MTH-2B71 : Mathematics for Geophysical Science I

1. Introduction: This Autumn Semester unit is designed for students of geophysical science who have already passed the first year applied mathematics pre-requisites Mathematical Methods I and Mathematical Methods II. This unit is the pre-requisite for Mathematics for Geophysical Science II, studied in the Spring. Mathematics for Geophysical Science I is taught by lectures and seminars, and computer laboratory sessions. The unit gives students some of the essential methods of applied mathematics. This unit covers matrix algebra for the solution of systems of linear equations and Markov chain modelling of competing populations. A variety of numerical methods are introduced and practiced in computer sessions using the algebra package Maple. Programming skills required to implement the methods in Maple are taught. The calculus of vector fields is also treated. Students are also introduced to the theory of complex functions.

2. Timetable Hours, Credits, Assessments: This is a 20 UCU unit of 28 lectures with 6 seminars and 5 computer lab sessions. There are 4 modules: matrix algebra and numerical methods (14 lectures + 3 seminars), and vector calculus (10 lectures + 2 seminars), Programming in Maple (4 lectures + 5 labs). Assessment is by 40% coursework and 60% examination.

3. Overview: Among other tools matrix algebra is essential for solving real physical problem numerically. For example, changes in the competing animal populations can be described by a set of recurrence relations, which may be naturally expressed as a matrix system. Mathematics allows us to predict the long-term fate of the different species. Gauss elimination provides an elegant tool for solving large matrix systems of linear equations. In practice, however, more devious techniques may be required in order to obtain accurate solutions on a computer. We will discuss a variety of computational numerical methods available to solve such problems. We will study methods to integrate differential equations on a computer. We will use them to solve the Lorenz equations, which have been applied in weather prediction, and plot chaotic trajectories in space. The numerical methods will be implemented using Maple in the computer lab sessions.

The vector calculus module is essential for modelling quantities or processes which vary in two or three dimensions. The vector operators div, grad and curl are defined, and various vector identities are explored. The divergence theorem and Stokes' theorem are introduced. These theorems are particularly useful in the study of geophysical fluid dynamics, dynamical meteorology and solid mechanics.

A structured approach to Maple programming is introduced and used to implement the numerical methods discussed in the Matrix Algebra lectures. This includes the use of functions and subroutines. The basic language elements are introduced, loops, conditional statements, arrays and input/output.

4. Recommended Reading:

E Kreyszig "Advanced Engineering Mathematics" (Wiley)

W. Cheney & D. Kincaid "Numerical Mathematics and Computing"

5. Lecture Contents:

Matrix Algebra and Numerical Methods Module

Introduction to matrices. Simple application (predator/prey population model). Systems of linear equations (introduced via two equations in two unknowns case). Solving systems of linear equations using elementary operations. Matrices and systems of linear equations. Gaussian elimination, row echelon form and back-substitution. Reduced row echelon form; use of rank to distinguish cases of no solution, unique solution, infinitely many solutions. **(4 lectures)**

Matrix inversion. Determinants and how to calculate them by expansion. Determinants and solutions to systems of linear equations (including homogeneous systems). **(3 lectures)**

Linear independence, calculation of eigenvalues and eigenvectors. Diagonalisation of matrices with linearly independent eigenvectors. Application of diagonalisation to population model. Long-term fate of populations. **(3 lectures)**

Numerical methods for solving matrix systems. The Thomas algorithm for tridiagonal systems. Iterative methods including Jacobi iteration and Gauss-Seidel iteration. Convergence criteria. Numerical integration of ODEs. **(4 lectures)**

Vector Calculus Module

Scalar fields, gradient, directional derivative, level surfaces. **(3 lectures)**

Vector fields, divergence, Laplacian, curl, divergence theorem, Stokes's theorem, solenoidal vector fields, irrotational vector fields. **(7 lectures)**

Maple Programming Module

Introduction to Maple. Defining and manipulating variables. Arrays. Data input and graphics output. Functions. If/Then statements and Do loops. **(4 lectures)**

MTH-2B72 : Mathematics for Geophysical Science II

1. Introduction: This is a sequel to **Mathematics for Geophysical Science I** and provides an introduction to Mathematical Modelling using the mathematical techniques developed in **Mathematics for Geophysical Science I**. It covers the same material as the course **Mathematics II** for Environmental Science students. The unit provides mathematical solutions that should be useful in ENV courses on Oceanography, Meteorology, Geophysics and Geophysical Modelling. It may be followed by advanced level 3 applied mathematics courses. A discussion of Fourier series arises from the solution of the partial differential equations of mathematical physics.

