MTH-2A23 : Algebra I

1. Introduction: Together with **Algebra II** this unit is one of the main second year units in Pure Mathematics. It has the 1st year unit **Linear Algebra II** as prerequisite. Its aim is to introduce students to algebraic structures and to axiomatic methods in pure mathematics. Its main topic is Group Theory. It will introduce the principal notions of groups, subgroups, homomorphisms and factor groups. While it can be taken on its own, this unit in conjunction with Algebra II is the prerequisite for several third year Pure Mathematics courses.

2. Hours, Credit, Assessment: The course is a 10 UCU unit of 20 lectures with three support classes. Assessment is by examination (80%) and coursework (20%) via assessed homework.

3. Overview: This course introduces students to groups, which are among the most important algebraic structures and which appear in most branches of mathematics. Modern algebra bases itself on axiomatic systems where theories are developed from axioms. This is true particularly for Group Theory. One of its roots and influential in its development has been the study of geometrical transformations and symmetry. Group Theory is therefore very close to these fundamental mathematical notions. It is also a subject where theory and axiomatics interacts with intuition and insight in fascinating ways. Axiomatic methods are not always easy to get used to. Group Theory is a field where one can see their full power and at the same time be close to concrete applications.

The course starts with the definitions of groups, subgroups and cosets. Lagrange's Theorem, saying that the order of a subgroup of a finite group divides the order of the group, is proved by showing that cosets partition the group. In order to compare groups one needs the notion of homorphism and isomorphism. This leads to normal subgroups in a group and to quotient groups. Groups occur as symmetries of other mathematical objects and so there is a section introducing the notion of group action and its relationship to homomorphisms. Each of these concepts is explored in a variety of examples. One of the most useful theorems on group actions is the Orbit Stabilizer Theorem, which will be used to count orbits and patterns.

With the theory developed so far it is possible to classify all commutative groups generated by a finite number of elements and this result is one of the principal results in the course.

4. Recommended Reading:

All of the following should be in the Library and there are many other books in the library which cover the material of the course.

I. N. Herstein "Topics in Algebra"

J.B. Fraleigh "A first course in abstract algebra"

R.A. Dean "Elements of abstract algebra"

5. Contents:

Binary operations, groups. Elementary properties. Examples: (a) the symmetric group, cycle notation and working with permutations; (b) symmetries of some familiar objects in $\mathbb{R}^3$; (c) the rotational symmetries of the regular n-gon and the definition of the dihedral groups. **(4 lectures)**

Subgroups, the powers of an element, classification of cyclic groups. Cosets and the theorem of Lagrange. Homomorphism, isomorphisms, and normal subgroups. Factor groups and the first isomorphism theorem. Examples. **(6 lectures)**

Group actions. The connection between a group acting on a set X and the homomorphisms from the group into the symmetric group on X. Examples: actions on cosets, various actions of the dihedral groups. Orbits and the Orbit Stabilizer Theorem. Applications. **(7 lectures)**

The Smith canonical form and applications to finitely generated abelian groups. **(3 lectures)**

MTH-2A24 : Algebra II

1. Introduction: This unit is one of the main second year units in Pure Mathematics. It leads on from **Algebra I** which is a prerequisite and continues with this introduction to contemporary algebra and axiomatic methods in mathematics. Its main topic is Ring Theory, a very powerful tool of mathematics. Apart from the general notions of rings, ideals and homomorphisms, the course deals with the theory of polynomials, an introduction to field theory and the theory of factorization. The notions of algebraic numbers, algebraic integers are introduced with minor comments. This unit is very important as it is a prerequisite for many third year Pure Mathematics courses: **Ring Theory**, **Number Theory, Finite Groups**, **Representation Theory**, **Linear Algebra and Linear Groups**.

2. Hours, Credit, Assessment: The course is a 10 UCU unit of 20 lectures and four seminars. Assessment is by examination (80%) and coursework (20%) via assessed home work.

3. Overview: The course is a further step towards an introduction to axiomatic systems. (The notions of a field and a vector space in **Pure Math I**, and of a group in **Algebra I**.) The idea of developing a theory from axioms is one of the central methods of contemporary mathematics. This requires developing new skills and, especially, the skill of working confidently in an axiomatic environment. The course starts by introducing the notions of ring, subring, ideal, homomorphism, isomorphism, quotient ring, and showing a variety of examples. After introducing the notion of a maximal ideal and a field, it becomes possible to demonstrate the method of constructing fields as quotients of commutative rings modulo maximal ideals. At this stage there will also be an introduction to finite fields. This is a starting point of commutative ring theory. The notions of principal ideal and divisibility are steps to higher arithmetics with divisibility theory, and to the theory of polynomial rings. The division algorithm opens a door to constructing non-prime finite fields by giving techniques for computations in finite fields. The next portion of abstract theory is the theory of principal ideal domains and the theory of Euclidean domains. At this stage will be introduced algebraic number and algebraic integers, the main figures of algebraic number theory. The course ends with discussion of polynomial rings in several variables and the Hilbert Basis theorem.

4. Recommended Reading: All of the following should be in the Library and there are many other books in the library which covers the material of the course.

1. D.A.R. Walace, "Groups, Rings and Fields", Springer, London, 1998;

2. J.B. Fraleigh, "A first course in abstract algebra";

3. R.A. Dean, "Elements of abstract algebra";

4. I.N. Herstein, "Topics in Algebra";

5. B. Hartley and T. Hawkes, "Rings, Modules and Linear algebra".

In addition, lecture notes may be provided to students and made available on the UEA intranet.

