MTH-3D38 : Advanced Mathematical Techniques

1. Introduction: This second semester course at level 3 follows on from the techniques stream courses of level 2 **Complex Analysis** and **Differential Equations**. It is designed to be equally suitable for students taking pure or applied units in their final year.

2. Timetable Hours, Credits, Assessments: The unit is of 20 UCU and is taught in the Spring Semester by means of 33 hours of lectures, three a week. A laboratory class using the computer software package MAPLE will also be timetabled. No formal support teaching is timetabled, but office hours are advertised and students are encouraged to seek help from the lecturer when required. Regular question sheets will be handed out, but assessment is only based on a three-hour examination (100%).

3. Overview: Calculus of Variations includes techniques for maximising integrals subject to constraints. A typical problem addressed is the curve described by a heavy chain hanging under the effect of gravity.

The Sturm-Liouville eigenvalue problem gives rise to the construction of sequences of mutually orthogonal functions. Functions can then be expressed as a sum of such orthogonal functions. In particular Legendre & Chebyshev polynomials will be studied.

Analysis of eigenfunction expansions leads naturally to Fourier series and hence to the concept of Fourier transforms. Indeed, the inversion of Fourier integrals can be investigated by analogy with convergence of Fourier series. Applications of Fourier and Laplace transforms include the solution of differential equations.

Techniques are available for the evaluation of integrals involving large or small parameters, a topic known as asymptotic analysis. Such techniques are particularly suitable for the inversion integrals arising from transform theory. In addition techniques for studying differential equations involving small parameters will be studied.

This unit will include illustration of concepts using numerical investigation with MAPLE.

4. Recommended Reading: No one book covers all the course. References are provided at the end of each section. Books which prove useful to several parts of the course include Arfken and Hinch.

Arfken 'Mathematical Methods for Physicists'

Lighthill 'An introduction to Fourier analysis & generalised functions'

Kreyszig 'Advanced Engineering Mathematics'

Hinch 'Perturbation Methods'

5. Lecture Contents:

Calculus of Variations
Variations subject to constraints. Eulers equation and applications. Lagrangian multipliers.
(4 lectures)

Sturm-Liouville Theory
Adjoint operators and orthogonality of eigenfunctions. Orthogonal polynomials. Examples including Legendre & Chebyshev polynomials. **(5 lectures)**

Fourier transforms
Including inversion theorems and convolution. Relevance of theory developed for Fourier series. Examples of applications to differential equations. **(5 lectures)**

Laplace transforms
Emphasis on inversions by contour deformation. Examples of application (including integral equations). Inversions involving branch cuts. **(5 lectures)**

Asymptotic series
Definitions and notation. Asymptotic solution of differential equations. Method of Multiple Scales. **(8 lectures)**

Asymptotic Evaluation of Integrals
Method of Stationary Phase. Contour deformation. Method of Steepest Descent. **(6 lectures)**

MTH-3D71 : History of Mathematics

1. Introduction: The course will concentrate on the conceptual development of Mathematics from earliest times and in different places to the present century. The influence of mathematical thinking on other disciplines will be constantly demonstrated.

2. Timetable hours, Credits, Assessments: A 20 UCU course of 33 lectures. The overall mark comes from coursework (20%) and one three hour examination (80%).

3. Overview: The course starts with exploration of the nature of Mathematics and how various early civilisations came to terms with it, culminating in the classical Greek period where the subject was recognised as fundamental in theory and practice. The reasons for the relative neglect of mathematical thinking in the so called Dark and Middle Ages is discussed with the revival of interest in Europe through issues of Astronomy, Navigation, Commerce and Art. The influence of Mathematics in the Scientific Revolution with focus on the work of Isaac Newton with huge advances through the development of the Calculus. The more modern approach to Mathematics is traced through the early nineteenth century with new ideas in Algebra and Geometry, the use of calculating devices culminating in the Computer and twentieth century directions and applications are finally considered.

4. Recommended Reading:

John Fauvel and Jeremy Gray, "The History of Mathematics-A Reader", Open University, 1987.

