MTH-3D16 : Theory of Finite Groups

1. Introduction: This third year course is a thorough introduction to Finite Groups with 2nd year **Algebra** as a prerequisite. Group theory is a large topic which interconnects with many branches of pure and applied mathematics. The unit is a good accompaniment to other pure mathematics units such as **Representation Theory**, **Ring Theory** and **Algebraic Number Theory**. (It is good advice where practicable to do these units concurrently or after the group theory course.)

2. Hours, Credits and Assessment: The course is a 20 UCU unit of 33 lectures and some additional hours for discussion in class. Assessment is by course work (20%) through assessed homework and examination (80%).

3. Overview: Group Theory has two main roots, one in geometry where groups of geometrical transformations were studied, the other in algebra and the theory of equations where groups of substitutions of variables (i.e. permutations) in polynomial functions were analysed. The revolutionary work of Galois (1820's) about the solvability of polynomial equations, for instance, made it necessary to study such groups of substitutions. Finite group theory evolved to a great extent from this second root. Abstract groups began to emerge with Jordan's seminal *Traité des substitutions et des equations algébriques* (1870) while the definition of abstract groups in general appears to be due to Weber (1882).

The course starts with a review of elementary facts such as the correspondence and isomorphism theorems, composition series and composition factors. The theorem of Jordan and Hölder will be proved which shows that a finite group is constructed in some fashion from simple groups.

The idea that a group acts on a set is fundamental. Therefore *group actions* will be studied in great detail. For finite groups the orbit stabilizer theorem, a relatively easy result on group actions, plays a central role and many theorems appear as a consequence of it. One instance is the theorem which determines the number of orbits of a permutation group which is used for pattern counting more generally. Other applications include the Class Equation which leads to an elementary introduction to groups whose order is a power of a prime. Group actions are also fundamental for the study of the Platonic bodies.

Sylow's Theorem is an early high point of finite group theory. It says that a finite group of order x. p^n with p a prime not dividing x has a subgroup of order p^n. It also says that any two such subgroups are conjugate and that their number is congruent to 1 modulo p. Several proofs of this theorem will be given, various strengthenings will be proved and a variety of numerical non-simplicity will be derived.

Commutative groups which are generated by a finite number of elements can be analysed completely. This theorem has many applications outside group theory, including in matrix theory, number theory and topology.

4. Recommended literature and references: While there is no single text book for the course there are a great number of books on group theory. The following list begins with some more basic texts which are useful to look up elementary facts about groups:

(1) Herstein: Topics in Algebra. (Useful for the beginner, does not cover all you need.)

(2) PM Cohn: Algebra. (Useful for the beginner, does not cover all you need.)

(3) J Rotman: Introduction to Group Theory, Springer Verlag. (Contains all of what you need and much more.)

(4) J Rose: A Course on Group Theory, CUP. (Contains a lot of exercises and examples.)

(5) M Hall: The Theory of Groups, Macmillan. (A thorough treatment, not in print anymore.)

(6) Burnside: Group Theory. (A classic text, not always easy to read.)

Not all of these books are still in print but all should be in the library.

5. Contents:

1. Factor groups, the isomorphism theorems and the correspondence theorems. Composition series, composition factors and the Jordan-Hölder theorem. Solvable and simple groups. **(9 lectures)**

2. Permutation groups, the sign function and definition of the alternating group. Permutation groups, group actions and applications: regular action, action on cosets, conjugation. Simplicity of alternating groups. Orbit stabilizer theorem, orbit counting theorem. Examples: Symmetry groups of combinatorial structures such as Petersen graph. The rotation groups of the cube and dodecahedron. The Class equation for a finite group. **(12 lectures)**

3. The proof of the three Sylow theorems with extensions and applications. Counting Sylow subgroups with applications to non-existence of simple groups of certain orders. Simple groups of small order. **(8 lectures)**

