Absolute Value*

A glimpse into what researchers say:

- Students' difficulties with the absolute value[1] might be because:
  - For them, a number measures a quantity. So, they see *x* as positive and $-x$ as negative.
  - The notion of absolute value as a "number without sign" may obstruct their meaning making of the formal definition. They may say, for example, $|x-2|<3$ is $x-2<\pm 3$, instead of $-3<x-2<3$.
  - Their "tendency to keep the form by which the absolute value was introduced to them"[1]. For example, they might be influenced by $|x| = x$ for $x \geq 0$ when they say $|x+2| = x+2$ for $x \geq 0$.
  - They believe that the absolute value is "just a symbol which must be taken away"[1]. For example, students may solve $|x+1| = -2x-5$ by transforming to $x+1 = -2x-5$ without verifying the solution.
- Visualisation with function graphs can help students understand the absolute value better but teachers sometimes prefer algebraic solutions which are more relevant to exams[2,3].
- Also, teachers may 'trust' difficult algebraic solutions more than quicker and more 'elegant' solutions. For example, they may suggest a long algebraic solution for the equation $|x|+|x-1|=0$ instead of seeing that there is no x that makes $|x|+|x-1|$ zero[3].

Practitioners suggest that the use of online applets help with visualization and exploration[5]:
https://www.geogebra.org/m/rTcFXNxb
https://www.geogebra.org/m/GsnbMMhx?mobile
https://www.geogebra.org/m/ukMRcSU4?mobile

Choose the definition that fits to you:

1. *Arithmetical definition[4]:*
   $|x| = x$, if $x>0$; $|x| = -x$, if $x<0$; and, $|0| = 0$
2. *Piece-wise function definition[4]:*
   $$|(\ )| = \begin{cases} (\ ) & \text{if } (\ ) \geq 0 \\ -(\ ) & \text{if } (\ ) < 0 \end{cases}$$
3. *Distance definition[1]:*
   $|x|$ is the distance of number *x* from zero on the number line.
4. *Maximum function definition[4]:*
   $$|x| = \max\{x, -x\}$$
5. Compound function definition[4]:
   $$|x| = +\sqrt{x^2}$$

Some problems you can try with your class …

1. Solve the equation: $|x+2| = 4$
2. Solve the equation: $|x+1|+|x-2| = 0$
3. Solve the equation: $|x+1| = |x-2|$
4. Find *x*, such as $|x-2|<6$

[1] Gagatsis, A., & Panaoura, A. (2014). A multidimensional approach to explore the understanding of the notion of absolute value. *International Journal of Mathematical Education in Science and Technology*, *45*(2), 159-173.
[2] Konyalioğlu, A.C., Aksu, Z., & Şenel, E.Ö. (2012). The preference of visualization in teaching and learning absolute value. *International Journal of Mathematical Education in Science and Technology*, *43*(5), 613-626.
[3] Biza, I., Zachariades, T., & Nardi, E. (2007). Using tasks to explore teacher knowledge in situation-specific contexts. *Journal of Mathematics Teacher Education*, *10*(4-6), 301-309.
[4] Wilhelmi, M.R., Godino, J.D. & Lacasta, E. (2007). Didactic effectiveness of mathematical definitions: The case of the absolute value. *International Electronic Journal of Mathematics Education, 2*(2).
[5] Stupel, M. & Ben-Chaim, D. (2014) Absolute value equations– what can we learn from their graphical representation? *International Journal of Mathematical Education in Science and Technology*, *45*(6), 923-928.

*** Resources selected and summarised by Rebecca Potiphar and Georgios Thoma. Let us know whether they are useful and how we can improve them at @mathtask or email Irene Biza at i.biza@uea.ac.uk**