5. Contents:

Main definitions and elementary properties: Subrings, commutative rings, units, fields, integral domains, simple rings, ideals, principal ideals. Matrix rings. **(4 lectures)**

Homomorphism and isomorphism, quotient ring, quotient ring of a maximal ideal, finite fields. **(2 lectures)**

Polynomial rings: evaluations, roots, division algorithm, remainder theorem, irreducible polynomials, coprime polynomials. **(2 lectures)**

Maximal ideals and fields. Construction of finite fields of non-prime order. Computations in finite fields. **(3 lectures)**

Algebraic numbers and algebraic integers. **(2 lectures)**

Principal ideal domains and Euclidean domains. Factorization. **(2 lectures)**

Polynomials with integer coefficients, content function, Gauss' lemma and Eisenstein's irreducibility criterion. **(2 lectures)**

Polynomial rings in several variables. **(3 lectures)**

MTH-2A31 : Advanced Calculus I

1. Introduction: This unit contributes to an ongoing development of continuous mathematics and Complex Analysis in particular while also including important methods. It has as prerequisites Mathematical Methods I, Mathematical Methods C, Pure Mathematics I and Linear Algebra II. The material is essential for a Mathematics degree and is of interest to pure and applied mathematicians.

2. Hours, Credits and Assessment: The course has 40 lectures and is 20 UCU unit with 6 seminars. Assessment is 20% coursework and 80% examination.

3. Overview: The unit consists of 3 modules.

Module 1 (Vector Calculus) Vector Algebra is designed to efficiently describe and operate with entities that have an intrinsic direction in three-dimensional space as well as a magnitude (such as line segments, displacements, velocities, power, etc.). Scalar and Vector quantities can be defined as "fields" throughout a three-dimensional space (for application in other level 2 units to fluid flows, for example). The Vector Calculus defines important differential and integral operations upon vector fields.

Module 2 (Complex Analysis) provides a basic introduction to the topology of the complex plane followed by an introduction to sequences, series and power series. Most of this theory lifts from the real case. One of the key ideas is differentiability and here we see the departure from the real case: Perhaps the biggest result in the module says that a convergent power series is differentiable.

The next topic of the module is geometric; the study of conformal mappings, in particular, Möbius transformations.

The basic theorems of the subject are covered. The concept of a path integral is central. The framework was primarily developed by Cauchy in the early 19th century, although under restrictive assumptions; there were contributions from Liouville, Laurent and Riemann. There followed work to relax assumptions and widen applicability of results which had profound implications in understanding the geometry of point sets (Bolzano, Weierstrass, Cantor) and on function theory. Weierstrass in particular developed the theory of complex functions but use of complex variable technique was widespread among 19th century mathematicians, physicists and engineers-many of this era combined the roles. Some terminology derives from electricity and magnetism. The surprising implications of the theory are covered in **Advanced Calculus II.**

Module 3 (Fourier Series & Separation of Variables) provides basic technique used in level 2&3 applied mathematics units. It introduces partial differential equations and elementary methods of analytical solution using level 1 techniques of integration and solving ordinary differential equations but also illustrating ideas from vector spaces.

The topics Fourier Series and Separation of Variables are linked and historically emerged in tandem in the 18th & early 19th centuries. B Taylor, Bernoulli (J&D), d'Alembert and Euler contributed in the context of waves on strings, but Fourier(1807) first put all the ingredients together, analysing heat

conduction. Laplace and Legendre developed aspects of the theory associated with the potential equation. Rigorous justification came later and is covered in a level 3 unit. The idea of Fourier coefficients as co-ordinates in an infinite dimensional vector space was due to Poincaré (1894) and led to new levels of mathematical abstraction.

Separation of variables when used with curvilinear co-ordinates leads to polynomial coefficient ordinary differential equations; their theory is developed in **Advanced Calculus II**.

4. Recommended literature and references:

Methods E Kreyzsig "Advanced Engineering Mathematics" (Wiley)

Churchill & Brown "Fourier Series & Boundary-value Problems" (McGraw-Hill)

Complex Analysis H A Priestly "Introduction to Complex Analysis" (OUP)

I. Stewart & D Tall "Complex Analysis" (CUP)

5. Contents:

Vector Calculus: Scalar fields, visualisation and level surfaces. Directional derivative and rate of change along a curved path. Gradient. as a differential operator. Interpretation of grad. Applications: 2 variable Taylor expansions and surface normals.

Vector fields, visualisation and field lines. Flux illustrated as volume flow and defined by integration over a curved surface with vector surface element. Divergence as a differential operator, defined in terms of components. Interpretation as flux per unit volume. Properties of the div operator. The Laplacian. Application to gradient fields: Laplace's and Poisson's equation.

Curl as a differential operator, illustrated for particular fields. Irrotational and solenoidal fields. Operators **u**… and ^{2} as applied to vector fields. Vector operator identities.

Integral theory. The Divergence Theorem. Application to express div, grad, curl as limits of integrals, and to derive a differential equation from an integral formulation. Stokes' Theorem. Path independent line integrals and finding a function with specified gradient.

Gradient, divergence, curl and Laplacian in spherical and cylindrical polar co-ordinates.

(10 lectures)

Fourier Series: Fourier series, for 2π periodic functions. Orthogonality integrals & the selection of Fourier coefficients. Odd and even functions. Fourier's Theorem stated. Behaviour of partial sums and Gibb's phenomenon - descriptive - reference to Maple worksheets. Application of Fourier series to the summation of series. Extension to Fourier series of functions of arbitrary period. Functions defined on a finite interval. Full-range Fourier series, half-range Fourier sine and Fourier cosine series. Integration and differentiation of series. Orthogonality and the linear independence of the functions {sin nx , cos nx} for integer values of n.

(5 lectures)

Elementary partial differential equations: Initial & boundary-value problems Linearity. Order. Derivation of the heat conduction equation. Advection and wave equations; explicit solutions involve arbitrary functions. The role of initial and boundary conditions. Uniqueness of solution for the 1D heat conduction initial-boundary-value problem as a means of deciding what conditions to apply.

Separation of variables: Rectangular Cartesian co-ordinates. Solution by separation for diffusion equation initial-boundary-value problems, including Fourier technique. Initial-boundary-value problems for the wave equation. Uniqueness stated. Solution by separation. Interpretation as the transverse vibration of a stretched string. Boundary-value problems for the potential equation in 2D. Solution by separation. Synthesis of solutions by superposition. Uniqueness stated. Repeated separation for a 3D problem in a box.