2. Timetable Hours, Credits, Assessments: This unit is of 20 UCU and is taught in the Spring Semester by 39 lectures in two parallel streams. It is supported by 5 example classes. Assessment is by set regular homework (20%) and an examination (80%).

3. Overview: This unit has four topics which all aim to develop skills in mathematical modelling. The topics are Fluid Dynamics, Fourier Series, Partial Differential Equations and Solid Mechanics. This course shows how mathematics can be applied to the environmental and geophysical sciences.

Fourier series are useful for analysing time series and solving partial differential equations. The concept of representing a function by a sum of appropriate trigonometric functions is developed, leading to examples of whole range and half range Fourier series. The range of validity, and the behaviour of Fourier series are explored in a laboratory class, using the computer algebra package MAPLE.

The solution of partial differential equations is one of the cornerstones of geophysical modelling. Analytic techniques based on the separation of variables, allied to Fourier series, are used to solve prototype problems in wave motion, unsteady heat conduction, and steady heat flow.

Fluid dynamics is the study of fluid motion and is a foundation of meteorology, oceanography and many branches of engineering.

4. Recommended Reading: The following books are recommend for different parts of the course.

E Kreyszig	"Advanced Engineering Mathematics" (Wiley) (This is available in the library)
A R Paterson	"A first course in fluid dynamics" (CUP 1983)
R O Davis & A P S Selvadurai	"Elasticity and Geomechanics", (CUP 1996)

5. Lecture Contents:

Fluid Dynamics: Introduction. Continuum hypothesis. Pressure. Material derivative. Buoyancy. Two-dimensional flows. Conservation of mass. Bernoulli's equation. Flow over weir. Hydraulic jumps. Lift. **(5 lectures)**

Three-dimensional flows. Euler's equations. Boundary conditions. Stream function. Vorticity. Vortex tubes. Circulation. Irrotational flow. Kelvin's circulation theorem. **(4 lectures)**

Velocity potential. Uniform stream. Line source. Dipole. Line vortex. Combined examples. Lift. **(3 lectures)**

Water waves. Linear surface gravity waves. Phase and Group speed. Deep water and shallow water approximation. Particle paths. **(4 lectures)**

Fourier Series : Taylor's series, convergence of series, Fourier's theorem for piece-wise smooth functions on $(-\pi , \pi)$, comments, examples, periodic extension outside $(-\pi , \pi)$, summing series, extension to the interval $(-\ell , \ell)$. Even and odd functions, half-range Fourier sine and cosine series on the interval $(0 , \ell)$, examples. **(5 lectures)**

Second-order partial differential equations : Basic concepts and definitions, wave equation, heat conduction equation, Laplace's equation. The method of separation of variables. Application of this method for solving initial/boundary value problems associated with the aforementioned P.D.E.'s. D'Alembert's solution for the wave equation. Application for solving initial value problems for the wave equation on infinite and semi-infinite domains. **(6 lectures)**

Solid Mechanics : Theory of stress. Traction on a surface, resultant force and torque, in terms of components of stress. Normal and shear stresses. Principle stresses and axes of stress. Equations of equilibrium and symmetry of stress matrix. Displacement, strain and small deformations. **(6 lectures)**

Elasticity. Linear stress-strain relations. Navier equations of equilibrium and motion. Elementary examples. Propagation of plane P and S waves. **(4 lectures)**

MTH-2B81 : Mathematics for Physical Scientists III

1. Introduction: This Autumn Semester unit is designed for students of physical science who have already passed the first year mathematics pre-requisites **Mathematics for Scientists I** and **II**. This unit is the pre-requisite for **Mathematics for Physical Scientists IV**, studied in the Spring. **Mathematics for Physical Scientitsts III** is taught by lectures and seminars. The unit goes on to cover the elements of complex variable theory. The calculus of vector fields is also treated.

2. Timetable Hours, Credits, Assessments: This is a 10 UCU unit of 16 lectures with 4 seminars. There are 2 modules : vector calculus (10 lectures + 2 seminars) and Matrix Algebra (6 lectures + 2 seminars). Assessment is by 100% coursework (marked homework and a test).

3. Overview: The vector calculus module is essential for modelling quantities or processes which vary in two or three dimensions. The vector operators div, grad and curl are defined, and various vector identities are explored. The divergence and Stokes' theorems are introduced. These theorems are particularly useful in the study of fluid dynamics. The Matrix Algebra module inroduces the concept of a matrix, its basic algebra, and its usefulness in solving applied mathematical problems. In particular, the power of matrices in solving large sets of linear equations in many unknowns will be demonstrated. Such problems arise in a host of practical problems such as weather prediction.

