@mathtask

Transforming Aspirations of Mathematics Teachers into Strategies in Context

Introduction

MathTASK is a collaborative research and development programme which explores mathematics teachers' pedagogical and mathematical discourses and the transformation of teacher aspirations into pedagogical practice. Research acknowledges the overt discrepancy between theoretically and out-of context expressed teacher beliefs about mathematics and pedagogy and actual practice (e.g. Speer, 2005) and a substantial body of work in mathematics education explores the use of specific teaching cases (e.g. Kersting, 2008, Markovits and Smith, 2008) in teacher education. Our research sets out from the assumption that teachers' mathematical and pedagogical discourses are better explored and developed in situation-specific contexts. To this aim, we design *situation-specific tasks (*thereafter *Tasks)* – presented in the format of a brief mathematics classroom scenario followed by a set of questions arising from said scenario – and use them for research and teaching purposes. These classroom situations have the following features: they are hypothetical but grounded on learning and teaching issues that previous research and experience have highlighted as seminal; they are likely to occur in actual practice; they have purpose and utility; and, they can be used both in (pre- and in-service) teacher education and research as they provide insight – and an opportunity to influence – teachers' perspectives and intended practices.

So far, mathematics educators from the UK, Greece and Brazil have been involved in MathTASK. The research we conduct is divided in four strands, of which the first dates back to 2005: (1) ***mathematical thinking*** focuses on pedagogical and didactical practices in relation to the teaching and learning of specific mathematical topics; (2) ***classroom management*** explores the interferences of classroom management challenges with the learning of mathematics; (3) *CAPTeaM*, a British Academy-funded strand, focuses on ***inclusion*** in the mathematics classroom and aims to **C**hallenge **A**bleist **P**erspectives on the **Tea**ching of **M**athematics, especially in relation to disabled learners; and (4) the role of ***digital technology and other resources*** in the teaching and learning of mathematics. As the programme develops, new strands emerge from the experiences and ideas of participating teachers.

The format of the *Tasks* varies across the programme – for example, a piece of prose that includes a dialogue between one or more students and / or with a teacher; written script with or without excerpts of students' written or online work; a video clip; etc. – reflecting the diversity of issues and situations that teachers may encounter in the mathematics classroom.

In the following sections, we describe each strand briefly.

Mathematical thinking

In the *Tasks* of this strand (Biza, Nardi & Zachariades, 2007, 2009, 2014, 2018; Nardi, Biza & Zachariades, 2012; Zachariades, Nardi & Biza, 2013) we invite teachers to:

- solve a mathematical problem;
- examine a (fictional, yet research- and practice-informed) solution proposed by a student (or students) in class and, in some versions, a (fictional, yet research- and practice-informed) teacher response to the student(s); and,
- describe the approach they themselves would adopt in this classroom situation.

When inviting responses to these *Tasks*, we aim to explore teachers' subject matter knowledge and their gravitation towards certain types of pedagogy and didactical practices (Biza et al., 2007).

So far, teacher responses to these *Tasks*, joined with post-*Task* individual semi-structured interviews, have allowed us insight into a range of teacher knowledge and beliefs (epistemological and pedagogical). For example, in (Biza *et al.,* 2009) we discuss the multiple didactical contracts on the role of visualisation in mathematics and mathematical learning that teachers are likely to offer their students under those influences (asking, for example, whether a graph-based argument is an acceptable argument in the mathematics classroom).

Additionally, teachers' responses to these *Tasks* and interviews with them have revealed a complex set of considerations that teachers take into account when they determine their actions (Nardi *et al.,* 2012) – what Herbst and colleagues (e.g. Herbst and Chazan 2003) describe as the *practical rationality of teaching.* Our analyses demonstrate how teacher arguments, not analysed for their mathematical accuracy only, can be reconsidered, arguably more productively, in the light of other teacher considerations and priorities – *pedagogical, curricular, professional* and *personal* – which influence the decisions teachers make in the classroom.

In the version of the *Tasks* which, apart from the student (fictional) response(s), a fictional response from a teacher has been added (Biza *et al.,* 2018; Zachariades *et al.,* 2013), we aim to explore, not only whether the teacher can diagnose mathematical errors or cope with the many and varied twists and turns that a mathematical situation may take in the classroom, and what their own pedagogical intentions are in response to this situation, but, also, how they evaluate the reaction of another teacher, the (fictional) teacher featured in the scenario. Analysis of written responses to these *Tasks* have revealed a great variation in teachers' competencies in diagnosing and addressing of teaching issues. We have described this variation in terms of a typology of four interrelated characteristics: *consistency* between stated principles and intended practice, *specificity* of the response to the given classroom situation, *reification of pedagogical discourses*, and *reification of mathematical discourses* (Biza *et al.,* 2018).