Separation for non-rectangular regions: The potential equation in plane polar co-ordinates. Solution of the Dirichlet problem in plane polar co-ordinates. Implied boundary conditions. Fourier techniques. The potential equation in cylindrical & spherical polar co-ordinates. Separation in the axisymmetric case and the emergence of polynomial coefficient ordinary differential equations i.e. Bessel and Legendre equations. Separation solutions stated.

(7 lectures)

Complex Analysis: Complex numbers, open and closed sets in the complex plane, sequences, limits and continuity

Differentiation, the Cauchy-Riemann equations, holomorphic functions, Power series, elementary functions.

Properties of paths and contours.

Conformal mappings, especially Möbius transformations.

Integration along a path, Fundamental Theorem of Calculus. The Estimation theorem.

Connectedness and regions. The covering theorem.

(18 lectures)

MTH-2A32 : Advanced Calculus II

1. Introduction: This second semester unit follows on from **Advanced Calculus I** which is a prerequisite. It is in the mainstream of methods teaching for degree programmes in mathematics. The unit divides into two parts namely Complex Analysis, and Ordinary Differential Equations and Special Functions.

2. Timetable Hours, Credits, Assessments: This unit is of 20 UCU and is taught in the Spring Semester by means of 20 lectures (Complex Analysis) and 20 lectures (Ordinary Differential Equations and Special Functions). Each of the two parts of the unit is supported by 3 seminars. Assessment is by set regular coursework (20%) and a three-hour examination (80%) towards the end of the semester.

3. Overview: Complex Analysis and Differential Equations provide mathematical tools with which the edifice of applied mathematics is constructed.

This unit applies the complex variable theory established in module 2 of **Advanced Calculus I**. The Complex Analysis module of this unit is centred on Cauchy's residue theorem and its application to the evaluation of contour integrals. Laplace transforms are introduced and the inversion integral quoted to provide further examples of contour integration.

The unit in parallel resumes the development of differential equations started in module 3 of **Advanced Calculus I**. Some basic theory of linear ordinary differential equations on the real line precedes methods which use complex series. These methods for solution of linear ordinary differential equations in the complex plane are applied to the solution of Legendre's and Bessel's equations and determine associated functions. These are used to complete the solution of Laplace's equation by separation of variables.

4. Recommended Reading: The notes taken in lectures are intended to be complete and self-contained in themselves. A particularly useful book that provides coverage of all aspects of the unit is "Advanced Engineering Mathematics" by E Kreyszig (Wiley). Another useful book is "Introduction to Complex Analysis" by H A Priestley (Oxford) which covers all the complex analysis in the unit.

5. Lecture Contents:

5A. Complex Analysis:

Logarithm and its principal value, argument. Riemann surface examples log, $z^{\frac{1}{2}}$, $z^{\frac{1}{3}}$. Integrals involving branch cuts. Cauchy's Theorem for a contour and its proof. The deformation theorem. Cauchy's Integral formula. Taylor's theorem. Cauchy's formula for derivatives. The Identity theorem. The Maximum-Modular theorem. **(10 Lectures)**

Contour integral in C . Integration of z^m , for integer m , round simple closed curve. Residues and Cauchy's residue theorem. Techniques for finding residues. Summation of series. Contour integrals with poles off and on contour. Integrals along real line, and technique of closing contour at infinity. Integrals of trigonometric functions. Number of zeros, minus number of poles, formula. **(10 Lectures)**

5B. Differential Equations And Special Functions:

Linear ordinary differential equations on the real line. Existence and uniqueness for the n_{th}-order equation. Linear dependence; the Wronskian, Liouville/Abel formula. The complete solution, method of variation of parameters. Second-order equations. **(4 Lectures)**

Second-order linear equations in the complex plane. Series solutions. Singular points and the method of Frobenius. The indicial equation; equal roots and roots differing by an integer. Singularities at infinity. **(5 Lectures)**

Legendre polynomials and Bessel functions. Motivation from separable solutions of Laplace's equation. Application to the solution of the heat conduction equation. **(6 Lectures)**

Laplace transforms. Definition, elementary functions. Derivatives, integrals and the convolution theorem. Application to differential equations, systems and integral equations. Differentiation and integration of transforms. Inversion integral without proof, demonstrated for poles. **(5 Lectures)**

MTH-2A43 : Hydrodynamics I

1. Introduction: This first semester course at level 2 is an introduction to mathematical modelling in the context of fluid flows. It follows on from the first year course **Mathematical Methods II** and provides an introduction to **Hydrodynamics II** in the second semester and to the level 3 courses **Fluid Dynamics**, **Geophysical Fluid Dynamics**, **Waves** and **Dynamical Oceanography.**

2. Timetable Hours, Credits, Assessments: This unit is of 10 UCU and is taught in the Autumn Semester by 20 lectures, supported by 3 hours of examples classes. Assessment is by set regular coursework (20%) and examination (80%). For students taking both Hydrodynamics I *and* Hydrodynamics II there is a combined 3 hour examination. For other students there is a $1\frac{1}{2}$ hour examination.

3. Overview: Hydrodynamics has been used as the topic to introduce Mathematical Modelling for many years in Britain. It provides a variety of simple situations which can be described mathematically and for which the mathematical solutions are realistic. The course begins with the development of the equations needed to describe the flow in channels, over weirs in rivers, and through pipes, and ends with the theory of water waves on the surface of lakes and the ocean.

4. Recommended Reading:

The two recommend text books are "Elementary Fluid Dynamics" by D Acheson (Oxford UP) and "A first course in Fluid Dynamics" by A Paterson (Cambridge UP). A useful book, consisting of just pictures of fluid flows, is "An album of fluid motion" by M Van Dyke (Parabolic Press). Further physical background is found in D J Tritton "Physical Fluid Dynamics" (Oxford UP).

