MTH-1B24 : Multivariable Calculus

Hours, Credits and Assessment: The course is a 10 UCU unit of 20 lectures taught in the Spring Semester. Support teaching is via seminars. Assessment is by coursework (20%) and a 2 hour examination (80%).

Overview: Partial derivatives, chain rule. Line integrals. Multiple integrals, including change of co-ordinates by Jacobians. Green's theorem in the plane. MAPLE integrated throughout.

Recommended Reading and References:

Guide to Mathematical Methods – Gilbert & Jordan (Palgrave Macmillan)

Lecture Contents:

Partial Differentiation: Real function of two or more real variables. Equation of a surface; contour plots. Definition of first partial derivative; geometric interpretation for two variables. Product and quotient rule. Higher order derivatives; equality of mixed derivatives. Directional derivative and total derivative. Rate of change along a curve.

Change of variables. Chain rule. Taylor's theorem for function of two variables. Stationary points; classification.

Multiple Integrals: Line Integrals. Double integrals interpreted as sum over region of plane: Limits which vary. Examples. Change of variables, Jacobians. Double integrals in polar coordinates e.g. exp $(-x^2)$. Green's Theorem in the plane. Triple integrals for calculating volume or mass - cylindrical and spherical coordinates. Surface integrals.

(20 lectures)

MTH-1B27: Calculus

1. Introduction: This unit is for students on joint courses with Mathematics, such as Meteorology and Oceanography or Geophysics. The unit is the first of a succession of level 1 and level 2 units that prepare for year 3 units in applied mathematics.

2. Hours, Credits and Assessment: This unit is of 20 UCU and is taught in the Autumn semester by means of 42 hours of lectures, four per week, supported by 6 hours of seminars. Assessment is by homework (20%) at fortnightly intervals and by a course test (80%) in the final week of the semester.

3. Overview: The origins of the integral calculus can be traced back to ancient times. The process of differentiation arose in modern times in the seventeenth century, to calculate the slope of a plane curve, and to define rates of change, such as the variable velocity of a falling particle. The inter-relation between differentiation and integration was discovered by Newton and Leibniz, to prepare for the role of the calculus as the language and principal tool of mathematical science. Newton's development of Calculus made the theory and use of Differential Equations possible. Vector algebra is designed to describe entities that have an intrinsic direction in three-dimensional space as well as a magnitude (such as line segments, displacements, velocities, forces etc). Complex numbers were introduced in the sixteenth century to help construct solutions of cubic equations, but are now a necessary to many scientific subjects at higher levels.

4. Recommended Reading and References: The notes taken in lectures from the blackboard are intended to be complete and self-contained in themselves, and do not as a rule follow closely the order, content or style of a particular printed textbook. There are a large number of books for sale or in the library. A particular book which is close in scope and style to the lecture course is "Guide to Mathematical Methods" by J. Gilbert and C. Jordan, the 2nd edition published by Palgrave Macmillan, as a paperback at £19.99, with ISBN 0-333-79444-3. Second-hand copies of the 1st edition (by J. Gilbert) are suitable as well.

5. Lecture Contents:

Differentiation of a function of one variable: Definition, motivated by rate of change and gradient of a curve. Maxima and minima, concavity, curve sketching. Standard elementary functions: exponential and logarithm, trigonometric, hyperbolic. Rules and examples for differentiation of a sum, product and quotient of functions, a function of a function, the inverse of a function, parametric and implicit relations, nth derivative of a product by Leibniz formula and by induction. **(8 lectures)**

Integration of a function of one variable: Indefinite integral as inverse of differentiation. Definite integral: area under curve, and properties, reversal of limits, infinite upper limit etc. Methods: change of variable, integration by parts, reduction formulae, partial fractions for rational integrand. Length of plane curves. Volume of a solid of revolution. **(9 lectures)**

Power Series: Geometric and general power series, examples, reference to radius of convergence. Properties: addition, multiplication, differentiation and integration of power series. Taylor and Maclaurin series of a function, applications. Indeterminate quotients, l'Hopital's rule. **(3 lectures)**