4. Recommended Reading:

E Kreyszig "Advanced Engineering Mathematics" (Wiley)

This is available in the library.

5. Lecture Contents:

Vector Calculus and Matrix Algebra Modules

Scalar fields, gradient, directional derivative, level surfaces. **(3 lectures)**

Vector fields, divergence, Laplacian, curl, divergence theorem, Stokes's theorem, solenoidal vector fields. **(7 lectures)**

Introduction to matrices. Simple application (predator/prey population model). Systems of linear equations (introduced via two equations in two unknowns case). Solving systems of linear equations using elementary operations. Matrices and systems of linear equations. Gaussian elimination, row echelon form and back-substitution. Reduced row echelon form; use of rank to distinguish cases of no solution, unique solution, infinitely many solutions. **(4 lectures)**

Matrix inversion. Determinants and how to calculate them by expansion. Determinants and solutions to systems of linear equations (including homogeneous systems). **(2 lectures)**

MTH-2B82 : Mathematics for Physical Scientists IV

1. Introduction: This is a sequel to **Mathematics for Physical Scientists III** and provides an introduction to Mathematical Modelling using the mathematical techniques developed in **Mathematics for Physical Scientists III**. The unit provides mathematical solutions for fluid flow problems.

2. Timetable Hours, Credits, Assessments: This unit is of 10 UCU and is taught in the Spring Semester by 16 lectures in two parallel streams. It is supported by 3 example classes. Assessment is by set regular homework (20%) and an examination (80%).

3. Overview: This unit aims to develop skills in mathematical modelling and in particular modelling of fluid flows with reference to the environment, aerodynamics and waves.

4. Recommended Reading:

A R Paterson	"A first course in fluid dynamics"

5. Lecture Contents:

Fluid Dynamics: Introduction. Continuum hypothesis. Pressure. Material derivative. Buoyancy. Two-dimensional flows. Conservation of mass. Bernoulli's equation. Flow over weir. Hydraulic jumps. Lift. **(5 lectures)**

Three-dimensional flows. Euler's equations. Boundary conditions. Stream function. Vorticity. Vortex tubes. Circulation. Irrotational flow. Kelvin's circulation theorem. **(4 lectures)**

Velocity potential. Uniform stream. Line source. Dipole. Line vortex. Combined examples. Lift. **(3 lectures)**

Water waves. Linear surface gravity waves. Phase and Group speed. Deep water and shallow water approximation. Particle paths. **(4 lectures)**

MTH-2C1Y: Analysis

1. Introduction: This unit continues the study of analysis of functions started in the first year. The first part gives a unified treatment of basic properties of real and complex functions, and the impact of the geometry of the complex plane on complex analytic functions is discussed. The second part introduces complex integration and exposes the remarkable rigidity that the property of differentiability imposes on a complex function. The material is of central importance to both pure and applied mathematicians.

2. Hours, Credits and Assessment: The course is a compulsory (MTH students) 20 UCU unit of 40 lectures, half in the Autumn Semester and half in the Spring Semester. Support teaching is via seminars. Assessment is by coursework (20%) and an examination (80%).

3. Overview: The study of Analysis is of central importance to Mathematics, in particular underpinning all of calculus. Complex Analysis, in particular the concept of a path integral, was primarily developed by Cauchy in the early 19th century (although under restrictive assumptions) and further contributions were made by Liouville, Laurent and Riemann. There followed work to relax assumptions and widen the applicability of results, which had profound implications in understanding the geometry of point sets (Bolzano, Weierstrass, Cantor) and on function theory. Weierstrass, in particular, developed the theory of complex functions, though the use of complex variable technique was already widespread among 19th century mathematicians, physicists and engineers (and many of this era combined these roles).

This unit continues the study from MTH-1C14 in two ways: first, functions of a complex variable are treated alongside real functions; second, the notion of convergence is strengthened to *uniform convergence*, and this allows us to include functions defined by power series, such as the exponential and trigonometric functions, in our setting. The unit then goes on to look at how the *topology* of the complex plane puts an extra rigidity on the complex theory, and sees how the theory of integration has some surprising implications and applications to problems in real analysis which are not solvable using only real analysis.

Results from the final part of this unit are used in MTH-2G42 (Aerodynamics).

4. Recommended literature and references: Books for Real Analysis:

Kopp, P.E., *Analysis*, Arnold. [QA303 KOP in short loan collection]

Protter, M.H., & Morrey, C.B., *A first course in real analysis*, Springer-Verlag. [QA300 PRO]

Stirling, D.S.G., *Mathematical analysis and proof*, Albion. [QA300 STI]

Books for Complex Analysis:

Priestley, H.A., *Introduction to Complex Analysis*, Oxford University Press. [QA331 PRI].