The University Library has an excellent collection in the History of Mathematics.

5. Lecture Contents:

Origins of Counting, Patterns and simple mathematical ideas. **(2 Lectures)**

Uses of Mathematics in the early civilizations of China, India, Egypt and Babylon. **(4 Lectures)**

Mathematics as pillar of Greek culture, philosophy and science. **(5 Lectures)**

Euclid, Archimedes and the nature of 'proof'. **(2 Lectures)**

Decline in Dark & Middle Ages and revival through Astronomy, Navigation, Art & Commerce. **(5 Lectures)**

The Scientific Revolution: the work of Isaac Newton. **(3 Lectures)**

Conceptual development of the Calculus. **(2 Lectures)**

New approach to Geometry, Algebra, Analysis in 19^{th} century. **(4 Lectures)**

Calculating devices and invention of the Computer. **(2 Lectures)**

Mathematical directions in 20^{th} century. **(4 Lectures)**

MTH-3E10 : Symbolic Dynamics

1. Introduction: This course will introduce students to the language and methods of symbolic dynamics and coding theory. This is a rapidly growing area of dynamical systems and has many applications within mathematics and in computer science. Second year algebra is useful but not essential. The course relates to graph theory, computing, linear algebra, and dynamical systems.

2. Timetable Hours, Credits, Assessments: A 20 UCU course of 33 lectures, supported by seminars to be arranged informally. The overall mark comes from coursework (20%) and one three-hour examination (80%).

3. Overview: The first part of the course will describe shift spaces, languages and sliding block codes. We will go on to study shifts that are of finite type and shifts that are sofic. These may be described via graphs and their associated matrices. The basic idea of state splitting will be described, and applications to data storage and coding outlined.

The second part of the course will concentrate on sofic shifts and presentations for them.

The last part of the course will study symbolic systems as dynamical systems. The entropy and zeta function invariants will be introduced and computed. If time permits, some recent work on the conjugacy problem will be described.

4. Recommended Reading:

1) "An introduction to Symbolic Dynamics and Coding",
Douglas Lind & Brian Marcus, Cambridge Univ. Press (1995).

2) Papers (these will be made available to participants):
R. Adler, "The torus and the disk", IBM J. of Research and Development 31 (1987), 224-234.
B. Marcus, "The impact of Roy Adler's work on symbolic dynamics and applications to data storage", Contemp. Math. 135 (1992), 33-56.
M. Morse & G. Hedlund, "Symbolic dynamics", Amer. J. Math. 60 (1938), 815-866.

5. Lecture Contents:

Shift spaces: full shifts, shift spaces, languages, higher block shifts, sliding block codes, convolutional encoders. **(6 lectures)**

Shifts of finite type: finite constraints, graphs and their shifts, graph presentations, state splitting, data storage. **(6 lectures)**

Sofic shifts: presentations, characterization, minimal right-resolving presentation. **(6 lectures)**

Entropy and growth: basic properties, Perron-Frobenius theory, computing entropy. **(6 lectures)**

Dynamical systems: shifts as model dynamical systems, periodic points and zeta functions. **(6 lectures)**

Additional topic: chosen from road colourings, sliding block decoders, conjugacy problems. **(3 lectures)**

MTH-3E18: Set theory

1. Introduction: This unit is concerned with foundational issues of mathematics and provides the appropriate mathematical framework in which to discuss 'sizes of infinity.' On the one hand we shall cover concepts such as ordinals, cardinals and the Zermelo - Fraenkel axioms with Choice. On the other, we shall see how these ideas come up in other areas of mathematics, such as graph theory and topology. Familiarity with and a taste for mathematical proofs such as would be seen in a rigorous first-year analysis or a second-year algebra unit will be assumed, because most of the theorems presented and the problems discussed will be accompanied by a proof. **Algebra I** is a desired prerequisite.

2. Timetable, Hours, Credits, Assessments: The unit is a 20 UCU unit of 33 lectures. Assessment is by exercises set throughout the unit (20%) and a written exam (80%).