5. Lecture Contents:

(1) Introduction: Examples of flows in science and nature. Liquids, gases and fluids. Pressure, hydrostatics and Archimedes principle - full and partial immersion. Compressibility and density. **(2 lectures)**

(2) Kinematics: Velocity, particle path, streamlines. Mass continuity: Incompressibility, streamtubes. Pipe flow example. **(3 lectures)**

(3) Dynamics: Material derivative, Euler's equations, vorticity and irrotational flows. The existence of a velocity potential. Bernoulli's equation for unsteady flow. Examples for steady flow: in pipe, leakage under pressure, waterfall, fountain. Open channel flows: over weir, under sluice gate, and in hydraulic jump (Severn Bore) including energy loss. **(4 lectures)**

(4) Circulation, Kelvin's Theorem, Helmholtz's vortex theorems. Tornado. **(2 lectures)**

(5) Velocity potential, ϕ . Examples. $^{2}\phi = 0$. Uniqueness theorems for $^{2}\phi = 0$ under various boundary conditions. **(4 lectures)**

(6) Water Waves. Full nonlinear b.v.p. on moving domain. Linearised equations for water in uniform depth. Simple solutions which are periodic in space and time. Dispersion relation with surface tension. Wave speed as a function of wavelength in deep or shallow water. Particle motion under progressive wave in finite depth. Deep water case. Group velocity - wave groups. Superposition of linear progressive waves to form standing waves in a tank of finite length. Normal modes. Particle paths beneath standing waves. **(5 lectures)**

MTH-2A44 : Hydrodynamics II

1. Introduction: This second semester course at level 2 follows on from **Hydrodynamics I** and extends the mathematical modelling of fluid flows. It provides a natural introduction to the level 3 courses **Fluid Dynamics**, **Geophysical Fluid Dynamics, Waves** and **Dynamical Oceanography**.

2. Timetable Hours, Credits, Assessments: This unit is of 10 UCU and is taught in the Spring Semester by 20 lectures, supported by 3 hours of examples classes. Assessment is by set regular coursework (20%) and examination (80%). For students taking both Hydrodynamics I *and* Hydrodynamics II there is a combined 3 hour examination. For other students there is a $1\frac{1}{2}$ hour examination.

3. Overview: This course develops the ideas of **Hydrodynamics I** and introduces techniques necessary to model more complicated flows. Motivation is provided by problems such as the flow around a cylinder or sphere, prediction of lift on an aircraft wing, and the motion of systems of vortices, such as may be generated when stirring a cup of tea. The course begins by revising the key elements of **Hydrodynamics I** before introducing the concept of a stream function as a means of plotting streamlines. Much of the course is concerned with two-dimensional flows. The complex potential is introduced, which together with the study of conformal transformations allows the analysis of flow around more complicated geometries. Using complex potentials provides a method for calculating the motion of systems of line vortices using induced velocities. Finally the idea of stream functions in axially symmetric geometries is introduced.

4. Recommended Reading: The two recommended text books are "Elementary Fluid Dynamics" by D Acheson (OUP) and "A first course in Fluid Dynamics" by A Paterson (CUP), the same books as recommended for **Hydrodynamics I**. Another suitable text is "An informal introduction to Theoretical Fluid Dynamics" by J Lighthill (CUP). Illustrations of some of the fluid flows studied are provided in "An album of fluid motion" by M Van Dyke (Parabolic Press).

5. Lecture Contents:

Revision of Hydrodynamics I: Euler's equations. Conservation of Mass. Circulation and Vorticity. Velocity Potential. Unsteady Pressure Equation for Irrotational Flows. **(3 lectures)**

General Theory of Irrotational and Incompressible Flows: Laplace's Equation. Simple examples of velocity potentials for 2D and 3D flows, Sources and Line Vortices. **(2 lectures)**

Two-Dimensional Flows: Stream functions. Complex Potentials. **(3 lectures)**

Force on a body using integrals of pressure. Blasius Formula. **(2 lectures)**

Induced Velocity and Image Systems, including image system for a circular boundary. **(2 lectures)**

Conformal Mappings. **(3 lectures)**

Joukowski Airfoils and Lift Generation. **(2 lectures)**

Axisymmetric Flows: General Axisymmetric Solution to Laplace's Equation. Sphere in a uniform flow. **(3 lectures)**

MTH-2B71 : Mathematics for Geophysical Science I

1. Introduction: This Autumn Semester unit is designed for students of geophysical science who have already passed the first year applied mathematics pre-requisites **Mathematical Methods I** and **Mathematical Methods II**. This unit is the pre-requisite for **Mathematics for Geophysical Science II**, studied in the Spring. **Mathematics for Geophysical Science I** is taught by lectures and seminars, and computer laboratory sessions. The unit gives students some of the essential methods of applied mathematics. This unit covers matrix algebra for the solution of systems of linear equations and Markov chain modelling of competing populations. The unit goes on to cover the elements of complex variable theory. The calculus of vector fields is also treated. Programming using the Fortran 90 computer language is introduced in preparation for the 3rd year Geophysical modelling course and project work.

2. Timetable Hours, Credits, Assessments: This is a 20 UCU unit of 31 lectures with 6 seminars and 5 computer lab sessions. There are 5 modules : matrix algebra (8 lectures + 2 seminars), complex variable (8 lectures and 2 seminars), and vector calculus (8 lectures + 2 seminars), Fortran programming (7 lectures + 5 labs). Assessment is by 100% coursework (marked homework and a test).

3. Overview: Among other tools matrix algebra is essential for solving differential equations numerically. In many mathematical models of physical problems the equations can be turned into a set of linear relations with perhaps hundreds of unknowns. For example, the changes in the populations of competing animal species can be described by a set of recurrence relations. These are naturally written in the form of a matrix multiplied by the current population vector, to obtain the expected population at some fixed time in the future. The mathematics help us determine the long-term fate of the different species. Solving systems of linear equations involves Gauss elimination, elementary row operations, the consideration of determinants, ways of computing the inverse of a matrix, and finding it if the inverse exists.

The theory of functions of a complex variable is useful for evaluating difficult integrals and has many applications in applied mathematics, e.g. aerofoil theory.