Spiegel, M.R., *Schaum's Outline of Theory and Problems of Complex Variables,* Mc-Graw Hill. [QA331 SPI]

Stewart, I., & Tall, D., *Complex Analysis*, Cambridge University Press. [QA331 STE]

5. Lecture Contents:

AUTUMN SEMESTER

The complex plane; the modulus as a measure of distance. **(2 lectures)**

Review of continuity for real and complex functions. **(2 lectures)**

Sequences of functions. Uniform and pointwise convergence for sequences of functions. **(4 lectures)**

Power series representing real and complex functions. Radius of convergence. **(4 lectures)**

Differentiability of real and complex functions. Pathological behaviour of real differentiable functions; Cauchy-Riemann equations. **(4 lectures)**

Differentiation of power series representing complex functions. **(4 lectures)**

SPRING SEMESTER

Topology of the complex plane: open, closed and compact sets. **(2 lectures)**

Review of differentiablility of complex functions, holomorphic functions, the Cauchy-Riemann equations and elementary functions defined by power series. **(2 lectures)**

Paths, contours, connectedness and simple-connectedness. **(2 lectures)**

Integration along a path, the Estimation Theorem, integration of power series, the Fundamental Theorem of Calculus. **(3 lectures)**

Cauchy's Theorem, Cauchy's Integral Formulae, Taylor's Theorem, Liouville's Theorem, the Identity Theorem, Laurent expansions. **(5 lectures)**

Singularities, residues and Cauchy's residue theorem, techniques for finding residues, summation of series and other applications. **(6 lectures)**

MTH-2C23 : Differential Equations II

1. Introduction: This Autumn semester unit is in the mainstream of methods teaching for degree programmes in mathematics. It studies important methods and differential equations that have many applications.

2. Hours, Credits and Assessment: This unit is of 10 UCU and is taught in the Autumn semester by means of 20 lectures, supported by 3 seminars. Assessment is by set regular coursework.

3. Overview: Fourier Series and Separation of Variables together provide a basic technique used in level 2 & 3 applied mathematics units, for the solution of Linear Partial Differential Equations. Ordinary differential equations with variable coefficients are solved by means of infinite series. The Legendre and Bessel differential equations are of particular importance especially for Laplace's partial differential equation.

4. Recommended literature and references:

A full coverage of the unit and much further reading are offered by "Advanced Engineering Mathematics" by E. Kreyszig and "Elementary Differential Equations and Boundary Value Problems" by W.E. Boyce and R.C. Diprima (both published by Wiley).

5. Lecture Contents:

A. Fourier Series: Fourier series, for 2π periodic functions. Orthogonality integrals & the selection of Fourier coefficients. Odd and even functions. Fourier's Theorem stated. Extension to Fourier series of functions of arbitrary period. Functions defined on a finite interval. Full-range Fourier series, half-range Fourier sine and Fourier cosine series and quarter-range. **(4 lectures)**

B. Elementary partial differential equations

Separation of variables: Rectangular Cartesian co-ordinates. Solution by separation for diffusion equation initial-boundary-value problems, including Fourier technique. Initial-boundary-value problems for the wave equation. Uniqueness stated. Solution by separation. Interpretation as the transverse vibration of a stretched string. Boundary-value problems for the potential equation in 2D. Solution by separation. Synthesis of solutions by superposition. Uniqueness stated.

Separation for non-rectangular regions: The potential equation in plane polar co-ordinates. Solution of the Dirichlet problem in plane polar co-ordinates. Implied boundary conditions. Fourier techniques. The potential equation in cylindrical & spherical polar co-ordinates. Separation in the axisymmetric case and the emergence of polynomial coefficient ordinary differential equations i.e. Bessel and Legendre equations. **(5 lectures)**

C. Differential Equations, Special Functions

Linear ordinary differential equations on the real line. Linear dependence; the Wronskian, Liouville/Abel formula. The complete solution, method of variation of parameters. **(3 Lectures)**

Second-order linear equations on the real line. Series solutions. Singular points and the method of Frobenius. The indicial equation; equal roots and roots differing by an integer. **(4 Lectures)**

Legendre polynomials and Bessel functions. Application to solutions of Laplace's equation.

(4 Lectures)

MTH-2C26 : Mathematical Algorithms

1. Introduction: An introduction to a variety of numerical methods. Solution of linear algebraic equations. Solution of nonlinear equations. Numerical integration. Numerical Solution of ODE's.