4. Introduction to p-groups. The centre intersect any normal subgroup non-trivially, normalizers of subgroups, the Frattini argument. Nilpotency and the Frattini subgroup. Generators for a p-group. Selected topics on linear groups **(4 lectures)**

MTH-3D23 : Mathematical Logic

1. Introduction: The course in concerned with foundational issues of modern pure mathematics. It is a rigorous introduction to first-order logic. Proofs will be given for most of the results discussed. Some degree of mathematical sophistication is called for and familiarity with (and a taste for) mathematical proofs, such as would be seen in a rigorous first-year analysis or algebra course, will be assumed. The prerequisite is 2nd year **Groups and Rings**.

2. Timetable Hours, Credits, Assessments: 33 one hour lectures; 20 UCU. Assessment: Coursework 20% via assessed homework; 3 hour examination 80%. There will be 4 problem sheets which will make up the coursework component of the unit. Sketch solutions will be distributed and consulting hours arranged.

3. Overview: Mathematical logic analyses symbolically the way in which we reason formally, particularly about mathematical structures. The subject was developed the 20th century with Alfred Tarski and Kurt Gödel as its major figures. Although it is highly abstract, its ideas are fundamental in theoretical computer science and artificial intelligence and its results have applications in other parts of Mathematics (via an area known as Model Theory).

The first level of the subject is the propositional calculus. We look at the way simple statements (propositions) can be built into more complicated ones using connectives (`or,' `and,' `not,' `implies') and make precise how the truth or falsity of the component statements influences the truth or falsity of the compound statement. This is done using truth tables and can be useful for testing the validity of various forms of reasoning. It provides a way of analysing deductions of the form `if the following statements are true: ..... ; then so is .... .' We then move on to a completely symbolic process of deduction and describe the formal deduction system for propositional calculus. The statements we consider (propositional formulas) are regarded as strings of symbols and we give rules for deducing a new formula from a given collection of formulas. We want these deduction rules to have the property that anything that could be deduced using truth tables (so by considering truth or falsity of the various statements), can be deduced in this formal way, and vice versa. This is the soundness and completeness of our formal system.

The next level of the subject is the predicate calculus. This is what is needed to analyse `real' mathematics and the extra ingredient is the use of quantifiers (`for all' and `there exists'). We introduce the notion of a first-order structure, which is general enough to include many of the algebraic objects you come across in mathematics (groups, rings, vector spaces). We then have to be precise about the expressions (formulas) which make statements about these structures, and give a precise definition of what it means for a particular formula to be true in a structure. This is quite intricate, and the clever part is in getting the definitions right (- this is due to Tarski), but it corresponds to ordinary mathematical usage. Once this is done, we set up a formal deduction system for predicate calculus. This parallels what we did for propositional calculus, but is much harder. Nevertheless, the end result is the same: the formulas which are produced by our formal deduction system (the `theorems') are precisely the formulas which are true in all first-order structures. This is Gödel's Completeness Theorem.

The final section of the unit is concerned with model theory: the study and classification of mathematical structures in terms of what can be said about them in 1st order languages.

4. Recommended Reading: The following can be found in the UEA library. An asterisk (*) indicates that a copy of the book has been placed in the Restricted Loan collection and can only be taken out for short periods of time. There may be another copy available and other suitable books.

Check the Library Catalogue.

1.* A G Hamilton "Logic for Mathematicians", (Cambridge University Press, 1988)

2. Elliott Mendelson "Introduction to Mathematical Logic" (2nd edition), (van Nostrand, 1979)

3. Wilfrid Hodges "A Shorter Model Theory", (Cambridge University Press)

4. Rene Cori and Daniel Lascar "Mathematical Logic" (Cambridge University Press)

5. Lecture Contents:

Propositional calculus: Truth tables; propositional formulas; adequacy of sets of connectives; disjunctive normal form. **(4 lectures)**

The formal system L for propositional calculus. Proofs and deduction in L. Soundness; the Deduction Theorem. Proof of the Completeness Theorem (Adequacy) for propositional calculus. **(5 lectures)**