The vector calculus module is essential for modelling quantities or processes which vary in two or three dimensions. The vector operators div, grad and curl are defined, and various vector identities are explored. The divergence and Stokes' theorems are introduced. These theorems are particularly useful in the study of geophysical fluid dynamics (see Geophysical Fluid Dynamics).

A structured approach to Fortran programming is introduced. This includes the use of functions and subroutines. The basic language elements are introduced, loops, conditional statements, arrays and input/output.

4. Recommended Reading:

T M R Ellis, I R Philips and T M Lahey "Fortran 90 Programming", (Eddison-Wesley)

E Kreyszig "Advanced Engineering Mathematics" (Wiley)

(This is available in the library)

5. Lecture Contents:

Matrix Algebra and Linear Equations Module

Introduction to matrices. Simple application (predator/prey population model). Systems of linear equations (introduced via two equations in two unknowns case). Solving systems of linear equations using elementary operations. Matrices and systems of linear equations. Gaussian elimination, row echelon form and back-substitution. Reduced row echelon form; use of rank to distinguish cases of no solution, unique solution, infinitely many solutions. **(4 lectures)**

Matrix inversion. Determinants and how to calculate them by expansion. Determinants and solutions to systems of linear equations (including homogeneous systems). **(2 lectures)**

Linear independence, calculation of eigenvalues and eigenvectors. Diagonalisation of matrices with linearly independent eigenvectors. Application of diagonalisation to population model. Long-term fate of populations. **(2 lectures)**

Complex Variable Module

Analytic functions and the Cauchy-Riemann equations. Conformal mapping, examples. Complex line integrals and Cauchy's integral theorem. Cauchy's integral formula, its derivatives and the evaluation of complex line integrals. Residues. **(8 lectures)**

Vector Calculus Module

Scalar fields, gradient, directional derivative, level surfaces. **(2 lectures)**

Vector fields, divergence, Laplacian, curl, divergence theorem, Stokes's theorem, solenoidal vector fields, irrotational vector fields. **(6 lectures)**

Fortran Programming Module

Introduction to computing and the UNIX operating system. Program, structure and documentation. Declaration of variables. Arithmetic, simple input-output. Structured programming. Intrinsic and external functions; Decision and repetition structures. Arrays. Formatted output. Subroutines. **(7 lectures)**

MTH-2B72 : Mathematics for Geophysical Science II

1. Introduction: This is a sequel to **Mathematics for Geophysical Science I** and provides an introduction to Mathematical Modelling using the mathematical techniques developed in **Mathematics for Geophysical Science I**. It covers the same material as the course **Mathematics II** for Environmental Science students. The unit provides mathematical solutions that should be useful in ENV courses on Oceanography, Meteorology, Geophysics and Geophysical Modelling. It may be followed by advanced level 3 applied mathematics courses. A discussion of Fourier series arises from the solution of the partial differential equations of mathematical physics.

2. Timetable Hours, Credits, Assessments: This unit is of 20 UCU and is taught in the Spring Semester by 39 lectures in two parallel streams. It is supported by 5 example classes and 1 computing laboratory session. Fourier Series (5 lectures + 1 seminar + 1 lab), partial differential equations (6 lectures + 1 seminar). Assessment is by set regular homework (20%) and an examination (80%).

3. Overview: This unit has four topics which all aim to develop skills in mathematical modelling. The topics are Fluid Dynamics, Fourier Series, Partial Differential Equations and Solid Mechanics. This course shows how mathematics can be applied to the environmental and geophysical sciences.

Fourier series are useful for analysing time series and solving partial differential equations. The concept of representing a function by a sum of appropriate trigonometric functions is developed, leading to examples of whole range and half range Fourier series. The range of validity, and the behaviour of Fourier series are explored in a laboratory class, using the computer algebra package MAPLE.

The solution of partial differential equations is one of the cornerstones of geophysical modelling. Analytic techniques based on the separation of variables, allied to Fourier series, are used to solve prototype problems in wave motion, unsteady heat conduction, and steady heat flow.

4. Recommended Reading: The following books are recommend for different parts of the course.

E Kreyszig "Advanced Engineering Mathematics" (Wiley) (This is available in the library)

D Tritton "Physical Fluid Dynamics"

R O Davis and A P S Selvadurai "Elasticity and Geomechanics", (CUP 1996)

5. Lecture Contents:

Fluid Dynamics : Introduction. Pressure. Material derivative. Two-dimensional flows. Conservation of mass. Bernoulli's equation. Flow over weir. Hydraulic jumps. Lift. **(5 lectures)**

Three-dimensional flows. Euler's equations. Boundary conditions. Stream function. Vortex tubes. Circulation. Irrotational flow. Kelvin's circulation theorem. **(5 lectures)**

Velocity potential. Uniform stream. Line source. Dipole. Line vortex. Combined examples. Lift. **(4 lectures)**

Water waves. Linear surface gravity waves. Phase and Group speed. Deep water and shallow water approximation. Particle paths. **(4 lectures)**

Fourier Series : Taylor's series, convergence of series, Fourier's theorem for piece-wise smooth functions on $(-\pi , \pi)$, comments, examples, periodic extension outside $(-\pi , \pi)$, summing series, extension to the interval $(-\ell , \ell)$. Even and odd functions, half-range Fourier sine and cosine series on the interval $(0 , \ell)$, examples. Lab session using Maple to illustrate Fourier Series. **(5 + 1 Lab)**

Second-order partial differential equations : Basic concepts and definitions, wave equation, heat conduction equation, Laplace's equation. The method of separation of variables. Application of this method for solving initial/boundary value problems associated with the aforementioned P.D.E.'s. D'Alembert's solution for the wave equation. Application for solving initial value problems for the wave equation on infinite and semi-infinite domains. **(6 lectures)**

Solid Mechanics : Theory of stress. Traction on a surface, resultant force and torque, in terms of components of stress. Normal and shear stresses. Principle stresses and axes of stress. Equations of equilibrium and symmetry of stress matrix. Displacement, strain and small deformations. **(6 lectures)**

Elasticity. Linear stress-strain relations. Navier equations of equilibrium and motion. Elementary examples. Propagation of plane P and S waves. **(4 lectures)**

MTH-2B81 : Mathematics for Physical Scientists III

1. Introduction: This Autumn Semester unit is designed for students of physical science who have already passed the first year mathematics pre-requisites **Mathematics for Scientists I** and **II**. This unit is the pre-requisite for **Mathematics for Physical Scientists IV**, studied in the Spring. **Mathematics for Physical Scientitsts III** is taught by lectures and seminars. The unit goes on to cover the elements of complex variable theory. The calculus of vector fields is also treated.