2. Hours, Credits and Assessment: The course is a 10 UCU unit of 20 lectures . Assessment is by 100% Coursework

3. Overview: In dealing with practical problems of applied science such as modelling fluid flow or predicting the motion of celestial bodies, a set of mathematical equations may be written down which are usually either very difficult or impossible to solve using analytical means. For example, a supercomputer is required in order to make accurate weather predictions. To make progress, approximations to a given set of equations may be written down and their solution sought by numerical computation. In this course, we will discuss some of the techniques which can be used for finding approximate solutions to difficult problems on a computer, from approximating the values of integrals to finding the roots of nonlinear algebraic equations and computing approximate solutions to nonlinear ordinary differential equations. Extensive use will be made of the algebra package Maple in computer classes to enable students to implement some of the techniques discussed in lectures.

4. Recommended Reading:

Numerical computation in Science and Engineering by C. Pozrikidis.

Numerical Mathematics and Computing by W. Cheney & D. Kincaid.

5. Contents:

Numerical error. Solution of linear equations; tridiagonal systems; Thomas' algorithm. Iterative methods for nonlinear equations; Newton's method. **(8 lectures)**

Numerical integration; Newton-Cotes formulae. **(4 lectures)**

Numerical integration of ODEs; Runge-Kutta integration. **(4 lectures)**

Several practical computer sessions with Maple.

MTH-2C2Y : Hydrodynamics (Autumn Semester)

1. Introduction: This first semester course at level 2 is an introduction to mathematical modelling in the context of fluid flows.

2. Timetable Hours, Credits, Assessments: This unit is of 10 UCU and is taught in the Autumn Semester by 20 lectures, supported by 3 hours of examples classes. Assessment is by set regular coursework (20%) and examination (80%).

3. Overview: Hydrodynamics has been used as the topic to introduce Mathematical Modelling for many years in Britain. It provides a variety of simple situations which can be described mathematically and for which the mathematical solutions are realistic. The course begins with the development of the equations needed to describe the flow in channels, over weirs in rivers, and through pipes, and ends with the theory of water waves on the surface of lakes and the ocean.

4. Recommended Reading:

The two recommend text books are "Elementary Fluid Dynamics" by D Acheson (Oxford UP) and "A first course in Fluid Dynamics" by A Paterson (Cambridge UP). A useful book, consisting of just pictures of fluid flows, is "An album of fluid motion" by M Van Dyke (Parabolic Press). Further physical background is found in D J Tritton "Physical Fluid Dynamics" (Oxford UP).

5. Lecture Contents:

(1) Introduction: Examples of flows in science and nature. Liquids, gases and fluids. Pressure, hydrostatics and Archimedes principle - full and partial immersion. Compressibility and density. **(2 lectures)**

(2) Kinematics: Velocity, particle path, streamlines. Mass continuity: Incompressibility, streamtubes. Pipe flow example. **(3 lectures)**

(3) Dynamics: Material derivative, Euler's equations, vorticity and irrotational flows. The existence of a velocity potential. Bernoulli's equation for unsteady flow. Examples for steady flow: in pipe, leakage under pressure, waterfall, fountain. Open channel flows: over weir, under sluice gate, and in hydraulic jump (Severn Bore) including energy loss. **(4 lectures)**

(4) Circulation, Kelvin's Theorem, Helmholtz's vortex theorems. Tornado. **(2 lectures)**

(5) Velocity potential, ϕ . Examples. $\nabla^2\phi = 0$. Uniqueness theorems for $\nabla^2\phi = 0$ under various boundary conditions. **(4 lectures)**

(6) Water Waves. Full nonlinear b.v.p. on moving domain. Linearised equations for water in uniform depth. Simple solutions which are periodic in space and time. Wave speed as a function of wavelength in deep or shallow water. Particle motion under progressive wave in finite depth. Deep water case. Group velocity - wave groups. Superposition of linear progressive waves to form standing waves in a tank of finite length. Normal modes. Particle paths beneath standing waves. **(5 lectures)**

MTH-2C2Y : Linear Elasticity (Spring Semester)

1. Introduction: This level-two unit follows on naturally from **Multivariable Calculus (MTH-1C24)** which is a prerequisite. The unit provides an introduction to continuum mechanics with special applications to linear elasticity.

2. Hours, Credits and Assessment: This unit is of 10 UCU and is taught in Semester 2 by 20 lectures, supported by seminars. Assessment is by set regular coursework (20%) and examination (80%).