3.Overview: Set theory has a dual role in mathematics. As well as providing the basic foundations and the language in which most of modern mathematics can be expressed, it also provides the means for studying infinite sets. Examples of such sets are the set of natural numbers, the set of integers and the set of real numbers. It turns out that there is a very natural way to assign the notion of size to such sets, providing us with more information than just 'infinite'. According to this notion (*cardinality)* the first two of the sets have the same size, which is smaller than that of the third. The foundational role of set theory is that it provides a reasonable set of assumptions (axioms) from which most mathematics can be expressed by proving theorems based on these axioms. We shall discuss these axioms, Zermelo-Fraenkel Axioms with the Axiom of Choice and demonstrate how they can be used to build the foundation of mathematics. We shall mention some alternatives and discuss some philosophical and foundational questions, including Gödel's Incompletness Theorem and the notion of consistency (these will be discussed without proofs). Finally, we shall see some concrete examples of infinite objects in mathematics, such as infinite graphs, topological spaces and groups and discuss some of their properties.

4. Recommended Reading: The unit is self-contained. The student is well advised to attend regularly, as the teaching will include a lot of discussion. Lecture notes will be produced as we go and will be clear and self-sufficient. For supplementary reading the following books are useful:

1. A. Hajnal and P. Hamburger, Set Theory, Cambridge, 1999.
2. P. Halmos, Naive Set Theory, Springer UTM.

Both books are in the library, as there are some others on similar topics.

5. Lecture contents:

Cardinality.	**(4 lectures)**
Operations with sets and cardinals	**(4 lectures)**
Orderings and well ordered sets, ordinals	**(4 lectures)**
Examples of infinite objects	**(2 lectures)**
The axioms of ZFC	**(5 lectures)**
Transfinite induction.	**(4 lectures)**

Some applications of the axiom of choice.	**(5 lectures)**
Foundational discussion	**(2 lectures)**
Continuum hypothesys	**(3 lectures)**

MTH-3E23 : Graph Theory

1. Introduction: This third year course is a thorough introduction on modern Graph Theory. This subject plays an important role in many branches of mathematics and the sciences generally. Graph Theory therefore has applications almost everywhere. There are no formal prerequisites from MTH Year 2 units; students from outside MTH with a good general mathematics background are welcome, please contact the lecturer.

2. Timetable Hours, Credits, Assessments: The course is a 20 UCU unit of 33 lectures and additional hours for group discussions on demand. Assessment is by Course Work (20%) and Examination (80%).

3. Overview: Graphs are among the most basic structures in mathematics. They consist of a set of points, the *vertices* of the graph, and a set of *edges* which link certain pairs of vertices. By their very simplicity it is therefore not surprising that graphs are important in many part of mathematics, computing and the sciences.

This course is designed as an introduction to the theory and applications of graphs. Thus we first develop the basic notions of connectivity and matchings. Graphs appear also in topology and these aspects will be investigated in a section on planarity. This is the question if a graph can be embedded in the plane in such a fashion that edges do not intersect. We shall aim to prove a famous theorem of Kuratowski which gives exact conditions for the planarity of a graph.

Of further interest will be graph colourings. These are assignments of colours to the edges of the graphs such that different edges of the same colour never meet at a vertex. There are also vertex colourings where vertices of the same colour may not be joined by an edge. Such questions lead to many beautiful results and interesting applications. One of the best known theorems in graph theory is the Four Colour Theorem. This result is not within our reach. However we shall prove a weakening of it, and this is Five Colour Theorem.

The course is based on Diestel's excellent book and will follow this text quite closely.

4. Recommended Reading: The literature on the subject is extensive. We will be using the first book as the main course text.