Predicate calculus: First-order structures. Construction of first-order languages and formulas. Interpretations. Satisfaction and truth of formulas. **(6 lectures)**

The formal system for predicate calculus. Soundness; the Deduction Theorem. Gödel's completeness theorem for countable first-order languages (Henkin's proof). The compactness theorem. Normal models and applications of the compactness theorem. **(9 lectures)**

Set theory: Countability and a brief discussion of cardinality and the Axiom of Choice. The General version of the completeness theorem. **(4 lectures)**

Model theory: Substructures and embeddings. elementary equivalence. The Löwenheim-Skolem theorems, categoricity and the Los-Vaught test. Applications: dense linear orders, equivalence relations. **(5 lectures)**

MTH-3D29: Representation Theory

Introduction: The course leads to the forefronts of one of the big achievements in 20th century mathematics: **Representation Theory** develops powerful applications of group theory via linear representations, which are fundamental in many parts of mathematics. **Linear Algebra II** and **Groups and Rings** are prerequisites, while **Group Theory** is a good accompaniment.

Hours, Credits and Assessment: A 20 UCU course of 33 lectures, supported by four or five seminars/office hours. The assessment will be from coursework (20%) and one three-hour examination (80%).

Overview: Group theory has many applications in science, especially in physics and crystallography. Most of these applications are realised via representation theory. In group theory itself, the famous classification of finite simple groups relies extremely heavily on representation theory. This course provides an introduction to the main ideas and notions of this powerful theory, explains some the machinery, and formulates its basic results.

In the first and second year units on group theory we have seen that abstract groups are quite complicated objects. One of the most fruitful approaches to studying these objects is to embed them into groups of matrices (to "represent" the elements of an abstract group by matrices). The advantage of this approach lies in the fact that matrices are concrete objects, and explicit calculations can easily be performed. Even more importantly, the powerful methods of linear algebra can be applied to matrices.

Representation Theory demonstrates the enormous power of linear algebra and builds upon what students have learned in **Linear Algebra I** and **II** and in **Groups and Rings**. Essentially, this topic is a symbiosis of group theory and linear algebra. This feature is important for understanding mathematics as a whole rather than as a union of disjoint theories.

The course is devoted to representations of finite groups by matrices with entries in the field of complex numbers.

Recommended literature and references: All of the following should be in the Library. There are many other books in the library that cover the material of the course.

G James & M Liebeck, Representation theory of finite groups, Cambridge University Press (A comprehensive yet very readable book covering most aspects of the course.)

J P Serre, Linear Representations of Finite Groups. (A classic introduction to representation theory.)

W Ledermann, The theory of group characters (A well written book for beginners.)

C Curtis & I Reiner, Methods of Representation theory, Wiley, 1986. (A comprehensive standard work on the subject.)

Lecture Contents:

Informal introduction to matrix representations. Vector space background. Inner product spaces. Review of group theoretical background. **(6 lectures)**

Definition and basic properties of complex representations of a finite group. Maschke's Theorem, characters, and character tables. **(6 lectures)**

The special cases of: cyclic groups, abelian groups, 1-dimensional representations. **(6 lectures)**

Schur's Lemma, orthogonality of characters, the number of irreducibles, the character degree divides the order of the group. **(6 lectures)**

Induced representations, Frobenius Reciprocity, double coset formula, methods of calculation. **(6 lectures)**

More examples and/or revision. **(3 lectures)**

MTH-3D38 : Advanced Mathematical Techniques

1. Introduction: This second semester course at level 3 follows on from the techniques stream courses of level 2 **Complex Analysis** and **Differential Equations**. It is designed to be equally suitable for students taking pure or applied units in their final year.