3. Overview: Continuum mechanics is the study of the properties of matter, regarded as a continuous medium, ignoring the effects of atoms and molecules. Newtonian fluids provide one example of this theory, considered in other units of the MTH programme, whilst elastic solids another example, which is introduced in this unit. Linear elasticity is a generalisation of Hooke's Law connecting the stress and strain for small displacements of an elastic material. It is a mathematical subject with many applications in science and engineering. From the stretching, bending and torsion (twisting) of rods to waves in steel, crystals and the solid earth.

4. Recommended Reading: There is no book ideal for the course and none, except perhaps D F Parker's, is recommended for buying and that for general purposes rather than for elasticity. Any of the books listed below are worth dipping into and others may be found in the University Library at QA808.2 or QA931, amongst other locations.

D F Parker	"Fields, Flows and Waves" (Springer, 2003)
R O Davies & A P S Selvadurai	"Elasticity and Geomechanics" (CUP, 1996)
L D Landau & E M Lifshitz	"Theory of Elasticity" (Pergamon, 1986)
A E H Love	"A Treatise on the Mathematical Theory of Elasticity" (Dover,1944)
R J Atkin & N Fox	"An Introduction to the Theory of Elasticity" (Longman)

5. Lecture Contents:

Strain: Deformation and deformation gradient, material time derivative. Linear strain tensor and interpretations. Homogeneous deformations: uniform dilatation, simple extension, simple shear. Conservation of mass, dilatation. **(3 lectures)**

Stress: Stress tensor and traction vector. Normal and shear stresses. Spherical, uniaxial and shear stresses. Balance of linear momentum. Symmetry of stress tensor. **(3 lectures)**

Linear elasticity: Generalised Hooke's law for isotropic elastic materials, Lamé moduli. Equilibrium equations. Bulk, shear and Young's elastic moduli, Poisson's ratio. Incompressible elasticity. **(2 lectures)**

Plane problems: Anti-plane strain. Plane strain and the Airy stress function. Plane stress. **(2 lectures)**

Torsion: Torsion of circular, elliptical and triangular rods. **(2 lectures)**

Beams: Bending by terminal couples. The Euler-Bernoulli law. **(2 lectures)**

Waves: Equations of motion. One-dimensional wave equation, plane waves. D'Alembert's solution. Longitudinal and shear waves. Elastic body waves, plane P-waves and S-waves. Reflection and refraction of plane waves. Rayleigh waves, Love waves. **(6 lectures)**

MTH-2C3Y: Algebra (Autumn Semester)

1. Introduction: This course follows on from Linear Algebra I which is a prerequisite. Linear Algebra underpins most of modern mathematics. Therefore this unit is a prerequisite for many mathematics units later on. The course looks at vectors and matrices from a more sophisticated viewpoint by introducing the concepts of vector space and linear transformation.

2. Hours, Credits and Assessment: The course is a 10 UCU unit of 20 lectures. Support teaching is via seminars. Assessment is by examination (80%) and coursework (20%) via assessed homework.

3. Overview: In Linear Algebra I we introduced matrix multiplication as a way of writing a system of linear equations as a single matrix equation. A more sophisticated point of view is to describe the multiplication of a vector by an m-by-n matrix as a mapping from R^n to R^m. To formalise this properly, we introduce the notion of an *abstract vector space*, and of a *linear transformation* from one vector space into another. One of the key concepts for vector spaces is that of a *basis* -- what we might informally have called a coordinate system. Any two bases of a vector space have the same number of elements and this number is the *dimension* of the vector space. We distinguish certain subsets of a vector space which can themselves be considered as vector spaces (subspaces), and relate the dimensions of subspaces to the dimension of the ambient space.

Linear transformations are determined by their effect on a basis and so can be described by a matrix. Changing bases changes this matrix, and this will be described precisely. We also investigate how 'simple' this matrix can be made by an appropriate choice of bases. This leads to the notions of *eigenvalues* and *eigenvectors* of (square) matrices.

In the last section we shall look at *inner products* on a vector space and how natural bases for a space with inner product can be computed.