R Diestel	"Graph Theory", Graduate Texts in Mathematics, 173, 1997;
NL Biggs	"Discrete Mathematics", OUP, (a very useful introductory text)
JA Bondy, USR Murty	"Graph Theory with applications", MacMillan
NL Biggs	"Algebraic Graph Theory", CUP

5. Lecture Contents:

Basics: Definitions, subgraphs, trees, Euler circuits, cycle.	**(9 lectures)**
Matchings: The theorems of König and Hall.	**(5 lectures)**
Connectivity : k-connected graphs, Menger's theorem.	**(5 lectures)**
Planarity: Basic topological ideas, introduction to Kuratowski's theorem.	**(6 lectures)**
Colourings: vertex colourings, edge colourings, the 5-colour theorem.	**(8 lectures)**

MTH-3E25 : Analytic Number Theory

1. Introduction: A basic introduction to the way techniques form real and complex analysis can be used to understand distribution properties of subsets of integers (such as the primes for instance.) The subject is over one hundred years old, time enough to have collected some beautiful results. The pre-requisites are **Advanced Calculus II** and **Groups and Rings.**

2. Timetable Hours, Credits, Assessments: A 20 UCU unit of 33 lectures with seminars to be arranged informally. Assessment is by coursework (20%) and a 3 hour examination (80%). Students will be encouraged to use www as background.

3. Overview: About 200 years ago, Gauss, based on tables of prime numbers he constructed, conjectured that the number of primes up to x (a large real number) should be asymptotically $x/\log x$. It was 100 year before a proof was found, and this, using 'analytic' techniques that had been developed during the nineteenth century. The 'Prime Number Theorem', as it is called, was one of the early successes of analytic number theory. Since then, analytic techniques have been used to understand a whole range of problems about subsets of integers. It is an especially useful approach where we want to prove asymptotic results.

It is important to stress that although the subject uses analytic tools, it is not analysis per se. It is only towards the end of the course, when we look at the Riemann zeta function (a basic tool in the proof of the Prime Number Theorem) that we require any real analytic finesse.

We will begin in quite a basic way, by discussing how to measure the growth of increasing functions. For example, we all know that the series is divergent, it is more interesting to know that the sum of the first N term is like $\log N$ for large N . We will go on to study the asymptotics of other number theoretic functions (like the Euler ϕ function for example).

A centre-piece of the course will be Dirichlet's Theorem on primes in arithmetic progression. For example, odd primes are $\equiv 1$ or 3 mod 4 . We will show that each class (1 or 3) gets approximately one half of all primes and study the more general phenomenon.

Perhaps the highlight of the course will be a proof of the functional equation for the Riemann zeta function. This was proved about one hundred years ago and has been enormously influential. One of the most notorious unsolved problems in mathematics has to do with the distribution of the zeros of this function.

4. Recommended Reading: The course book is:

Graham Everest and Tom Ward “An Introduction to Number Theory” (Springer-Verlag)

See also: Tom Apostol "Introduction to Analytic Number Theory" (Springer-Verlag)

Also students will be encouraged to use the www as a resource.

5. Lecture Contents:

Elementary results on distribution of primes.	**(3 lectures)**
Arithmetic functions, Dirichlet multiplication.	**(4 lectures)**
Möbius inversion, Mangoldt function. Averages of arithmetic functions.	**(3 lectures)**
Characters of finite abelian groups.	**(2 lectures)**

Dirichlet's Theorem of primes in arithmetic progressions. **(5 lectures)**

Dirichlet Series, Euler products, integral formulae. **(3 lectures)**

The Riemann zeta function, the Gamma function, functional equation, zero-free regions, theta functions. **(6 lectures)**

Partitions, geometric representation, pentagonal number theorem, Jacobi triple product identity, Ramanujan. **(5 lectures)**

Revision, problem classes. **(2 lectures)**

MTH-3E28: Galois Theory

1. Introduction: This course is an introduction to Galois Theory, which beautifully brings together the notion of a group with the notion of a field from **Groups and Rings** (which is a prerequisite). In particular, the ideas developed will be applied to looking at the question of solving polynomial equations.

2. Hours, Credits and Assessment: A 20 UCU course of 33 lectures, supported by office hours. The assessment will be from coursework (20%) and one three-hour examination (80%).