2. Timetable Hours, Credits, Assessments: The unit is of 20 UCU and is taught in the Spring Semester by means of 33 hours of lectures, three a week. A laboratory class using the computer software package MAPLE will also be timetabled. Support teaching is timetabled in the form of informal problem classes. Office hours are advertised and students are encouraged to seek help from the lecturer when required. Regular question sheets will be handed out, but assessment is only based on a three-hour examination (100%).

3. Overview:
This unit provides a selection of techniques applicable to mathematical problems in a wide range of applications.

Calculus of Variations includes techniques for maximising integrals subject to constraints. A typical problem addressed is the curve described by a heavy chain hanging under the effect of gravity.

Techniques are available for the solution of algebraic equations, differential equations and evaluation of integrals involving large or small parameters, a topic known as asymptotic analysis. This provides approximate solutions when
exact solutions can not be found and when numerical solutions are difficult.

Integral transforms are useful for solving a range of problems including
the solution of integro-differential equations.

This unit will include illustration of concepts using numerical investigation with MAPLE.

4. Recommended Reading: No one book covers all the course. References are provided at the end of each section. Books which prove useful to several parts of the course include Arfken and Hinch.

Arfken	'Mathematical Methods for Physicists'
Lighthill	'An introduction to Fourier analysis & generalised functions'
Kreyszig	'Advanced Engineering Mathematics'
Hinch	'Perturbation Methods'

5. Lecture Contents:

The unit will include a selection of topics from those listed below:

Calculus of Variations
Variations subject to constraints. Eulers equation and applications. Lagrangian multipliers.

Sturm-Liouville Theory
Adjoint operators and orthogonality of eigenfunctions. Orthogonal polynomials. Examples including Legendre & Chebyshev polynomials.

Green's Functions
Use of Green's functions to solve ordinary differential equations, with applications to partial differential equations.

Integral transforms
Fourier and Laplace transforms including inversion theorems and convolution. Evaluation of integrals using residue theorem. Examples of applications to differential and integro-differential equations.

Asymptotic series
Definitions and notation. Asymptotic solution of algebraic equations.

Matched Asymptotic Expansions
Solution of singular differential equations.

Method of Multiple Scales.
Oscillatory problems involving two or more different time scales.

Asymptotic Evaluation of Integrals
Contour deformation. Method of Steepest Descent.

MTH-3D41 : Fluid Dynamics

1. Introduction: This course is a natural successor to **Hydrodynamics** part of MTH-2C2Y. It is available to students with that background, or other appropriate background as provided, for example, for students in the School of Environmental Sciences, through MTH-2B72/2B82/ENV-2A22.

2. Timetable Hours, Credits, Assessments: This unit is of 20 UCU and is taught in the Autumn Semester by means of 33 lectures at a rate of three lectures per week. The Assessment is by set regular coursework (20%) and a three-hour examination (80%).

3. Overview: Modern fluid mechanics has its roots in the 19th Century. But the rapid developments over the last 150 years, with their impact on flight, ocean engineering and meteorology, demonstrate that fluid mechanics has contributed to the shaping and understanding of our world on a par with the other advances of mathematical physics. The purpose of this unit is to introduce you to the Navier-Stokes equations, which describe the motion of viscous fluids. These will be derived from fundamental principles and some remarkable, exact solutions will be discussed. Simplifications to the full equations will be studied in the context of slow, creeping flow encountered, for example, when a liquid drop moves slowly in an ambient fluid, or during the deposition of a chemical film in a microelectronic device. The use of asymptotic methods for high speed flows will lead naturally to the notion of a boundary layer and its importance in aerodynamic design. The course will conclude with an introduction to hydrodynamic stability.

