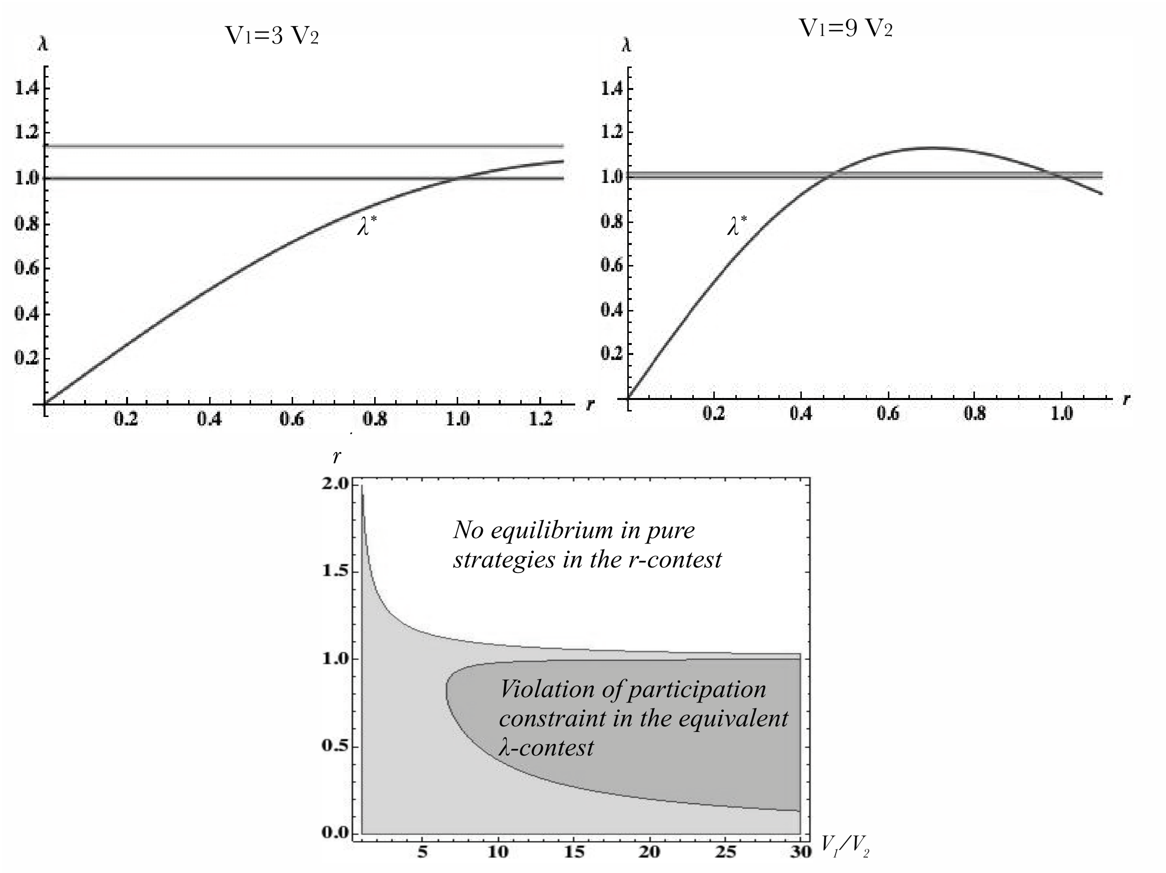

Figure 3: Effort equivalence value of λ and participation constraint.

does not satisfy the participation constraint depends on the specific combination of cost asymmetry and noise level.

In the two upper panels of Figure 3, we provide a graphical representation of two different scenarios regarding the satisfaction of the participation constraint. The lower horizontal line at value 1 reminds the reader that above this value the λ equivalent effort contest requires the presence of transfers. The upper horizontal line of these two first panels, represents the upper bound on the values of λ for which the participation constraint is satisfied. In the left upper panel, for a low level of asymmetry such that $V_2 = 3V_1$, for any r we can find an equivalent λ-contest that satisfies the participation constraint, even if the latter requires some transfers. In contrast, in the right upper panel, we observe how the effort equivalent λ-contest violates the participation constraint for some values of r when the level of asymmetry is high enough such that $V_2 = 9V_1$.