First-order differential equations of the following type: separable, homogeneous, quasi-homogeneous, linear, numerical solutions using Maple. A geometric view of solutions using Maple. Second-order (and higher order) linear differential equations with constant coefficients: complementary function, particular integral (by trial substitution). **(12 lectures)**

Basic properties of vectors, notation, algebra, midpoints, Unit vectors, scalar product and geometric interpretation, Vector product and geometric interpretation, Equations of lines and planes, Orthogonal projections. **(5 Lectures)**

Real and Imaginary numbers, standard notation, algebra, multiplicative inverses. The Argand Diagram, describing subsets of the complex numbers graphically. The triangle inequality Polar form, de Moivre's theorem, The exponential function, Finding nth roots. **(5 Lectures)**

MTH-1B81 : Mathematics for Scientists I

1. Introduction: This unit is designed for candidates from outside the School of Mathematics who have reached the A-level or equivalent standard in mathematics and who are candidates for degrees in scientific or other subjects in other Schools.

2. Timetabled Hours, Credits, Assessment: The unit is of 10 UCU and is taught in the Autumn Semester by means of 20 hours of lectures supported by 4 hours of seminars in which candidates are given help with and answers to set homework questions. Assessment is normally by means of set homework and a one-hour course test in week 12. There is no formal university examination.

3. Overview: The course revises briefly the A-level calculus and proceeds to more advanced topics in the calculus and in the algebras of complex numbers and vectors. It is the first of a sequence of four level 1 and 2 units: **Mathematics for Scientists I, II** and **III** that introduce the analytical techniques necessary for the mathematical modelling of physical phenomena, such as the fluid flows of **Mathematics for Scientists IV**, for example.

4. Recommended reading: The notes taken in lectures from the blackboard are intended to be complete and self-contained in themselves, for the purposes of the Assessment. The lecture topics are basic, long established ones that can be found in a large number of books for sale or in the library. The particular book "Guide to Mathematical Methods" by J. Gilbert and C. Jordan (2002, published by Palgrave) can be recommended as close in scope and style to the lecture contents in six of its chapters, while the remaining three chapters are relevant to much of the succeeding unit, Mathematics for Scientists II. It is helpful also to refer to a formula book such as "Mathematical Formulae for Engineering and Science Students" by S Barnett and T M Cronin (Longmans).

5. Lecture contents:

A. Differentiation of a Function of One Variable : Definition; application to gradient of a curve, maxima and minima, rate of change. Differentiation of exponential, log, circular, hyperbolic functions and their inverses. Rules for product, quotient, function of a function, parametric relation. **(3 lectures)**

B. Integration of a Function of One Variable : Indefinite (inverse of differentiation) and definite (as limit of sum, area under curve) integration and their relationship. Properties (reversal of limits etc). Methods: change of variable, parts, partial fractions. Arc length of plane curve. Volume of a solid of revolution. **(4 lectures)**

C. Vector Algebra : Addition law, components. Scalar and vector products. Relative velocity problems. **(4 lectures)**

D. Power Series Expansions : Geometric series. Maclaurin series for exponential, logarithmic, binomial, circular and hyperbolic functions. Taylor's series. L'Hôpital's rule for limit of indeterminate quotient. **(3 lectures)**

E. Partial Differentiation : Definition. Total derivative, errors. Chain rules. **(3 lectures)**

F. Complex Numbers : Solution of quadratic equations to motivate. Definition and operations (equality, sum, product, quotient, conjugate, modulus and argument). Argand diagram. polar form. De Moivre's theorem; application to trigonometric formulae. Square root of a complex number. **(3 lectures)**

MTH-1B82 : Mathematics for Scientists II

1. Introduction: This is a level 1 Spring Semester unit which follows on **from Mathematics for Scientists I** which is a prerequisite. The unit aims to introduce ideas and techniques which are particularly relevant to the physical sciences. It includes: ordinary differential equations, line and multiple integrals, matrices and linear equations, and partial differential equations.

2. Hours, Credits and Assessment: The unit is 10 UCU consisting of 20 lectures plus 4 support classes which are primarily devoted to exercises issued in the lectures. It consists of 4 modules on the topics listed above. Assessment is 100% Coursework.