4. Recommended Reading: The notes taken in lectures are intended to be complete and self-contained. Recommended books (in order of increasing technical detail) are:

D J Acheson	Elementary Fluid Dynamics, (Oxford UP)
G K Batchelor	An Introduction to Fluid Dynamics, (Cambridge UP)
C Pozrikidis	Introduction to Theoretical and Computational Fluid Dynamics (Oxford UP)

5. Lecture Contents:

Lectures 1-10:
Fundamental principles. Stress, strain, and vorticity. Derivation of the Navier-Stokes equations. Boundary conditions. **(10 lectures)**

Lectures 11-15:
Exact solutions to the Navier-Stokes equations. Oscillating flows and Stokes layers. Axisymmetric flow. **(5 lectures)**

Lectures 16-22:
Creeping (Stokes) flow. Uniqueness of Stokes flows. Minimum dissipation theorem. Thin film flow. **(7 lectures)**

Lectures 23-29:
Boundary layer theory. High Reynolds number flow. Blasius' solution. **(7 lectures)**

Lectures 30-33:
Hydrodynamic stability theory. Rayleigh's criterion. **(4 lectures)**

MTH-3D43 : Continuum Mechanics and Elasticity

1. Introduction: The prerequisite is **MTH-2C2Y Fluids and Solids**, replaced by **MTH-2B72/ENV 2A21 Mathematics for Geophysical Science II/Mathematics 2** for ENV students, which ensures a little prior knowledge of linear elasticity. Continuum mechanics underpins all theoretical work in both solid and fluid mechanics. It concerns the deformation and motion of a continuous body, the balance laws and the formulation of constitutive equations, which distinguish one sort of material from another. These general ideas are applied in particular to nonlinear elasticity.

2. Timetable Hours, Credits, Assessments: 33 lectures; 20 UCU; assessment is 20% coursework and 80% examination. Example sheets are handed out containing many examples to support and illustrate the lecture material. Some are set as coursework and detailed solutions to most exercises are handed out.

3. Overview: Continuum mechanics is the mechanics of continuous materials, i.e. materials which occupy every point of a continuous region of space. Real materials are not like this, they consist of atoms and molecules and are mostly empty space! Continuum mechanics approximates real materials by smearing out the atoms and molecules uniformly in space. The justification is that it works! Those theoretical and experimental phenomena of engineering and physics which take place over a length scale greater than the interatomic spacing in solids or the mean free path in gases are found to be well described by continuum mechanics.

It took a hundred years to get from Newton's laws for particles and rigid bodies to the equivalent integral balance law formulations of Euler and Cauchy for continuous materials, leading to the definition of a stress tensor and the partial differential equations governing the motion of a body. It is the precise nature of the dependence of the stress upon the motion of the body, termed the constitutive equation, that distinguishes one sort of material from another.

The resurgence of modern continuum mechanics, and of nonlinear elasticity in particular, began c. 60 years ago with Rivlin's discovery of nonlinear exact solutions which are valid for all incompressible isotropic elastic materials (e.g. rubber). In this course we examine the general foundations of continuum mechanics, setting up the basic equations for both fluid and solid mechanics. In the second half we consider various aspects of nonlinear isotropic elasticity, including Rivlin's work.

4. Recommended Reading:

G A Holzapfel	"Nonlinear Solid Mechanics" (Wiley)
A J M Spencer	"Continuum Mechanics" (Longman)
R W Ogden	"Non-Linear Elastic Deformations" (Dover)
P Chadwick	"Continuum Mechanics" (George Allen & Unwin)
R J Atkin & N Fox	"An Introduction to the Theory of Elasticity" (Longman)

5. Lecture contents:

CONTINUUM MECHANICS

Introduction: The nature and structure of continuum mechanics

1. **Kinematics of a continuum**
   Basic description of motion: two illustrations, velocity and acceleration. The referential and spatial descriptions: material time derivative, particle paths. Mass: referential equation of continuity. Some kinematical lemmas. Notes on tensors. The spatial equation of continuity: Reynold's transport formula. **(3 lectures)**

2. **Dynamics of a continuum**
   Momentum, angular momentum, force and torque: Euler's laws of motion. The theory of stress: the Euler-Cauchy stress principle. Cauchy's equations of motion: equilibrium of a body. Principal stresses and principal axes of stress. **(4 lectures)**