Classroom management

The motivation for creating and deploying *Tasks* in this strand of MathTASK came directly from teachers and teacher educators who had responded to Mathematical Thinking *Tasks* and who stressed repeatedly that classroom management often interferes with working towards commendable learning goals in mathematics (e.g. Biza, Joel & Nardi, 2015; Biza, Nardi & Joel, 2015).

The *Tasks* we design for this strand feature classroom situations in which this type of interference proliferates, often in the shape of conflict between students or between students and teacher. For example, in one of these *Tasks* a class is asked to solve the problem: "When p=2.8 and c=1.2, calculate the expression: $3c^2+5p-3c(c-2)-4p$". Two students reach the result (10) in different ways: Student A substitutes the values for p and c and carries out the calculation; Student B simplifies the expression first and then substitutes the values for p and c. When Student A acknowledges her difficulty with simplifying expressions, Student B retorts offensively ("you are thick") and dismissively ("what can I expect from you anyway?"). Both solutions are correct, Student B's approach particularly demonstrates proficiency in important algebraic skills but Student B's behaviour is questionable. We typically ask teachers engaged with this *Task* to write about, and then discuss, how they would handle this classroom situation. Responses across educational contexts vary but, for example, a use of this *Task* with 21 prospective mathematics teachers in the UK (Biza, Nardi & Joel, 2015) revealed a series of commendable social and sociomathematical norms (Cobb and Yackel, 1996) that these teachers aspire to establish in their classroom, such as peer respect, value of discussion and investigative mathematical learning. However, analysis of the responses often revealed that the opportunity to engage students with metamathematical discussions around the value and efficiency of various methods of solving a mathematical problem was often missed, as respondents focused primarily on behavioural issues or endorsed dichotomous – e.g. instrumental vs relational understanding (Skemp, 1976) – and simplistic views of mathematical learning.

Inclusion

Since 2014, one strand of MathTASK has been dedicated to inclusive education and teacher perspectives on the mathematical learning of students with disabilities (in our studies so far deaf, blind or with Down syndrome). The project in this strand is called *CAPTeaM* (Challenging Ableist Perspectives on the Teaching of Mathematics) and is funded by the British Academy.

According to the ableist world-view, the able-bodied are the norm in society and disability is an unfortunate failing, a disadvantage that must be overcome. Within education, ableism results in institutional and personal prejudice against learners with disabilities, and has a drastic effect on approaches to teaching (Nardi, Healy & Biza, 2015; Nardi, Healy, Biza & Fernandes, 2016, 2018). Our project investigates how ableist perspectives impact on the teaching of mathematics, a discipline where public perceptions of ability as innate often shape pedagogical perspectives and practice.

This strand brings together the inclusive mathematics education expertise of a team of researchers based in Brazil (Rumo à Educação Matemática Inclusiva, REMI) with the MathTASK approach to task design (Biza *et al.,* 2007). Together, we develop and trial *Tasks* that invite teachers to reflect upon the challenges of mathematics teaching in inclusive classrooms. The *Tasks* in this strand are of two types.

In Type I *Tasks*, a learning incident which typically involves the mathematical contribution of a disabled learner, is inserted (often this is a video clip from the REMI archives) into a brief narrative about a mathematics classroom situation. We then invite participants (prospective or in-service mathematics teachers) to assume the role of the class teacher and evaluate the interactions of the disabled students present in the incident – first individually and in written responses to a set of questions, and then in a group discussion (which we also typically video-record).

In Type II *Tasks*, which aim to provoke reflections about how access to mediational means differently shapes mathematical activity, participants work in groups of three. Two members of the group are asked to solve a mathematical problem whilst, temporarily and artificially, deprived of one sensory or communication channel each. The third member of the group observes and records the actions of the other two. Discussion of the groups' experiences, setting out from the observers' highlights, follows (Nardi *et al.,* 2016; 2018).