4. Recommended literature and references:

S Lang, Linear Algebra, UGT, Springer Verlag

H Anton Elementary Linear Algebra, (Wiley)

5. Lecture Contents:

Motivation and examples of vector spaces: linear equations, matrices, functions. **(1 lecture)**

Vector spaces: Definition of a vector space. Linear combinations and subspaces. Linear independence. Definition of basis and dimension. Dimensions of subspaces and the modular law. Further examples. **(6 lectures)**

Linear Transformations: Definition and examples (the linear transformation arising from a matrix; examples in analysis; geometric examples such as rotation and projection). Kernel and image of a linear transformation. The rank-nullity theorem. Examples from geometry, derivatives and integrals of polynomials, linear equations; applications. **(4 lectures)**

The matrix of a linear transformation and change of basis formulae. **(2 lectures)**

Eigenvalues and eigenvectors: The characteristic and minimum polynomial. Eigenspaces. Diagonalising matrices. Connection with eigenvectors. Examples of non-diagonalisability. Proof that 'n distinct eigenvalues' implies 'A diagonalisable'. Examples. Computing powers and roots of diagonalisable matrices. **(2 lectures)**

Inner product and orthogonal vectors in R^n . The Gram-Schmidt process. Diagonalisability of real symmetric matrices. **(5 lectures)**

MTH-2C3Y : Algebra (Spring Semester)

Hours, Credits and Assessment: The course is a 10 UCU unit of 20 lectures with three support classes. Assessment is by 100% coursework.

Overview: This course introduces the theory of groups and rings, which are among the most important algebraic structures. They appear in many branches of mathematics and are fundamental examples of axiomatic systems. Development of the associated theory is methodical and self-contained and encourages working formally in an axiomatic environment.

At the heart of group theory is the study of geometric transformations and symmetry. The course remains close to such fundamental mathematical notions whilst also introducing a theory with enough generality to be applied elsewhere. This includes structural results such as isomorphism theorems as well as results on integer congruences and intriguing methods of counting developed using group actions on sets.

The second part of the unit starts by introducing rings, using the Integers as a model. The subsequent theory is developed with a variety of examples, giving new insight into familiar concepts such as substitution and factorisation. In contrast to the first section of the unit, a commutative setting is soon adopted. Important examples of commutative rings are fields and domains. New constructions of fields are introduced using quotients of rings modulo maximal ideals. The concepts of divisibility and factorisation are tackled in domains. The related notion of a valuation on a domain exposes some fascinating links with other areas of mathematics typically studied at this level.

Recommended literature and references: All of the following should be in the Library, which also has numerous other texts covering the course material.

Joseph J. Rotman, "A first course in abstract algebra"
R.B.J.T. Allenby, "Rings, Fields and Groups: Introduction to Abstract Algebra"
J.B. Fraleigh, "A first course in abstract algebra"

Lecture Contents:

Groups (10 Lectures)

i) Finite and infinite examples, groups of symmetries, normal subgroups, cosets and Lagrange's Theorem
ii) Homomorphisms, quotient groups, and isomorphism theorems
iii) Integer congruences, theorems of Fermat, Euler and Wilson
iv) The Platonic Solids
v) Group actions and counting

Rings (10 Lectures)

i) Rings and their elementary properties, including polynomial rings and quadratic rings
ii) Examples where Euclid's algorithm works
iii) Euclidean Domains and consequences including unique factorization
iv) Units
v) Ideals, quotient rings and ring homomorphisms

MTH-2G34: Combinatorics

1. **Introduction:** Combinatorics is one of the most applicable and accessible parts of mathematics, yet it is also full of challenging problems. In this course, we shall cover many basic combinatorial concepts. These will include graphs, colourings and Ramsey theory, partitions and Young diagrams and Latin squares.

2. **Timetable, Hours, Credits, Assessments:** The unit is a 10 UCU unit of 15 lectures. Assessment is by exercises set throughout the unit (20%) and a written exam (80%).

3. **Overview:** A graph consists of a set V of vertices and a set E of edges between these vertices. Two vertices may or may not be connected by an edge, or they may have multiple edges between them. For example, the relation of 'being acquainted with' in a class of students can be represented by an edge if the two students that these vertices represent are acquainted with each other. We shall consider several kinds of graphs and discuss their basic properties. A vertex colouring of the graph is a function from the set of vertices to a finite set of colours, say {red, blue, green}, in which each vertex gets a distinct colour. One can similarly consider edge colouring, in which edges are coloured rather than vertices. We shall use this idea to prove, e.g. that in any group of 6 people, there are either 3 who all know each other, or there are 3 who do not know each other. This is a simple example of Ramsey's theorem for finite sets, which we will describe and prove. We will then move on to discuss partitions and their properties, before concluding with a brief introduction to Latin squares.

4. **Recommended Reading:** The unit is self contained. Lecture notes will be provided after the course is complete. A useful text is

    1. J.H. van lint and R.M. Wilson: A course in combinatorics, Cambridge University Press 1992,

    which is available at the library.

MTH-2G35: Metric and Topological Spaces

1. Introduction: This unit is suitable for second or third year Mathematics students. Some background knowledge of elementary real analysis (as in MTH-1C14) is assumed.