The darkest area in the lower panel plots the combinations of asymmetry V_1/V_2 and r for which the effort equivalent λ-contest does not satisfy the participation constraint. While the equivalent λ-contest satisfies the participation constraint for any level of r when players asymmetry is low, for higher levels of asymmetry the region of r's for which an equivalent λ-contest satisfies the participation constraint shrinks.

3.2 Extension to N players

Well known difficulties, in terms of non-existence of a closed form solution for the pure strategy Nash equilibrium, arise while extending the r-contest to set-ups with more than two heterogeneous players and $r \neq 1$. In contrast, the biggest advantage of employing a λ-contest is that closed form solutions are still obtained by expression (4). Given that for values of $\lambda \in [0, 1]$ no transfers are involved, parameter λ can be interpreted as a measure of the noise level. Hence, when the effect of noise is of interest one can employ the λ-CSF as a way of modelling contests with more than two asymmetric players. The following proposition summarizes the properties of the λ-contest with more than two-players.

Proposition 3. *In any N-player λ-contest with $N > 2$:*

1. *A unique pure strategy equilibrium with a closed form solution, given by (4), exists.*
2. *In equilibrium individual and aggregate equilibrium effort are increasing in λ.*
3. *A strategically or effort equivalent r-contest may not exist (except for $r = \lambda = 0$ and $r = \lambda = 1$ when the two contests coincide).*

Proof: *See the appendix.*

Representing and solving the N-player asymmetric contests with non-proportional noise level (i.e., $r \neq 1$) in a tractable way constitutes an important advantage of the λ-contest. Another alternative in this area was proposed by Amegashie (2006) while proposing a CSF with tractable noise parameter. He employed the structure of Dasgupta and Nti (1998) in which they propose the α-CSF:

$$f_i^{\alpha}(e_1, ..., e_N) = \frac{e_i + \alpha}{\sum_{j=1}^{N} e_j + N\alpha}$$

where $\alpha > 0$ is the introduced "tractable" noise level parameter. The higher α is, the more noise is introduced, with the case of $\alpha \to \infty$ representing a fair lottery ($r = 0$ for a Tullock CSF and $\lambda = 0$ for the nested contest presented above).

Note that the α-contest and the λ-contest coincide for the pairs ($\alpha \to \infty,\ \lambda = 0$) and ($\alpha = 0,\ \lambda = 1$). Therefore the λ-contest without transfers (i.e., $\lambda \in [0, 1]$) can represent the same levels of noise as the α-contest.[7] As it turns out the α-contest and the

[7] Fu and Lu (2007) show that the three types of contests considered here are strategically equivalent to a noisy ranking contest with a type I extreme-value (maximum) distributed noise. However, a different production technology of effort, $g(e_i)$ is associated with each type of contest. In particular, $g(e_i) = e_i^r$ in the r-contest, $g(e_i) = e_i$ in the λ-contest and $g(e_i) = e_i + \alpha$ in the α-contest.

λ-contest are never effort equivalent for such intermediate levels of noise (i.e., $\lambda \in (0,1)$ and $\alpha \in (0,\infty)$). In the following corollary we highlight the main differences that arise in terms of equilibrium results.

Corollary 1.

- *For $N = 2$ there exists a λ such that the λ-contest is effort equivalent to an r-contest for any r that guarantees an equilibrium in pure strategies (Proposition 1). An α-contest is never effort equivalent to an r-contest.*

- *For $N > 2$ and symmetric players there exists a λ such that the λ-contest is effort equivalent to an r-contest for any r that guarantees an equilibrium in pure strategies (Proposition 3). In the α-contest this is only true for $r \in [0,1]$.*

- *In an N-symmetric-players contest adding an additional player increases total effort in the r-contest with an equilibrium in pure strategies and in the λ-contest while it may decrease total effort in the α-contest.*

- *The λ-contest and r-contest can not sustain an equilibrium where all players are inactive while this may occur in the α-contest.*

The statements in Corollary 1 arise directly from Proposition 1 and 3 and Amegashie (2006). Amegashie (2006) also discusses the axiomatic properties satisfied by his proposal compared to the ones of Tullock (1980) as provided by Skaperdas (1996) and Clark and Riis (1998). For our proposal and when the λ-contest represents a contest success function (i.e., $\lambda \in [0,1]$) the following characteristics are of interest.