2. Timetable Hours, Credits, Assessments: This is a 10 UCU unit of 16 lectures with 4 seminars. There are 2 modules : complex variable (8 lectures + 2 seminars) and vector calculus (8 lectures + 2 seminars). Assessment is by 100% coursework (marked homework and a test).

3. Overview: The theory of functions of a complex variable is useful for evaluating difficult integrals and has many applications in applied mathematics, e.g. aerofoil theory.

The vector calculus module is essential for modelling quantities or processes which vary in two or three dimensions. The vector operators div, grad and curl are defined, and various vector identities are explored. The divergence and Stokes' theorems are introduced. These theorems are particularly useful in the study of fluid dynamics.

4. Recommended Reading:

E Kreyszig "Advanced Engineering Mathematics" (Wiley)

This is available in the library.

5. Lecture Contents:

Complex Variable Module

Analytic functions and the Cauchy-Riemann equations. Conformal mapping, examples. Complex line integrals and Cauchy's integral theorem. Cauchy's integral formula, its derivatives and the evaluation of complex line integrals. Residues. **(8 lectures)**

Vector Calculus Module

Scalar fields, gradient, directional derivative, level surfaces. **(2 lectures)**

Vector fields, divergence, Laplacian, curl, divergence theorem, Stokes's theorem, solenoidal vector fields, irrotational vector fields. **(6 lectures)**

MTH-2B82 : Mathematics for Physical Scientists IV

1. Introduction: This is a sequel to **Mathematics for Physical Scientists III** and provides an introduction to Mathematical Modelling using the mathematical techniques developed in **Mathematics for Physical Scientists III**. The unit provides mathematical solutions for fluid flow problems.

2. Timetable Hours, Credits, Assessments: This unit is of 10 UCU and is taught in the Spring Semester by 18 lectures in two parallel streams. It is supported by 3 example classes. Assessment is by 100% coursework (marked homework).

3. Overview: This unit aims to develop skills in mathematical modelling and in particular modelling of fluid flows with reference to the environment, aerodynamics and waves.

4. Recommended Reading:

D Tritton "Physical Fluid Dynamics"

5. Lecture Contents:

Fluid Dynamics: Introduction. Pressure. Material derivative. Two-dimensional flows. Conservation of mass. Bernoulli's equation. Flow over weir. Hydraulic jumps. Lift. **(5 lectures)**

Three-dimensional flows. Euler's equations. Boundary conditions. Stream function. Vortex tubes. Circulation. Irrotational flow. Kelvin's circulation theorem. **(5 lectures)**

Velocity potential. Uniform stream. Line source. Dipole. Line vortex. Combined examples. Lift. **(4 lectures)**

Water waves. Linear surface gravity waves. Phase and Group speed. Deep water and shallow water approximation. Particle paths. **(4 lectures)**

MTH-2F11 : Discrete Mathematics I

1. Introduction: Discrete Mathematics underpins many branches of mathematics, computing and the sciences more generally. This introduction has no formal prerequisites and is therefore suitable for students from outside the School.

2. Hours, Credits and Assessment: The course is a 10 UCU unit of 15 lectures with 3 seminars. Assessment is by coursework (20%) consisting of 3 homework sheets and examination (80%).

3. Overview: We introduce the idea of error-correcting codes and develop the necessary tools from algebra, finite fields in particular. Error-correcting codes are fundamental in all data transmission systems: Transmission through noisy channels, from telephone cables to computer links or radio signals travelling huge distances, inevitably introduce errors to the signal sent. If these were allows to go undetected whole systems would come to a halt quite quickly. The idea behind error correcting codes is to encode the message with redundancy sufficient to detect errors of certain kinds. This theory is now very well developed and has interesting applications in mathematics, computing and the natural sciences

Much of this is based on finite fields. These are finite analogues of the more familiar rational or real number systems in which the basic algebraic operations of addition and multiplication work in the usual way. Finite fields, however, have some remarkable properties which make them quite distinct from other number systems. For instance, every map from a finite field into itself is given by a polynomial, and so functions can be studied effectively by algebraic tools. For the same reason systems of equations are given by polynomials and this leads to many practical applications of finite fields.

4. Recommended Reading: The literature on the subject is extensive, the following are more closely related to the course and can be found in the library:

NL Biggs, Discrete Mathematics, OUP (several chapters from this book can be used for this course, a useful all-round text);

JB Fraleigh, Abstract Algebra, (several chapters can be used for this course);

PJ Cameron, Introduction to Algebra, Oxford Science Publications, (several chapters can be used for this course);

O Pretzel, Error Correcting Codes and Finite Fields: Student Edition, Oxford Applied Mathematics and Computing Sciences Series, (comprehensive introduction);

J van Lint, Introduction to Coding Theory, Springer Verlag, (a comprehensive treatment of coding theory);

R Lidl and H Niederreiter, Introduction to Finite Fields and their Applications, CUP (advanced text on finite fields, codes, linear recurring sequences),

5. Contents:

5.1 Introduction:

Examples of error correcting codes and their use. A mini-project. **(2 Lectures)**

5.2 Algebraic Tools:

Finite Fields I: Modular arithmetic, the Euclidean algorithm for integers and the construction of GF(p). Computations and examples. **(1 Lecture)**

Finite Fields II: Polynomials over a finite field and the use of the Euclidean algorithm to construct fields of prime power order. Do fields of order 4, 8, 9 and work out their arithmetic. Primitive elements and subfields of a finite field. **(2 Lectures)**

Polynomials and functions: Prove that all functions of the field into itself are polynomial. Find a general formula about representing a given function as a polynomial. **(2 Lectures)**

Polynomial equations: Counting the number of solutions of certain polynomial equation, (Mention the theorem of König-Rados or of Warning-Chevalley if time allows.) **(2 Lectures)**

5.3 Error correcting codes

Block codes: Hamming distance. Nearest neighbour decoding and general bounds on codes. Hamming Bound, Singleton Bound. **(2 Lectures)**

Linear codes: Generator and check matrix, minimum weight. Hamming codes. **(2 Lectures)**

Cyclic Codes: Construction as ideals in factor rings. BCH codes. **(2 Lectures)**

MTH-2F12 : Discrete Mathematics II

1. Introduction: This unit provides an introduction to Computability: a topic in Mathematical Logic. It is largely self-contained and is suitable for non-Mathematics students with an appropriate background.