3. Overview: Galois theory is one of the most spectacular mathematical theories. It gives a beautiful connection of the theory of polynomial equations and group theory. In fact, many fundamental notions of group theory originated in the work of Galois. For example, why some groups are called "solvable"? Because they correspond to the equations which can be solved! (Meaning by a solution some formula based on the coefficients and involving algebraic operations and extracting roots of various degrees.) Galois theory explains why we can solve quadratic, cubic and quartic equations, but no similar formulae exist for equations of degree greater than 4. In modern exposition, Galois theory deals with "field extensions", and the central topic is the "Galois correspondence" between extensions and groups.

4. Recommended literature and references: The best book for the course is probably

Stewart, I., *Galois Theory*, Chapman and Hall. [QA214STE]

A nice concise (and cheap) book is

Artin, E., *Galois Theory*, Dover.

Alternatively, you could try

Cohn, P.M., *Algebra Vol. 1*, Wiley. [QA154COH]

Herstein, I.N., *Topics in Algebra*, Wiley. [QA154HER]

Snaith, V.P., *Groups, rings and Galois theory*, World Scientific. [QA171SNA]

5. Lecture Contents:

Fields and polynomial rings; irreducibility of polynomials and irreducibility criteria for polynomials over Q; maximal ideals and construction of algebraic field extensions; degree; tower law; splitting fields. **(9 lectures)**

Artin's Extension Theorem, separability, inseparability, Primitive Element Theorem. **(5 lectures)**

Normal and Galois extensions, the Fundamental Theorem of Galois Theory, examples of the explicit computation of Galois groups. **(7 lectures)**

Radical extensions, solvable groups, proof that a polynomial can be solved using radicals if and only if the associated Galois group is a solvable group, radical solution to general quadratic, cubic and quartic equations, explicit examples of polynomials which are not solvable by radicals. **(7 lectures)**

Finite fields: basic structure of finite fields, all extensions of finite degree are Galois with cyclic Galois group. **(5 lectures)**

MTH-3E42 : Waves

1. Introduction: This is a 3rd year course running in conjunction with MTH-ME42. The two courses are offered in two levels of difficulty: as MTH-3E42, which is geared towards a third year student rather than a MMath student. With additional topics offered as a reading element of the course, MTH-ME42 is geared toward a more advanced student. The course describes various ideas in wave motion. The aim is to cover well established ideas and to introduce students to active research in the field. Nonlinear theory is emphasised. There are usually no known exact solutions of the equations modelling the problems. Asymptotic and numerical techniques are hence presented to derive appropriate approximations. This course is suitable for those with an interest in Applied Mathematics and with some experience in hydrodynamics.

2. Timetable Hours, Credits, Assessments: 20 UCU assessed by course work (20 percent, taken from problem sheets) and examination (80 percent). There will be 33 lectures.

3. Overview: A wave is any recognisable signal that is transferred from one part of a medium to another with a recognisable velocity of propagation. Wave motion is an extremely broad subject, which covers water waves, propagation of light and sound, traffic flow, and many other phenomena. Over the last 150 years many mathematical techniques have been developed and applied to understand wave motion. Great success has been achieved, but there are still many open questions. We will introduce some of the most powerful analytical and numerical techniques to study wave motion, describe what has been achieved, point out limitations, discuss open questions, and attempt to answer them. Both dispersive and hyperbolic waves will be covered.

4. Recommended Reading:

G.B. Whitham	"Linear and nonlinear waves"
G.D. Crapper	"Introduction to water waves"
James Lighthill	"Waves in Fluids"
Randall J. LeVeque	"Numerical Methods for Conservation Laws"

In addition, material would be taken from recent journal articles and appropriate references will be given during the lectures.

5. Lecture Contents:

Introduction: Hyperbolic waves and dispersive waves. Review from hydrodynamics, the water wave equations, nonlinear boundary conditions, the concept of surface tension. **(3 lectures)**

The theory of gravity-capillary waves in water of infinite depth. Linear theory and the concept of group velocity. Nonlinear theory: the Stokes expansion, Wilton ripples and numerical techniques for periodic waves. **(7 lectures)**

The theory of gravity-capillary waves in water of finite depth, the fifth order Korteweg de Vries equation, generalized solitary waves and solitary waves with oscillatory tails. Introduction to asymptotics beyond any order. **(7 lectures)**

Numerical techniques to solve time dependent free surface flow problems. Stability and applications to standing waves. **(4 lectures)**

Ship waves and the radiation condition. **(6 lectures)**

Hyperbolic waves and conservation Laws. Analytical and numerical techniques. **(6 lectures)**

Other topics might be considered, as time permits.