3. Overview: Line and multiple integrals are concerned with summation of a continuously distributed variable. Thus they include volumes and surface area for curved as well as flat surfaces; but they also cover quantities with non-uniform concentration, material flux and work following a curved path. There is a calculus for manipulation of these integrals.

Matrices are linked to arrays of coefficients of linear algebraic equations. These can be illustrated geometrically but have many applications. Although exercises involve small arrays of numbers some concepts extend readily to very large arrays. Rules for matrix manipulation are developed; determinants are also covered.

Differential equations are equations concerning rates of change and ordinary differential equations involve only one independent variable. For equations involving only first derivatives both linear and non-linear equations are covered. For equations involving higher derivatives only linear equations are covered; linearity allows synthesis of solutions and this greatly simplifies procedures.

Partial differential equations involve two or more independent variables, typically space and time. The main focus is on reducing the solution of linear partial differential equations to the solution of ordinary differential equations and then synthesising solutions. Applications include diffusion and wave processes.

4. Recommended literature and references: The first text covers the material, the second text develops topics to a higher level and is used in subsequent units:

G Stephenson	" Mathematical Methods for Science Students" (Longman)
E Kreyzsig	"Advanced Engineering Mathematics" (Wiley)

5. Contents:

Ordinary Differential Equations
First order. Separation of variables. Reductions to separable form. Homogeneous type. Linear, integrating factor. Second order, constant coefficients. Homogeneous equation, exponential solutions, real and complex conjugate roots of auxiliary equation. Repeated roots. Non-homogeneous equation, particular integral construction. **(5 lectures)**

Further Integration
Double integrals. Evaluation by repeated integration. Polar co-ordinates. Evaluation of volume. Triple integrals. Line integrals. Length integrals. Area of surface of revolution. **(5 lectures)**

Linear Algebraic Equations
Solution by elimination. Determinants and Cramer's rule. Non-homogeneous and homogeneous systems. Determinant expansion rules. Matrices: addition, multiplication by a scalar, matrix multiplication, transpose. Identity matrix and powers of square matrices. Non-singular and singular matrices. Inverse. **(5 lectures)**

Introduction to partial differential equations
Heat conduction as motivation. Linearity. Integration of advection (1st order) and wave equation (2nd order) to illustrate the arbitrary functions in general solutions; initial and boundary value problems are required for determined solutions. **(1 lectures)**

Separation of variables
Application to the diffusion equation and the wave equation, with trigonometric values prescribed initially and various homogeneous boundary conditions. Illustration of waves on a stretched string. The relation between standing waves and progressive waves. Application to the potential equation, with trigonometric values prescribed, exploiting superposition. Orthogonality technique for non-trigonometric prescribed values presented as a 'drill'. **(4 lectures)**

MTH-1C14 : Analysis and Algebra

Hours, Credits and Assessment: The course is a compulsory (MTH students), 15 UCU unit of 30 lectures taught in the Spring Semester. Support teaching is via seminars. Assessment is by coursework (20%) and a 2 hour examination (80%).

Overview: Analysis is that part of mathematics concerned with functions and graphs. Throughout the 17th and 18th centuries, problems in mathematics and physics were being successfully solved using ideas that we now call Differential Equations, Fourier Analysis, and Fixed Point Theorems. Despite these successes, much of the work was not accurate and many basic questions were left unanswered. For example: What are the real numbers? Which functions have Taylor expansions? Do all differential equations have solutions?

During the 19th century these problems were formulated precisely and solved. The study of analysis begins with a deep understanding of the real numbers, and then goes on to study functions and how they behave. These developments resulted in the "rigorisation" or "arithmetisation" of the calculus. The course will provide you with the basic tools to rigorously examine functions, graphs, and differential equations.

Part I. Sequences and Series: The study of analysis begins with a deep understanding of the real numbers **R**. In the first semester, we saw that **R** satisfies the axioms for a field but it also has an additional property: any set of real numbers all of which are less than some fixed number has a *least upper bound*. (This says that **R** is a *complete ordered field*.) This basic property is the key tool to unlock properties of *sequences* (infinite lists $x_1, x_2, x_3, \ldots$ of numbers) and *series* (infinite sums, like 1+1/2+1/3+1/4+... or 1+1/4+1/9+1/16+...). We give an exact definition of what we mean by *convergence* of a sequence (that is, the notion of tending towards a limit) and develop tests for convergence of sequences and rules for manipulating convergent sequences. Using these results we develop a precise language for talking about series and tests to determine when they converge.

