

## Iteration\*

### A glimpse into what practitioners say:

- Spreadsheets can be used to make iteration as part of the UK A-level mathematics course more interesting. Smith (1990) explains how this can be done when using the Newton-Raphson method or linear interpolation<sup>1</sup>.
- Graphing calculators can be very useful for solving equations using iteration. Hayes (1990) explains how the calculator 'updates' itself by using its memory function to remember the result of the most recent calculation<sup>2</sup>.

### Some useful links:

- You can find a description of how iteration is introduced to the UK GCSE syllabus and how it would be examined in the blog below:  
<https://colleenyoung.wordpress.com/2015/05/24/gcse-new-content-iterative-methods-for-numerical-solution-of-equations/>
- AQA advice and resources about helping students understand iterative procedures:  
<https://allaboutmaths.aqa.org.uk/attachments/5309.pdf>
- You can find an explanation on how iteration work in the video below:  
<https://m.youtube.com/watch?v=du3cJ3MmR>

### Some problems you can try with your class ...

1. An approximate solution to an equation is found using this iterative process:

$$x_{n+1} = \frac{x_n^3 - 3}{8} \quad \text{and} \quad x_1 = -1$$

- (a) Work out the values of  $x_2$  and  $x_3$ .
  - (b) Work out the solution to 6 decimal places.
2. (a) Show that the equation  $x^3 + 4x = 1$  can be

$$\text{rearranged into } x = \frac{1}{4} - \frac{x^3}{4}.$$

- (b) Starting with  $x_0 = 0$ , use the iteration formula

$$x_{n+1} = \frac{1}{4} - \frac{x_n^3}{4} \text{ twice, to find an estimate of the solution}$$

of  $x^3 + 4x = 1$ .

3. Find correct to 3 d.p. a root of the equation

$$f(x) = x^3 - 2x + 3$$

given that there is a solution near  $x = -2$ .

<sup>1</sup> Smith, M. (1990) Iteration with spreadsheets. *Mathematics in School*, 19(1), 38-39.

<sup>2</sup> Hayes, T. (1990) The old box and ladder problem, revisited. *Mathematics in School*, 19(2), 42-43.