MTH-3E48 : Dynamical Oceanography

1. Introduction: This level 3 course covers modelling the large scale ocean circulation and structure, internal waves and coastal flows. It requires prior completion of second level courses, either **Hydrodynamics** or **Mathematics for Geophysical Science II** or the ENV course **Mathematics II**.

2. Timetabled Hours, Credits, Assessment: This is a 20 UCU unit of 33 lectures supported by occasional problem classes. Assessment is by set homework (20%) and examination (80%).

3. Overview: The mathematical modelling of the oceans in this course provides a demonstration of how the techniques developed in second year units on fluid dynamics and differential equations can be used to explain some interesting phenomena in the real physical world. The course begins with a discussion of the effects of rotation in fluid flows. The dynamics of large scale ocean circulation is discussed including the development of ocean gyres and strong western boundary currents. The thermal structure associated with these flows is examined. These large scale currents are responsible for the variation in climate between land on the eastern and western side of major ocean basins. The dynamics of equatorial waves are examined. Such waves are intimately linked with the El nino phenomena which affects the climate throughout the globe.

4. Recommended texts: The recommended reference books are:

A Gill	"Atmosphere-Ocean Dynamics", (Academic Press)
J Pedlosky	"Ocean Circulation Theory", (Springer)

5. Lecture Contents:

Introduction: Equations of motion. Rotation. Geostrophic flow. Ekman boundary layer. Ekman transport and suction. Rossby and Ekman numbers. Geostrophic and hydrostatic balances. Time scales. **(4 lectures)**

Gyre models: Wind-driven ocean circulation. Surface Ekman layer. Sverdrup interior circulation. Stommel boundary layer. Fofonoff gyre. **(6 lectures)**

Stratification: The effect of density (temperature) variations on ocean dynamics. **(6 lectures)**

Southern Ocean Dynamics: Antarctic Circumpolar current. Steady circulation with topography. Unsteady circulation without topography. Ertel's theorem. **(6 lectures)**

Internal waves: Coastal and equatorial Kelvin Waves. Rossby Waves. Continental Shelf Waves for various shelf topography. **(6 lectures)**

Equatorial dynamics: Equatorial jet. Kelvin and Rossby waves. **(4 lectures)**

MTH-3E61 : Mathematical Biology

1. Introduction: Mathematics finds wide-ranging applications in biological systems: including population dynamics, epidemics and the spread of diseases, enzyme kinetics, some diffusion models in biology including Turing instabilities and pattern formation, and various aspects of physiological fluid dynamics.

2. Timetable Hours, Credits, Assessments: A 20 UCU unit of 33. Assessment is by coursework (20%) and a 3 hour examination (80%).

3. Overview: The application of mathematics to problems in biology is one of the hottest topics of current research. From investigating the development of animal coat patterns (how did the leopard really get its spots?) to studying blood flow in the cardiovascular system with a view to predicting danger zones for the development of arterial disease, mathematical biology is replete with interesting problems and applications. In this course, we will discuss a selection of topics from this rapidly burgeoning field, including the modelling of cancerous tumours and studying the enzyme kinetics of chemical reactions in the human body. No significant biological knowledge is required.

4. Recommended Reading: The course book is:

Mathematical Biology (Vols I and II) by J.D.Murray (Springer)

Essential Mathematical Biology by N.F.Britton (Springer)

5. Lecture Contents:

Population dynamics and epidemics (spatial variation and wave-like solutions; Fisher's equation). Reaction-diffusion systems. Animal coat pattern formation and Turing instability. Molecular biology and enzyme kinetics. Mathematical models of cancer; avascular diffusion-limited tumour growth. Blood flow in arteries and veins.