Part II. Real Analysis: We now go on to study functions and how they behave. In particular, we look at the notions of limits, continuity, differentiability and integration, which are the basic tools needed to examine rigorously functions, graphs, and differential equations.

Part III. Algebra: The final part of the course introduces the notion of a *group*, which is among the most important algebraic structures and which appears in most branches of mathematics. Modern algebra bases itself on axiomatic systems where theories are developed from axioms. This is true particularly for Group Theory. One of its roots, and influential in its development, has been the study of geometrical transformations and symmetry. The solution of algebraic equations was also influential in the historical development of group theory. Group Theory is therefore very close to these fundamental mathematical notions. It is also a subject where theory and axiomatics interacts with intuition and insight in fascinating ways. Axiomatic methods are not always easy to get used to. Group Theory is a field where one can see their full power and at the same time be close to concrete applications.

Recommended literature and references: There are many books in the library which will cover the material in the course; the classmarks for Analysis are QA300 and QA303, and those for Abstract Algebra are QA154, QA162 and QA266.

For the analysis, the following books are suitable:
Hart, F.M., *Guide to Analysis,* Basingstoke: Palgrave. [QA303HAR]
Kopp, P.E., *Analysis*, Arnold. [QA303 KOP in short loan collection]

Spivak, M., *Calculus*, Publish or Perish. [QA303 SPI]

If you want to buy one then Hart's book is the more appropriate: Spivak's book is excellent but very expensive. For the algebra, you might try (from the library – these are all rather expensive):

Smith, G., *Introductory Mathematics: Algebra and Analysis*, Springer.[QA 154.2 SMI]
Herstein, I.N., *Topics in Algebra*, Blaisdell. [QA154 HER on shelf/short loan collection]
Herstein, I.N., *Abstract Algebra*, Wiley. [QA162 HER on shelf/short loan collection]
Fraleigh, J.B., *A first course in abstract algebra*, Addison-Wesley. [QA266 FRA]
Dean, R.A., *Elements of abstract algebra*, Wiley. [QA266 DEA]

Lecture Contents:

Introduction: The real numbers as a complete ordered field. Use of the modulus $| \ |$ to give exact definitions for convergence of a sequence of real numbers. A bounded increasing sequence converges. **(2 lectures)**

Sequences: Examples of convergent sequences. Algebra of convergent sequences. Euler's number and e as an example. **(3 lectures)**

Series: Convergence for series. Basic tests: comparison, limit comparison, ratio and integral. Standard examples: harmonic and geometric series; series of powers. **(4 lectures)**

Limits of functions: Neighbourhood, punctured neighbourhood. Bounded functions (above and below). Limit at a point. Standard theorems, including the sandwich theorem. One-sided limits and connection with full limit. Other types of limits. Examples throughout. **(3 lectures)**

Continuity: Definition and standard theorems including equivalent sequential characterisation of continuity. Results concerning function continuous on closed, bounded intervals. Intermediate-Value theorems. **(4 lectures)**

Differentiability: Definition and standard properties. Monotone functions. Local maxima, minima, stationary points. Rolle's theorem, Mean-Value theorem. Applications include connections with monotone functions, Taylor's theorem. **(3 lectures)**

Riemann integral: Partition; norm; upper and lower sums; Riemann integral. Examples of existence and non-existence of integrals. Standard theorems on integrals. Continuity of "an integral". The fundamental theorem of the Integral Calculus. Integration using primitives. Infinite integrals. **(2 lectures)**

Abstract Algebra: Examples from algebra and geometry; permutations. Definition of group; basic properties; abelian and cyclic groups; subgroups; order of a group and of an element; statement of Lagrange's Theorem; isomorphisms. Examples throughout. **(9 lectures)**

MTH-1C17 : Pure Mathematics

1. Introduction: This unit is compulsory at level 1 for all single subject, three and four year, first degree programmes in mathematics and for the joint Computing and Mathematics programme. The unit consists of two parts:
Part A: Sets, Numbers and Proofs
Part B: Linear Algebra I.

