

Tangent-N Task

In a class of Year 12 students specializing in mathematics, the teacher gave to the students the following problem:

“Examine whether the line with equation $y = 2$ is tangent to the graph of function f , where

$$f(x) = 3x^3 + 2.”$$

A student responded as follows:

“I will find the common points between the line and the graph solving the system:

$$\begin{cases} y = 3x^3 + 2 \\ y = 2 \end{cases} \Leftrightarrow \begin{cases} 3x^3 + 2 = 2 \\ y = 2 \end{cases} \Leftrightarrow \begin{cases} 3x^3 = 0 \\ y = 2 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y = 2 \end{cases}$$

The common point is $A(0, 2)$.

The line is tangent of the graph at the point A because they have only one common point (which is A).”

The following dialogue then took place in the classroom:

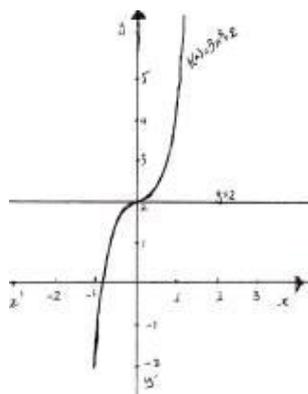
TEACHER: The parabola $y = x^2$ and the line $x = 0$ have only one common point, the point $(0, 0)$. Is the line $x = 0$ tangent of the parabola at this point?

The student sketches the parabola and the line on the board and answers:

STUDENT: No, it isn't, because the line cuts the parabola at this point.

TEACHER: OK. In our case [the teacher shows the problem in question] what is happening?

The student sketches the following graph and answers:



STUDENT: As we see from the graph, the line $y = 2$ cuts the curve $y = 3x^3 + 2$ at the point $(0, 2)$. So, the line is not a tangent of this curve.

TEACHER: This is correct but you also need to justify it algebraically. Even if a graphical understanding of functions is particularly useful, you should not forget that it is not always possible to use graphical representations and that you should learn to solve problems also algebraically.

Questions

1. How do you evaluate the teacher's management from
 - a) a mathematical perspective?
 - b) a didactical perspective?
 - c) an epistemological perspective, especially regarding the teacher's beliefs about the role of visualization in mathematics?
2. If you were the teacher, how would you manage the situation following the student's answer to the problem?

Justify your responses.

Publications with reference to the *Tangent Task*

Biza, I., Nardi, E., & Zachariades, T. (in press). Competences of mathematics teachers in diagnosing teaching situations and offering feedback to students: Specificity, consistency and reification of pedagogical and mathematical discourses. In K. Philipps, T. Leuders & J. Leuders (Eds.), *Diagnosing Competence of Mathematics Teachers*. Springer.

Zachariades, T., Nardi, E., & Biza, I. (2013). Using multi-stage tasks in mathematics education: Raising awareness, revealing intended practice. In A.M. Lindmeier & A. Heinze (Eds.). *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education (PME)* (Vol. 4, pp. 417-424). Kiel, Germany: PME.

Acknowledgements

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Let us know whether this task is useful at @mathtask or email Irene Biza at i.biza@uea.ac.uk. For more tasks, visit [MathTASK](#).