3. **Constitutive equations**
   Introduction: basic constitutive assumption. Examples of constitutive equations: inviscid fluids, viscous fluids, elastic materials. Principles governing the formulation of constitutive equations: (determinism, local action, objectivity). Observer transformations - superposed rigid-body motions: objective fields, objectivity of density and stress. The principle of objectivity applied to constitutive equations: inviscid fluids, viscous fluids, elastic materials. Material symmetry: symmetry of elastic materials. Reiner-Rivlin fluids. First and second Piola-Kirchhoff stresses. Energy balance. The strain energy function and constitutive forms. Incompressibility: inviscid fluids, viscous fluids, isotropic elastic solids.
   (6 lectures)

LINEAR ISOTROPIC ELASTICITY

4. **Linear isotropic elasticity**
   The infinitesimal strain tensor: the infinitesimal deformation approximation. The generalized Hooke's law, equations of motion and equilibrium. Homogeneous deformations: simple dilatation, simple extension, simple shear (bulk, Young's and shear moduli, Poisson's ratio). Elastic constants and physically reasonable response.
   (3 lectures)

NON-LINEAR ISOTROPIC ELASTICITY

Compressible isotropic elasticity:

5. **Constitutive equation**
   Equations of motion, equilibrium equations. Interpretation of left and right Cauchy-Green strain tensors. **(1 lecture)**

6. **Homogeneous deformations**
   Dilatation: pressure a monotonic decreasing function of stretch (example, foam rubber), incremental bulk modulus. Simple extension: axial tension a monotonic increasing function of axial stretch (example, Hadamard material), incremental Young's modulus and Poisson's ratio. Simple shear: generalized shear modulus is positive, Kelvin effect, universal relations, Poynting effect, stresses on the inclined faces, linear simple shear. **(5 lectures)**

Incompressible isotropic elasticity:

7. Constitutive equation **(1 lecture)**

8. Homogeneous deformations
Dilatation (impossible). Simple extension, incremental Young's modulus and Poisson's ratio. Stretching of a plane sheet. Simple shear. Examples of constitutive equations (neo-Hookean, Mooney-Rivlin). **(2 lectures)**

9. Non-homogeneous deformations
The five families. Family 1: bending and extension of a rectangular block. Family 2: straightening and extension of an annular wedge. Family 3: extension and torsion of a circular cylinder.

Appendix: strain, stress and equilibrium equations in orthogonal curvilinear coordinate systems. **(5 lectures)**

10. Bell-constrained isotropic elasticity

Bell's constraint. The reaction stress, constitutive equations. Generalized shear with normal stretch. Inequalities among the response functions. Uniaxial loading. The general universal relation. Shear without shear stress! **(3 lectures)**

MTH-3D51 : Dynamical Meteorology

1. Introduction: Introduction: This level 3 course covers modelling the large-scale circulation of the atmosphere. It requires prior completion of second level courses, either Hydrodynamics or Mathematics for Geophysical Science II or Mathematics for Scientists IV or the ENV course Mathematics II.

2. Timetable Hours, Credits, Assessments: This is a 20 UCU unit of 33 lectures supported by occasional problem classes. Assessment is by set homework (20%) and examination (80%).

3. Overview: The mathematical modelling of the atmosphere in this course provides a demonstration of how the techniques developed in second year units on fluid dynamics and differential equations can be used to explain some interesting phenomena in the real physical world. Dynamical meteorology is a core subject on which weather forecasting and the study of climate change and climate variability (such as El Nino) are based. This unit applies fluid dynamics to the study of the circulation of the Earth's atmosphere. The fluid dynamical equations on a rotating planet and some basic thermodynamics for the atmosphere are introduced. These are then applied to topics such as hydrostatic balance, geostrophic flow and weather systems, cyclostrophic flow and tornadoes, thermal wind and the jet streams, vorticity and potential vorticity, Rossby waves, boundary layers, gravity waves, the Hadley circulation, and equatorial waves. Emphasis will be placed on fluid dynamical concepts as well as on finding analytical solutions to the equations of motion.