2. Hours, Credits and Assessment: This 20 UCU unit of 40 lectures is taught in the Autumn Semester and is compulsory for first-year MTH-students. Support teaching is via seminars. Assessment is by 100% coursework (including a 2 hour coursetest).

3. Overview:

Part A: The unit provides a thorough introduction to the systems of numbers commonly found in Mathematics: natural numbers, integers, rational numbers, real numbers and complex numbers. It also introduces common set theoretic notation and terminology and a precise language in which to talk about functions. There is emphasis on precise definitions of concepts and careful proofs of results. Styles of mathematical proofs discussed include: proof by induction, direct proofs, proof by contradiction, contrapositive statements, equivalent statements and the role of examples and counterexamples.

Part B:
This unit gives an introduction to linear algebra. We start off by considering systems of linear equations in an arbitrary number of variables (the coefficients in the equations will be taken to be real numbers, but the theory developed will be applicable to arbitrary fields). The main result is the Gaussian elimination procedure, which is an algorithm for finding the solutions of such systems. This will be illustrated in examples, and we will also deduce some theoretical consequences of it (such as the fact that a system of homogeneous linear equations with more variables than equations has a non-trivial solution).

Consideration of the array of coefficients in a system of linear equations leads to the notion of a matrix, and the Gaussian elimination process is most conveniently described in terms of elementary row operations on such a matrix. We treat matrices as mathematical objects in their own right, and develop the basic algebraic operations (addition and multiplication) on them. Then we describe how elementary row operations can be used to compute the inverse of an (invertible) square matrix, and relate this back to solving linear equations.

Further, we interpret the solutions of systems of linear equations in terms of vectors. We will see that, for homogeneous systems, the solution set has many of the same properties as the whole space of vectors $\mathbf{R}^n$ and this helps to motivate the definition of abstract vector spaces. We will see that such vector spaces have a well-defined notion of dimension and that this is related to the invertibility of square matrices.

Another subject is about the determinant of a square matrix. This is a polynomial function of the entries of the matrix. It has many interesting properties and applications: in particular, the determinant is non-zero precisely when the matrix is invertible, and one can even give a formula for computing the inverse in terms of certain determinants.

The final part is concerned with the notions of eigenvalue and eigenvector of a (square) matrix. The idea here is that, by choosing a coordinate system appropriate to a given matrix, we can (sometimes) bring it to a diagonal form which is much easier to work with. We will discuss how to compute eigenvalues and eigenvectors and applications of the diagonalisation of matrices.

4. Recommended Reading:

Part A:
Geoff Smith, Introductory Mathematics: Algebra and Analysis, Springer Undergraduate Mathematics Series, Springer-Verlag, London, 1998. (Price: about 20 pounds)

Also useful:

S. Lipschutz, Set Theory and related topics, Schaum's Outline Series, 1998.

Part B:
There are many books which cover the material in the course. Many books are available in the library, the classmark for Linear Algebra is QA251. It is strongly recommended that students consult books in addition to their lecture notes and coursework material.

Anton, H., *Elementary Linear Algebra*, Wiley. [QA251 ANT] This book is the recommended reading.

Lipschutz, S. and Lipson, M., *Schaum's outline of linear algebra*, Mc-Graw Hill. [an older version is at QA251 LIP]

Morris, A.O., *Linear algebra: an introduction*, Van Nostrand Reinhold. [QA251 MOR]

In addition, much of the material on matrices and determinants is in chapter 4 of Smith, G., *Introductory Mathematics: Algebra and Analysis.*

5. Lecture Contents:

Part A:
Basic set-theoretic notation. Mathematical induction. **(3 lectures)**

Euclidean algorithm. Greatest common divisors. Prime numbers, the Fundamental Theorem of Arithmetic. **(3 lectures)**

Rational and irrational numbers: irrationality of root 2. **(1 lecture)**

Review of styles of proof and the role of definitions. **(1 lecture)**

Real numbers. Manipulation of inequalities. Absolute value and triangle inequality. Introduction to the idea of 'least upper bound.' **(2 lectures)**