2. Timetable Hours, Credits, Assessments: The course is a 10 UCU unit of 15 lectures with 3 seminars. Assessment is by coursework (20%), consisting of 3 homework sheets, and an examination (80%).

3. Overview: Mathematical Logic is concerned with the foundations of Mathematics. It analyses symbolically the way in which we reason formally, particularly about mathematics. The subject was developed last century, and although its origins are in Pure Mathematics, it is fundamental in computer science and artificial intelligence.

The course is concerned with **computability**. It addresses issues such as which functions can be computed by a mechanical process or computer program. This is idealised in the very precise notions of an **unlimited register machine (URM)** or a **Turing machine**. A related issue is whether it is theoretically possible to write a computer program which will decide whether any mathematical statement is true or false. The course will give examples to show that this is not possible: there are mathematical problems which are **undecidable**.

4. Recommended Reading:

1) A G Hamilton "Logic for Mathematicians", CUP

2) E Mendelson "Introduction to Mathematical Logic", Chapman and Hall

3) N Cutland "Computability", CUP

4) R Soare "Recursively enumerable degrees" (Chapters 1-3)

The lectures will follow (3) quite closely with some material from (1). (2) and (4) are more advanced.

5. Lecture Contents:

URMs and some examples of functions computable by them. Subroutines, recursion, minimalisation. **(7 lectures)**

Goedel numbers, universal programs and some undecidable problems. **(4 lectures)**

Other approaches: Turing machines, recursive functions. The Church-Turing thesis. **(2 lectures)**

Undecidable problems in mathematics. The word problem for semigroups. **(2 lectures)**

MTH-2F41 : Introduction to Wave Motion

1. Introduction: This unit is an optional unit at level 2 which is available not only to single subject, three and four year, first degree programmes in mathematics, but also joint programmes and students with a suitable mathematical background.

2. Timetable Hours, Credits, Assessments: This unit is of 10 UCU and is taught in Semester II by means of 15 lectures and is supported by three seminars. Assessment is by set regular coursework (20%) and examination (80%).

3. Overview: Waves are everywhere. Water waves on a pond are easily generated. Large ships, storms and tides generate waves on the ocean. Sound waves due, for example, to speech or to a stringed or wind instrument, propagate through the atmosphere. Waves also propagate in solid materials; for example seismic waves in the earth's crust due to earthquakes. In this unit transverse waves on strings and membranes, longitudinal waves in solid bars and 3D elastic waves, are studied. Much of the underlying mathematics, for small amplitude waves, is common to all of these systems. As a consequence particular attention is paid to waves on strings.

4. Recommended Reading:

(1) J Billingham & A C King "Wave Motion" (CUP)

(2) E Kreyszig "Advanced Engineering Mathematics" (Wiley)

5. Lecture Contents:

Waves on a stretched string. Equation of motion. Solution by separation of variables to motivate d'Alembert's solution. Initial conditions. Kinetic and potential energies. Wave reflection from a density discontinuity. Finite string, solution represented by an infinite series of normal modes. Energies. End and initial conditions, including fixed-fixed and fixed-free, for example string attached to a ring, end conditions. Effect of air resistance. **(7 lectures)**

Waves in membranes. Equation of motion, solution by separation of variables, representation of displacement as a doubly-infinite series of normal modes. Degenerate modes. Kinetic and potential energies. **(3 lectures)**

Longitudinal waves in bars. Equation of motion for a Hookean solid. Solution by separation of variables, representation of displacement as superposition of normal modes. Initial and end conditions for finite length including fixed-fixed and fixed-free conditions. **(2 lectures)**

Elastic waves in three dimensions. Primary and secondary waves. Reflection and refraction. Rayleigh surface waves. Love waves. **(3 lectures)**

MTH-2F42 : Mathematics of Diffusion

1. Introduction: Taught by lectures and seminars. This is an applied mathematics unit for students who have already studied first-year university-level mathematics. The work derives appropriate partial differential equations for the flow of heat or diffusion of matter, and solves these linear equations in one, two or three space dimensions and time. The work uses separation of variables, Fourier series, integral representations of solutions and other main stream parts of applied mathematics.

2. Timetable Hours, Credits, Assessments: This is a 10 UCU unit and is given in Semester II by means of 15 hours of lectures and supported by 3 hours of seminars. The assessment is by 20% coursework and 80% examination. The coursework consists of written answers to set exercises, submitted by given deadlines.

3. Overview: The contents have been chosen to show how the phenomena of diffusion can be translated into a precise mathematical description or model. This was first done by Joseph Fourier in 1820, in the form of a linear partial differential equation. Fourier invented Fourier analysis to help solve the heat equation on a finite length of conducting bar. We examine these initial-boundary value problems and extend ideas into solutions for infinite and semi-infinite domains, primarily in one and three space dimensions. The mathematical solutions can be applied to a wide range of problems from the spread of industrial contaminants to the movement of perfume in still air.

4. Recommended Reading:

(i) E Kreyszig "Advanced Engineering Mathematics", Wiley.

§11.5 and §11.6 introduces the material well.

(ii) J Crank "The Mathematics of Diffusion", Oxford University Press.