4. Recommended Reading: The recommended reference books are:

J Holton "An introduction to Dynamic Meteorology" (Academic Press)

A Gill "Atmosphere-Ocean Dynamics", (Academic Press)

5. Lecture Contents:

Introduction: Observed atmospheric structure. Weather systems. Scales of motion. **(1 lecture)**

Atmospheric dynamics: Gravitational force. Pressure gradient force. Hydrostatic balance. Material derivative. Acceleration. Rotating frame of reference (centrifugal and Coriolis forces). Equation of motion. Geostrophic balance. Natural coordinates. Isobaric coordinates. Thermal wind. Mass conservation. **(13 lectures)**

Atmospheric thermodynamics: First law of thermodynamics. Adiabatic processes and potential temperature. Thermodynamical equation. Static stability. **(4 lectures)**

Vorticity and Rossby waves: Relative vorticity. Planetary vorticity. Absolute vorticity. Vorticity equation. Barotopic potential vorticity. Free Rossby waves. Forced topographic Rossby waves. **(7 lectures)**

Tropical meteorology and equatorial waves: Governing equations. Vertical structure equation. Shallow water equations. Equatorial beta-plane approximation. Equatorial Kelvin wave. Equatorial Rossby wave. Forced equatorial wave response to tropical heating. **(8 lectures)**

MTH-3D56: Fermat's Last Theorem

1. Introduction: This course provides an introduction to Algebraic Number Theory, motivated by Fermat's famous "Last Theorem". Second year **Algebra** is a prerequisite.

2. Hours, Credits and Assessment: A 20 UCU course of 33 lectures, supported by weekly office hours. The assessment will be from coursework (20%) and one three-hour examination (80%).

3. Overview: In the 17^{th} century, Pierre de Fermat famously wrote in the margin of his copy of Diophantus' "Arithmetic" that he could prove that the equation

$$x^n + y^n = z^n$$

has no non-zero integer solutions whenever $n > 2$ – but, alas, there was too little space there for his truly remarkable proof. This led to a 300 year search for a proof, starting with elementary means – for example, Femat's own method of infinite descent, which gives a proof for $n = 4$ – but getting increasingly sophisticated; this culminated in Wiles's proof of the general case in 1993/4, using the full force of three centuries' developments in Number Theory.

In this course, we will take a roughly historical approach to the search for a proof, but from a modern viewpoint. We will see how the property of unique factorization of integers (every integer can be factorized as a product of primes in an essentially unique way) is a crucial tool. We will also see how we are led to consider larger fields of numbers, but how the property of unique factorization is lost when we do this. It was through trying to recover this that Kummer was led to define his "ideal numbers" and thence to prove a great many cases of the theorem.

4. Recommended literature and references: Any textbook on Algebraic Number Theory would be useful – the classmark in the Library is QA247. Possibilities include:

Stewart, I., and Tall, D., *Algebraic number theory*, Chapman and Hall. [QA247 STE]

Swinnerton-Dyer. H., *A brief guide to algebraic number theory*, CUP. [QA247 SWI]

Alaca, S., and Williams, K., *Introductory algebraic number theory*, CUP. [QA247 ALA]

There is also a historical account of the development of Fermat's Last Theorem in:

Edwards, H., *Fermat's Last Theorem*, Springer.

though I find it rather difficult to read. As always, there are also very many useful web resources including lecture notes for Algebraic Number Theory courses.

5. Lecture Contents:

Overview, the Fundamental Theorem of Arithmetic, the method of infinite descent, Pythagorean triples, the case $n = 4$. **(4 lectures)**

Quadratic reciprocity, binary quadratic forms, the case n=3. Sophie Germain's Theorem. **(7 lectures)**

Quadratic fields, Pell's equation, Ideals and the recovery of unique factorization, class number. **(9 lectures)**

Cyclotomic extensions. Regular primes, Kummer's proof for regular primes. **(10 lectures)**

Revision. **(3 lectures)**

MTH-3D71 : History of Mathematics

1. Introduction: The course will concentrate on the conceptual development of Mathematics from earliest times and in different places to the present century. The influence of mathematical thinking on other disciplines will be constantly demonstrated.