2. Timetable, Hours, Credits, Assessments: The unit is a 10 UCU unit of 15 lectures. Assessment is by exercises set throughout the unit (20%) and a written exam (80%).

3. Overview: Topology is the mathematical study of properties which are preserved under continuous transformation. Clearly we need to make precise what is being transformed, and what is meant by continuity. For the first, we introduce the idea of a *metric space* : this is just a set together with a function telling us the 'distance' between any two points. This idea is pervasive in mathematics: from situations such as the usual distance in n-dimensional space, to the Hamming distance between codewords in an error-correcting code, to the useful idea of the distance between functions when one is approximating a given function by a sequence of functions. Once we have the idea of a metric space we can define the notion of a *continuous* function between two metric spaces. This is an easy but very powerful generalisation of the notion of continuity in real analysis. We also introduce the more general notion of a *topological space*: here the fundamental concept is not so much the precise distance between points, but rather the idea of points being 'close' to each other.

The unit fits well with the second-year units on real and complex anaysis, and indeed provides a general framework in which to view these subjects.

4. Recommended Reading:

W. A. Sutherland: Introduction to Metric and Topological Spaces, Oxford University Press, 1975. (Price: about 20 pounds) UEA library classmark: QA713 SUT

5 Lecture contents

Metric spaces; examples. Convergence of sequences; continuity. Open and closed sets; examples; continuity in terms of open sets. Topological spaces; examples; continuity.
(7 lectures)

Compactness; Heine - Borel Theorem; Haudorff property. Connectedness; examples and applications. Completeness of metric spaces. Product and quotient spaces. **(8 lectures)**

MTH-2G41 : Dynamical Systems

1. Introduction: This unit is an optional unit at level 2 which is available to students on single subject degree programmes in mathematics and to other students with a suitable mathematical background..

2. Timetable Hours, Credits, Assessments: This unit is of 10 UCU and is taught in Semester I by 15 lectures, supported by 3 seminars. Assessment is by set regular coursework (20%) and a 2 hour examination (80%).

3. Overview: The study of the growing interdisciplinary field of dynamical systems and chaos is necessary for understanding the frequently unexpected behaviour of many dynamical processes of great interest within physics, chemistry, biology, engineering, medicine and economics.

This introductory course covers some of the fundamental ideas, definitions and results on dynamical systems. Primary emphasis will be given to the mathematical aspects of the theory, but some important applications will be developed as well.

4. Recommended Reading:

S H Strogatz "Nonlinear Dynamics and Chaos" (Addison-Wesley)

5. Lecture contents:

The phase plane and phase portraits. Second order equations as two first-order systems. General first order systems in two variables. Classification of equilibrium points. Applications to physics, biology and chemistry. **(7 lectures)**

Periodic orbits. Limit cycles. Oscillators with a small parameter. Method of averaging. **(4 lectures)**

Bifurcation theory. Saddle-node, transcritical, pitchfork and Hopf bifurcations. Stability of limit cycles. **(4 lectures)**

MTH-2G42 : Aerodynamics

1. Introduction: This second semester course at level two follows on naturally from Hydrodynamics (MTH-2C2Y first part), which is a pre-requisite for this course. The emphasis in this unit is the application of theory to the more practical problems of aerodynamics.

2. Timetable Hours, Credits, Assessments: This unit is of 10 UCU and is taught in Semester 2 by 15 lectures, supported by 3 hours of seminars or examples classes. In addition, computer classes will also be arranged. Assessment is by set regular coursework (20%) and examination (80%).

3. Overview: This unit combines the theory of incompressible, irrotational flow with applications to lift and drag forces on bodies such as cars, wings and sports balls. The ultimate aim of the course is to understand why aircraft wings generate lift, why different car shapes lead to different drag forces and why cricket balls swing through the air.

This course will also include flow visualisations using MAPLE.

4. Recommended Reading:

Two recommended books which cover the basic hydrodynamics required for this course are "Elementary Fluid Dynamics" by D Acheson (OUP) and "A first course in Fluid Dynamics" by A Paterson (CUP). A list of books applicable to the more practical parts of the course will be provided by the lecturer.

5. Lecture contents:

Revision of basic principles of Hydrodynamics - velocity potential and stream function.

Complex Potential

Slender airfoil theory - modelling an airfoil by a distribution of sources and sinks.

Image Systems - with applications to ground effect.

Conformal Mappings - theory and application to Joukowski airfoils.

Calculation of lift coefficients using Blasius formula.

Viscous effects and wakes.