1. As Clark and Riis (1998) show the axioms of imperfect discrimination (A1), monotonicity (A2), Luce's axiom (A4') and homogeneity of degree zero (A6) hold if and only if $f_i(e_1,...,e_N) = a_i e_i^r / \sum_{j=1}^{N} a_j e_j^r$ where $r > 0$ and $a_i, a_j > 0$ are constants (i.e., an augmented version of the r-CSF). While the axioms of imperfect discrimination and monotonicity are satisfied both by the λ- and α-CSF, Luce's axiom is satisfied only by the α-CSF and homogeneity of degree zero is satisfied only by the λ-CSF.

2. Both for the λ- and α-CSF and in contrast to the r-CSF, **(i)** it does not hold that if $e_i = 0$ for any player i then the probability of winning the price is zero; and **(ii)** these functions are continuous at $e_i = e_j = 0$ for all $i \neq j$.

In terms of axiomatic properties the λ-CSF does not satisfy Luce's axiom while it satisfies homogeneity of degree zero. Whereas, the α-CSF satisfies Luce's axiom, but it does not satisfy homogeneity of degree zero. Hence, while in terms of equilibrium results the λ-contest is closer to the results of an r-contest than the ones of an α-contest, the latter performs better than the λ-CSF in terms of axiomatic properties.

4 Discussion

In this study we provide with a theoretical link between the individual contest (Tullock, 1980) and the collective contest (Nitzan, 1991); and derive the sufficient conditions for effort equivalence among the two. Since an equilibrium in pure strategies always exists with a collective contest framework, this link allows one to to implement the same as an appropriate mechanism for N-asymmetric-players individual contests. We further provide the relationship with the contest proposed by Amegashie (2006) in solving the same issue.

The λ-CSF we propose can be implemented in applications where the absence of closed form solutions induces focus only on $r = 1$. Franke (2012), for instance, analyzes the effect of affirmative action policies on aggregate effort. While in the two-player case different noise level is allowed, the analysis is restricted to $r = 1$ for the N-players contest. By considering the λ-CSF one can generalize the results to investigate whether the affirmative action condition to maximize effort are also true for lower levels of noise.

A typical feature of the λ-CSF is that exerting zero-effort does not necessarily result in a zero payoff. A similar result is obtained in multi-winner contests for which a noisy winner selection mechanism is implemented (Berry, 1993; Chowdhury and Kim, 2014). That is because in these multi-winner contests only one prize is allocated through the effort outlays whereas others are allocated randomly - which resembles the nested prize allocation feature of the λ-CSF. It would be of much interest to understand and analyze the links between these two types of contest mechanisms.

The λ-CSF can also be of interest for experimental work and the effort equivalence result can be tested. Since $\lambda \in [0, 1]$ can be interpreted as noise, one can study the effect of the latter on individual behavior (Millner and Pratt, 1989). The attractiveness of the λ-CSF comes from the intuitive manner it can be introduced in the laboratory. Experimenters could split $(1 - \lambda)$ fraction of the prize in an egalitarian manner and let subjects compete for the remaining part λ through a standard Tullock contest.

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5 Appendix

5.1 Proof of Proposition 1

1. When $N = 2$ the condition for a player being active active in the λ-contest is always satisfied. From (4) the equilibrium effort of player i is $e_i = \frac{V_i^2 V_j \lambda}{(V_1+V_2)^2}$ for $i = 1, 2$, $j \neq i$. To prove effort equivalence we just need to equalize these equilibrium efforts with the ones of the r-contest as presented in expression (5). Equilibrium efforts of the λ-contest coincide with the ones of the r-contest for $\lambda = \frac{r(V_1V_2)^r(V_1+V_2)^2}{V_1V_2(V_1^r+V_2^r)^2}$.