2. Timetable hours, Credits, Assessments: A 20 UCU course of 33 lectures. The overall mark comes from coursework (20%) and one three hour examination (80%).

3. Overview: The course starts with exploration of the nature of Mathematics and how various early civilisations came to terms with it, culminating in the classical Greek period where the subject was recognised as fundamental in theory and practice. The reasons for the relative neglect of mathematical thinking in the so called Dark and Middle Ages is discussed with the revival of interest in Europe through issues of Astronomy, Navigation, Commerce and Art. The influence of Mathematics in the Scientific Revolution with focus on the work of Isaac Newton with huge advances through the development of the Calculus. The more modern approach to Mathematics is traced through the early nineteenth century with new ideas in Algebra and Geometry, the use of calculating devices culminating in the Computer and twentieth century directions and applications are finally considered.

4. Recommended Reading:

John Fauvel and Jeremy Gray, "The History of Mathematics-A Reader", Open University, 1987.

The University Library has an excellent collection in the History of Mathematics.

5. Lecture Contents:

Origins of Counting, Patterns and simple mathematical ideas. **(2 Lectures)**

Uses of Mathematics in the early civilizations of China, India, Egypt and Babylon. **(4 Lectures)**

Mathematics as pillar of Greek culture, philosophy and science. **(5 Lectures)**

Euclid, Archimedes and the nature of 'proof'. **(2 Lectures)**

Decline in Dark & Middle Ages and revival through Astronomy, Navigation, Art & Commerce. **(5 Lectures)**

The Scientific Revolution: the work of Isaac Newton. **(3 Lectures)**

Conceptual development of the Calculus. **(2 Lectures)**

New approach to Geometry, Algebra, Analysis in 19^{th} century. **(4 Lectures)**

Calculating devices and invention of the Computer. **(2 Lectures)**

Mathematical directions in 20^{th} century. **(4 Lectures)**

MTH-3D78 : Free Surface Flows

1. Introduction: This course provides an introduction to the theory of free surface flows, in particular of liquid jets, drops and liquid sheets. It requires some knowledge of hydrodynamics and multivariable calculus

2. Timetable Hours, Credits, Assessments: 20 UCU assessed by course work (20 percent, taken from problem sheets) and examination (80 percent). There will be 33 lectures.

3. Overview: Free surface problems occur in many aspects of science and everyday life. Examples of free surface problems are waves on a beach, bubbles rising in a glass of champagne and pouring flows from a container. In these examples the free surface is the surface of the sea, the interface between the gas and the champagne and the boundary of the pouring flow. The phenomena of the breakup of liquid jets and sheets into drops has many industrial applications. We will study first the dynamics and breakup of liquid jets using stability analysis and some numerical simulations. We will then study the liquid sheets and the waves on these liquid sheets.

4. Recommended literature and references:
S. Middleman, *Modeling Axisymmetric Flows: Dynamics of Films, Jets, and Drops.*
S.P.Lin, *Breakup of Liquid Sheets and Jets*.

In addition, material will be taken from recent journal articles and appropriate references will be given during the lectures.

5. Lecture Contents:

Introduction and overview.

Theory of liquid jets. Breakup of liquid jets.
Linear stability analysis (infinite jet).
Spatial analysis of the linear stability(semi-infinite jet).
Nonlinear instability. Drops, satellite droplets.
One-dimensional approximations.
Numerical simulations.

Liquid sheets.
Inviscid and viscous liquid sheets; temporal instability, convective/absolute instability.
Waves on liquid sheets.

Other free surface flows may be considered, as time permits.

(33 lectures)