(33 lectures)

MTH-3E73 : Electricity and Magnetism

1. Introduction: This course is an introduction to the mathematical theory of electricity and magnetism. It requires knowledge of partial differential equations and multivariable calculus. No prior knowledge of electricity or magnetism is required.

2. Timetable Hours, Credits, Assessments: A 20 UCU unit of 33 lectures. Assessment is by coursework (20%) and examination (80%).

3. Overview: The behaviour of electric and magnetic fields is fundamental to many features of life we take for granted yet the underlying equations are surprisingly compact and elegant. This course looks at the mathematical theory of those equations. We will begin with an historical overview of electrodynamics to see where the governing equations (Maxwell's) come from. We will then use these equations as axioms and apply them to a variety of situations including electro- and magneto-statics problems and then time-dependent problems (eg electromagnetic waves). We shall also consider how the equations change in an electromagnetic media and look at some simple examples.

4. Recommended Reading:

Two relevant books are Feynman, Feynman lectures on Physics, Vol II and Cottingham & Greenwood, Electricity and Magnetism. Further books will be recommended during the course.

5. Lecture contents:

Charges, forces and fields.

Charge distributions (discrete and continuous).

Electrostatics (Gauss's Theorem) and magnetostatics (Biot-Savart law).

Faraday's law.

Maxwell's Equations in vacuum.

Waves, potentials. Boundary value problems. Energy.

Special relativity and its role in electromagnetism.

Maxwell's equations in conducting matter.

(33 lectures)

MTH-3E75 : Financial Mathematics

1. Introduction: This unit is primarily concerned with the valuation of certain financial instruments known as derivatives. It has great application to finance and banking. The unit assumes some knowledge of differential equations and MTH-2C23 Differential Equations II, or equivalent, is a prerequisite. Some of the mathematical modelling is probabilistic but no prior knowledge of probability or statistics is assumed. Neither is any previous background in finance necessary.

2. Timetable Hours, Credits, Assessments: This unit is of 20 UCU and is taught in the Autumn Semester by 33 lectures in two parallel streams.. Assessment is by coursework (20%) and an examination (80%).

3. Overview: The Mathematical Modelling of Finance is a relatively new area of application of mathematics yet it is expanding rapidly and has great importance for world financial markets. The unit is concerned with the valuation of financial instruments known as derivatives. Introduction to options, futures and the no-arbitrage principle. Mathematical models for various types of options are discussed. We consider also Brownian motion, stochastic processes, stochastic calculus and Ito's lemma. The Black-Scholes partial differential equation is derived and its connection with diffusion brought out. It is applied and solved in various circumstances.

4. Recommended Reading:

Any of the following should prove useful:

1) Paul Wilmott introduces quantitative finance
   P. Wilmott. Wiley, 2001.

2) The mathematics of financial derivatives
   P. Wilmott, S. Howison, J. Dewynne. CUP, 1995.

3) An elementary introduction to mathematical finance, 2nd ed.
   S.M. Ross. CUP, 2003.

4) A course in financial calculus
   A. Etheridge. CUP, 2002.

5) The concepts and practice of mathematical finance
   M. Joshi. CUP, 2003.

5. Lecture Contents:

Introduction to options, futures and the no-arbitrage principle -- using this to calculate fair delivery prices for futures. **(5 lectures)**

Models for the movement of stock prices, efficient markets, Brownian motion and geometric Brownian motion. Stochastic and deterministic processes. **(3 lectures)**

Basics of stochastic calculus and Ito's lemma. **(3 lectures)**

The Black-Scholes analysis. Derivation of the Black-Scholes partial differential equation and the assumptions behind it. Formulating the mathematical problem, determining boundary conditions for option pricing problems. **(6 lectures)**

Solving the Black-Scholes equation. Connection with heat conduction equation, solution of the heat conduction equation - similarity solutions and the Dirac delta function. Derivation of the price of European options. **(7 lectures)**

Extension to options on assets paying dividends and American options, free boundary problems. **(6 lectures)**

Examples and revision. **(3 lectures)**