(iii) H S Carslaw "Conduction of heat in solids", Oxford University Press.

& J C Jaeger

5. Lecture Contents:

§1.1 Introduction. Examples of diffusion. The concept of flux. Derivation of the general unsteady heat equation in 3-space. Importance of the diffusion coefficient. Boundary conditions. **(2 lectures)**

1.2 Uniqueness of solutions via integral theorems. Conservation of heat (or mass of substance). Boundary conditions related to physical concepts. **(1 lecture)**

§2.1 1-space dimension. Steady solutions are linear with constant heat flux. Double glazing and piecewise linear solutions. **(1 lecture)**

2.2 Unsteady solutions for x [0,L] obtained by separation of variables. Eigenvalues obtained from boundary conditions. **(1 lecture)**

2.3 Revision of Fourier analysis. Series solutions on x [0,L] for initial value problems. **(2 lectures)**

2.4 Examples of §2.2 and §2.3. **(1 lecture)**

2.5 Solution of initial delta function on x (–•,•) . **($\frac{1}{2}$ lecture)**

2.6 General solutions for x (–•,•) for initial-value problems. **($\frac{1}{2}$ lecture)**

2.7 Semi-Infinite bar with time harmonic boundary condition (diurnal heating). Use of Laplace Transform shown by example. **(1$\frac{1}{2}$ lecture)**

§3.1 Separation of variables in 2 and 3 space dimensions. Steady heat flow and Laplace's equation briefly covered. **(1 lecture)**

3.2 Spherical polar coordinates. Spherically-symmetric solutions in r and t . Reduction to the 1D heat equation in r and t . Steady temperature distributions. **(1$\frac{1}{2}$ lectures)**

3.3 Presence or absence of singularity at r = 0 . Internal and external problems with appropriate conditions at r = 0 , on a sphere's surface and at r = • . **(2 lectures)**

MTH-2S53 : Statistics I

1. Introduction: A course in basic statistical inference. **Statistics I** covers estimation and **Statistics II** (2S55) testing. They have to be taken in sequence. The aim is to introduce some essential concepts of inference.

2. Timetable Hours, Credits, Assessments: The course is a 10 UCU unit of 14 lectures. Assessment is coursework (20%) and examination (80%). Students are given an extensive sheet of sample questions with worked solutions on each module.

3. Overview: In Statistics I we introduce some further distribution theory and then move to the idea of the likelihood function. We discuss estimation of population parameters in some detail.

4. Recommended Reading: Students should buy *one* of the following texts: Mathematical Statistics with Applications by Wackerly, Mendenhall and Scheaffer (Duxbury) and Applied Statistics and Probability for Engineers by Montgomery and Runger (Wiley). These are also useful for **Applied Statistics**.

5. Lecture Contents:

Joint Distributions. **(1 lecture)**

Conditional/Marginal distributions. **(1 lecture)**

Moments. **(1 lecture)**

Correlation. **(1 lecture)**

Moment generating functions and sums of independent random variables. **(2 lectures)**

Sums of random variables-Central limit theorem. **(1 lecture)**

Estimates, simple properties. **(1 lecture)**

Likelihood, likelihood based ideas. **(1 lecture)**

Maximum likelihood estimates (mles). **(1 lecture)**

Cramer-Rao bound and the asymptotic distribution of mles. **(2 lectures)**

Confidence sets. **(2 lectures)**

MTH-2S55: Statistics II

1. Introduction: A course in basic statistical inference. **Statistics II** covers testing and follows on from **Statistics I** (2S53) which covered estimation. They have to be taken in sequence. The aim is to introduce some essential concepts in inference.

2. Timetable Hours, Credits, Assessments: The course is a 10 UCU unit of 16 lectures. Assessment is coursework (20%) and examination (80%). Students are given an extensive sheet of sample questions with worked solutions on each module.

3. Overview: Much of classical statistics was developed in the context of "testing". In this course we look at the idea of tests and study ways of constructing tests. As a comparison we also consider a Bayesian view.

4. Recommended Reading: Students should buy *one* of the following texts: Mathematical Statistics with Applications by Wackerly, Mendenhall and Scheaffer (Duxbury) and Applied Statistics and Probability for Engineers by Montgomery and Runger (Wiley).

5. Lecture Contents:

Ideas of testing. **(1 lecture)**

Neyman Pearson, UMP tests. **(3 lectures)**

G.Likelihood ratio. **(3 lectures)**

Applications to contingency tables. **(3 lectures)**

Derivation of standard tests. **(2 lectures)**

Bayesian Inference. **(4 lectures)**

MTH-2S56 : Applied Statistics

1. Introduction: An introduction to regression and analysis of variance. The prerequisite is **Statistics II**.

2. Timetable Hours, Credits, Assessments: The course is a 20 UCU unit of 30 lectures. Assessment is via an extended project (40%) and examination (60%). Each project is presented by an introductory session given by the lecturer. This is a fairly practical course with one hour a week practical class.

3. Overview: This is a course on statistical modelling - primarily using the linear model. The aim is fairly practical - we aim to use the techniques discussed on realistic data sets.

4. Recommended Reading: **Statistics I** provides some background. A reading list is provided at the beginning of the course together with some advice on report writing. The reading list changes to reflect the topics in the courses.

5. Lecture Contents:

Conditional expectation. The regression function. **(1 lecture)**

Simple linear regression. **(1 lecture)**

Vector random variables (notation). **(2 lectures)**

Setting up regression in matrix form, examples. **(1 lecture)**

Solution in full rank case. **(1 lecture)**

Examples. Questions of inference. **(1 lecture)**

Tests based on linear forms in coefficients. **(2 lectures)**

Decompositions of sums of squares, F tests. **(2 lectures)**

Diagnostics. **(1 lecture)**

Factors. **(1 lecture)**

Non-full rank case (ANOVA). **(1 lecture)**

Solutions of normal equations. **(1 lecture)**

Inference in ANOVA. **(2 lectures)**

Examples. **(1 lecture)**

Analysis of covariance. **(2 lectures)**

Project. **(10 lectures)**