For example, in one of the Type I *Tasks* (Nardi *et al.,* 2016) the class in the scenario is working on how they would describe a square-based pyramid. André, who is blind, and has been working with 3D solids, offers a description (seen in a video clip) shaped around the idea of a square-based pyramid being built out of squares gradually shrinking to a point at the top. André's approach differs substantially from the "counting faces, edges and vertices" approach found in many textbooks and the teacher is faced with the conundrum of what to do next. We typically ask prospective and in-service teachers engaged with this *Task* to write about, and then discuss, how they would handle this classroom situation. Responses across educational contexts vary but, for example, a use of this *Task* with 81 participants in the UK and in Brazil (ibid.) revealed that, while these teachers overall see mathematical value in André's proposition for a dynamic description of a square-based pyramid, they also tend to prioritise switching his perspective towards the textbook description of a square-based pyramid as a composition of fixed shapes (four triangles and a square).

In one of the Type II *Tasks,* trios of participants were invited to assume the following roles: one participant, who could not speak (the teacher), had to communicate a mathematical problem to a participant who could not see (the student). Then both had to work on a solution of the problem. The third participant observed their work. Analysis of the data from trials of this *Task* in the UK and Brazil (Nardi *et al.,* 2018) revealed at least two discursive shifts that take place during the teachers' engagement with these *Tasks*, labelled in our analysis as *attunement* and *resignification*. Initial feelings of helplessness and frustration evolve into excitement, pleasure and surprise at the success of solving the mathematical problem – which, in most occasions, participants do solve. Engaging teachers with this type of *Task* makes the tracing of these shifts possible. Our analyses also reveal that, in the process of establishing new ways to communicate mathematically – a necessity emerging from the temporary deprivation of access to hearing or seeing by some members of the group – participants redefine roles traditionally associated with those of the teacher and the student, and find a new balance in which every group member's needs are treated as equally important.

Digital technology and other resources

In this fourth strand of MathTASK, we focus on resources and digital technology in the teaching and learning of mathematics. Specifically, we design *Tasks* that invite teachers to reflect and discuss classroom situations in which teachers and students use educational software (e.g. Dynamic Geometry, Computer Algebra Systems, etc.) and online or other resources (e.g. search engines, e-books, textbooks, paper and pencil, etc.) to introduce a mathematical topic or to solve a problem. These *Tasks* discuss a range of ways to approach mathematics – visually, symbolically or in words – and potential connections between these different ways. They also aim to address issues such as affordances and limitations of digital technologies and conflicts between different forms of communicating mathematically. The design of these *Tasks* draws on research literature and teaching practice (e.g. Giraldo, Gaetano & Mattos, 2013). Recently, *Tasks* are also inspired by classroom observations conducted towards the ongoing doctoral research of MathTASK team member Lina Kayali (Kayali & Biza, 2017, 2018).

Emerging Strands

Another direction of MathTASK focuses on teachers' creation of their own tasks and is in resonance with works such as Zazkis, Sinclair and Liljedahl's (2013) in which teachers create what these authors call "lesson plays". A first attempt at working on this direction took place in 2014, when we invited prospective teachers to write up brief teaching/learning incidents from their experiences of school placements during the first months of training. We collected 12 incidents, we grouped them thematically and we invited trainees to discuss these in groups, produce posters of the key points of the discussion and then share these points with the whole group. The trainees raised issues closely associated with the teaching and learning of mathematics – such as student misconceptions, instrumental and relational understanding in the mathematics classroom – as well issues often seen as more generic – such as classroom management, student engagement and prospective teachers' relationships with more experienced teachers. Discussions were audio-recorded and transcribed. Later, we also discussed these classroom incidents with practising teachers. We see writing these incidents as opportunities for teachers' reflection on their practice. Furthermore, we see the benefits of the collaboration of researchers and teachers in the analysis of these incidents in both research and professional development.

Inspired by the often urgently expressed focus in several trainee narratives on classroom management – and supported by the 2015-16 Ian Hunter Prize at UEA – we created a team consisting of secondary mathematics teachers (newly qualified and experienced), researchers in mathematics education (faculty and doctoral students) and mathematics teacher educators to explore how to achieve balance between creating opportunities for high quality mathematical thinking and attending to classroom management and behaviour issues.

The team designed and deployed *Tasks* that offered opportunities to explore what propels and what impedes achieving this balance. As an instrument in these discussions, we deployed Terry Haydn's (2012) ten-point scale on the working atmosphere in the classroom, a construct that was not devised specifically for the mathematics classroom but is being used widely in teacher education programmes. Outcomes of this work (Biza *et al.,* 2016) include professional development sessions for mathematics teachers in which teachers are invited to reflect on their classroom experiences and give examples of classroom situations by using the language of the Haydn scale.

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