2. Note that when $r = \lambda = 0$ or $r = \lambda = 1$ the λ-contest and the r-contest coincide, hence strategic equivalence follows immediately in these cases. The best response for player i in the λ-contest is $e_i(e_j) = \max\{-e_j + \sqrt{e_j V_i \lambda}\ ,\ -e_j - \sqrt{e_j V_i \lambda}\}$ while it is not possible to find a closed form solution for the best response of the r-contest. However, as shown in Chowdhury and Sheremeta (????) effort equivalence is a necessary condition for strategic equivalence. Therefore, strategic equivalence is guaranteed only if the first order conditions of the r-contest are satisfied for any value of e_j after substituting the best responses of the λ-contest with $\lambda = \frac{r(V_1V_2)^r(V_1+V_2)^2}{V_1V_2(V_1^r+V_2^r)^2}$. This is true if and only if $\frac{e_j^r r V_i (A)^{r-1}}{(e_j^r+(A)^r)^2} = 1$, where $A = -e_j + \sqrt{e_j V_i}\sqrt{\frac{r(V_iV_j)^{r-1}(V_i+V_j)^2}{(V_i^r+V_j^r)^2}}$ which is not true for all values of e_j (only for the equilibrium one).

3. By plugin equilibrium efforts in the payoff of player 1 we obtain that the λ-contest induces the same payoff as the one in the r-contest for $\lambda = \frac{(V_1+V_2)^2(V_1^{2r}-V_2^{2r}-2r(V_1V_2)^r)}{(V_1^2-2V_1V_2-V_2^2)(V_1^r+V_2^r)^2} = \lambda_1$. Similarly, the λ-contest induces payoff equivalence for player 2 if and only if $\lambda = \frac{(V_1+V_2)^2(V_1^{2r}-V_2^{2r}+2r(V_1V_2)^r)}{(V_1^2+2V_1V_2-V_2^2)(V_1^r+V_2^r)^2} = \lambda_2$. Normalizing $V_2 = 1$ and $V_1/V_2 = v$ we see that $\lambda_1 = \lambda_2$, i.e., payoff equivalence, is only obtained for $V_1 = V_2$ or $r = \{0, 1\}$ (when the two contests coincide).

5.2 Proof of Proposition 2

1. The existence of an equilibrium in pure strategies for any λ arises directly from the conditions in Nti (1999). Uniqueness is shown in Matros (2006).

2. Total effort in the λ-contest is $\frac{V_1V_2\lambda}{V_1+V_2}$. Consequently, aggregate equilibrium effort is increasing in λ.

3. Assuming without loss of generality that $V_1 \geq V_2$ and by normalizing $V_1 = 1$ and $v = V_2/V_1 \in [0, 1]$, we obtain that the derivative of total effort with respect to v is $\frac{\lambda}{(1+v)^2} > 0$, therefore aggregate equilibrium effort is strictly increasing in symmetry.

5.3 Proof of Proposition 3

Parts 1 and 2 of the proposition arise directly from (4) and the arguments presented by Stein (2002). For part 3 if an effort equivalent λ-contest exists then we should be able to find a relationship between λ and r such that the first order conditions of all active players in the r-contest are satisfied by the equilibrium expressions in the λ-contest. However, we can show that this is not true with a counterexample. Assume that $V_1 = 4$, $V_2 = 3$ and $V_3 = 2$. Then, $\lambda = 0.8046$ guarantees that the first order condition of player 1 in the r-contest is satisfied. However, this value of λ does not satisfy the first order conditions of players 2 and 3 in the r-contest. As effort equivalence is not guaranteed, strategic equivalence is neither. When $r = \lambda = 0$ or $r = \lambda = 1$ the λ-contest and the r-contest coincide, hence strategic and effort equivalence follows immediately in these cases.