More on sets. Venn diagrams, union and intersection, distributivity. Difference of 2 sets, complement, De Morgan's laws. Inclusion-exclusion principle and applications. Power set. Ordered pairs. **(3 lectures)**

Functions. Examples. Injectivity and surjectivity (and alternative terminologies). Examples. Composition. Equivalence of bijection and inverse being a function. **(2 lectures)**

Complex numbers. Solution of quadratic equations. Definition and basic algebraic operations: sum, product, quotient, conjugate, modulus. Geometrical interpretation: Argand diagram, parallelogram rule of addition, triangle inequality, parallelogram equality, argument. Exponential (polar) form of complex numbers. Argument of product and quotient, roots. De Moivre's theorem. **(5 lectures)**

Part B:
Systems of linear equations (over **R**): equivalent systems; the augmented matrix; elementary row operations; Gaussian elimination. Examples. The theorem that a homogeneous system of linear equations with more unknowns than equations has a non-trivial solution. **(4 lectures)**

Addition and multiplication of matrices (over **R**). The inverse of a square matrix. Connection with solving linear equations. Singular matrices and non-invertibility. Method for inverting a square matrix. **(5 lectures)**

Vectors in $\mathbf{R}^n$ and interpretation of systems of linear equations in terms of vectors; geometric interpretation in $\mathbf{R}^3$. Definition of (abstract) vector space (over **R**). Subspaces and homogeneous systems of linear equations. Linear independence and spanning. Basis and dimension. Proof that a square matrix is invertible if and only if its columns are linearly independent. **(4 Lectures)**

Determinant of 2×2 and 3×3 matrices. Determinant of $n\times n$ matrices, defined via expansion. Properties and equivalent ways of computing the determinant. Examples (including Vandermonde). Determinant of a product and of a transpose. Inverting matrices by using determinants. **(3 lectures)**

Eigenvalues and eigenvectors. The characteristic polynomial. Diagonalising matrices. Examples of non-diagonalisability. Proof that 'n distinct eigenvalues' implies 'A diagonalisable'. Examples. Computing powers and roots of diagonalisable matrices. **(4 lectures)**

MTH-1C24 : Multivariable Calculus

Hours, Credits and Assessment: The course is a compulsory (MTH students), 15 UCU unit of 30 lectures taught in the Spring Semester. Support teaching is via seminars. Assessment is by coursework (20%) and a 2 hour examination (80%).

Overview: Partial derivatives, chain rule. Line integrals. Multiple integrals, including change of co-ordinates by Jacobians. Green's theorem in the plane. Divergence, gradient and curl of a vector field. Scalar potential and path independence of line integral. Divergence and Stokes' theorems. MAPLE integrated throughout.

Recommended Reading and References:

Guide to Mathematical Methods – Gilbert & Jordan (Palgrave Macmillan)

Lecture Contents:

Partial Differentiation: Real function of two or more real variables. Equation of a surface; contour plots. Definition of first partial derivative; geometric interpretation for two variables. Product and quotient rule. Higher order derivatives; equality of mixed derivatives. Directional derivative and total derivative. Rate of change along a curve.

Change of variables. Chain rule. Taylor's theorem for function of two variables. Stationary points; classification.

Multiple Integrals: Line Integrals. Double integrals interpreted as sum over region of plane: Limits which vary. Examples. Change of variables, Jacobians. Double integrals in polar coordinates e.g. exp $(-x^2)$. Green's Theorem in the plane. Triple integrals for calculating volume or mass - cylindrical and spherical coordinates. Surface integrals.

Vector Calculus: Scalar fields, visualisation and level surfaces. Directional derivative and rate of change along a curved path. Gradient. ∇ as a differential operator. Interpretation of grad. Applications: 2 variable Taylor expansions and surface normals.

Vector fields, visualisation and field lines. Flux illustrated as volume flow and defined by integration over a curved surface with vector surface element. Divergence as a differential operator, defined in terms of components. Interpretation as flux per unit volume. Properties of the div operator. The Laplacian. Application to gradient fields: Laplace's and Poisson's equation.

