

Differential Equation with Technology (2)

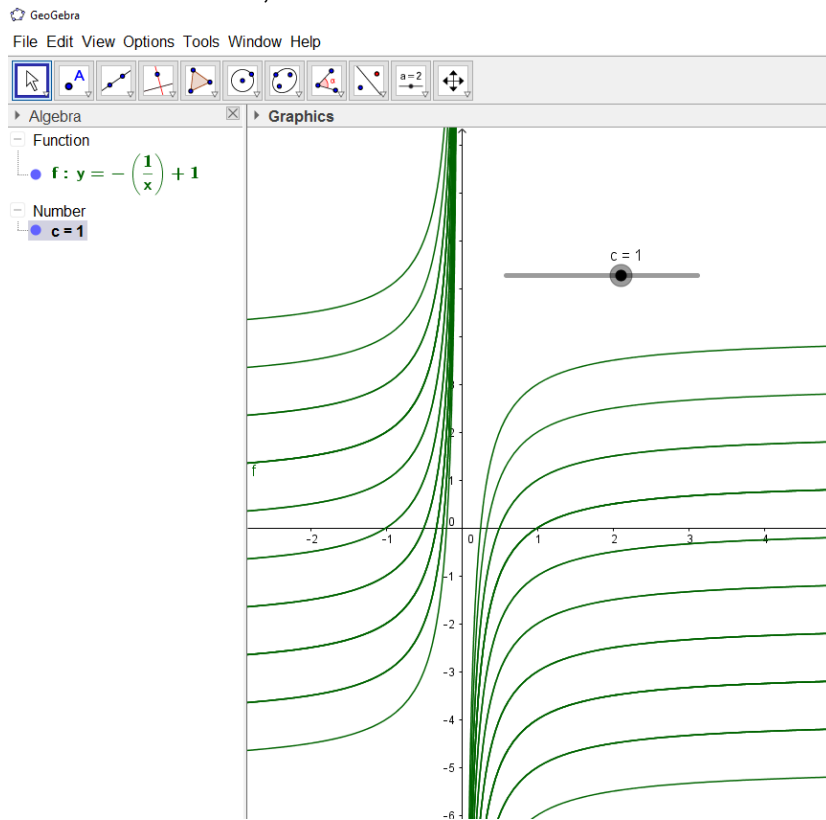
In a Year 13 class, students are asked to solve the following problem:

“It is given that $y = -\frac{1}{x}$ is a solution to the differential equation $\frac{dy}{dx} = y^2$.

Find the general solution to this differential equation.”

The conversation between Students A and B follows.

Student A: If $y = -\frac{1}{x}$ is a solution, then $y = -\frac{1}{x} + C$ should be the general solution, where C is a constant. It seems to me that the graphs of all these functions are vertical translations of the $y = -\frac{1}{x}$ graph. Let me sketch these graphs in GeoGebra, I can make a slider for C and keep the trace of the graphs for the different values of C , look:



Student B: Hold on, you say that $y = -\frac{1}{x} + 1$, for example, is a solution of the equation

$\frac{dy}{dx} = y^2$. Let me check [he writes]: $\frac{dy}{dx} = \frac{1}{x^2}$, but $y^2 = \left(-\frac{1}{x} + 1\right)^2$ is not $\frac{1}{x^2}$.

What's the problem here?

You are the teacher and you just heard this conversation.

Questions:

- What is a solution to this mathematical problem?
- What are the aims of using this problem in class?
- What are the issues emerging from the exchange between Students A and B?
- How would you respond to the two students and to the whole class?