Curl as a differential operator, illustrated for particular fields. Irrotational and solenoidal fields. Operators $\mathbf{u}\bullet\nabla$ and ∇^2 as applied to vector fields. Vector operator identities.

Integral theory. The Divergence Theorem. Application to express div, grad, curl as limits of integrals, and to derive a differential equation from an integral formulation. Stokes' Theorem. Path independent line integrals.

(30 Lectures)

MTH-1C27: Calculus

1. Introduction: This unit is compulsory at level 1 for all single subject, three and four year, first degree programmes in mathematics. It is an optional unit for other candidates who have successfully attained A-level or an equivalent standard in mathematics. The unit is the first of a succession of level 1 and level 2 units on Calculus that prepare for subsequent units in applied mathematics.

2. Hours, Credits and Assessment: This unit is of 20 UCU and is taught in the Autumn semester by means of 40 hours of lectures, four per week, supported by 6 hours of seminars. Assessment is by homework (20%) at fortnightly intervals and by a course test (80%) in the final week of the semester.

3. Overview: The origins of the integral calculus can be traced back to ancient times, in the calculation by limiting processes of the areas of plane figures enclosed by curved boundaries. The process of differentiation arose in modern times in the seventeenth century, to calculate the slope of a plane curve, and to define rates of change, such as the variable velocity of a falling particle. The inter-relation between differentiation and integration was discovered by Newton and Leibniz, to prepare for the role of the calculus as the language and principal tool of mathematical science. Newton's development of Calculus made the theory and use of Differential Equations possible.

4. Recommended Reading and References: The notes taken in lectures from the blackboard are intended to be complete and self-contained in themselves, and do not as a rule follow closely the order, content or style of a particular printed textbook. The subject is a basic, long established one that can be found in a large number of books for sale or in the library. A particular book which is close in scope and style to the lecture course is "Guide to Mathematical Methods" by J. Gilbert and C. Jordan, the 2nd edition published by Palgrave Macmillan, as a paperback at £19.99, with ISBN 0-333-79444-3. Second-hand copies of the 1st edition (by J. Gilbert) are suitable as well.

5. Lecture Contents:

Differentiation of a function of one variable: Definition, motivated by rate of change and gradient of a curve. Maxima and minima, concavity, curve sketching. Standard elementary functions: exponential and logarithm, trigonometric, hyperbolic. Rules and examples for differentiation of a sum, product and quotient of functions, a function of a function, the inverse of a function, parametric and implicit relations, nth derivative of a product by Leibniz formula and by induction. **(8 lectures)**

Integration of a function of one variable: Indefinite integral as inverse of differentiation. Definite integral: area under curve, and properties, reversal of limits, infinite upper limit etc. Methods: change of variable, integration by parts, reduction formulae, partial fractions for rational integrand. Length of plane curves. Volume of a solid of revolution. **(9 lectures)**

Power Series: Geometric and general power series, examples, reference to radius of convergence. Properties: addition, multiplication, differentiation and integration of power series. Taylor and Maclaurin series of a function, applications. Indeterminate quotients, l'Hopital's rule. **(3 lectures)**

First-order differential equations of the following type: separable, homogeneous, quasi-homogeneous, linear, numerical solutions using Maple. A geometric view of solutions using Maple. Second-order (and higher order) linear differential equations with constant coefficients: complementary function, particular integral (by trial substitution). **(12 lectures)**

Euler type differential equations. Reduction of order. Solution of differential equations by substitution. Nonlinear differential equations. Phase space as a means to analyse the behaviour of differential equations. Elementary oscillations (linear and nonlinear). Equilibrium points and stability. Limit cycles. Nonlinear oscillations and chaos. **(8 lectures)**

MTH-1C31 : Geometry

Hours, Credits and Assessment: The module is a compulsory (MTH students), 10 UCU module of 16 Lectures taught in the Autumn Semester. Support teaching is via seminars. Assessment is by 100% coursework (includes 1 hour test).

Overview: We begin with a short introduction to Euclidean Geometry based on Euclid's Elements. Theorems in geometry are not founded on experience but require proof and this is one of the first instances in the development in mathematics where this form of reasoning occurs systematically and naturally. We will then move on to the more modern treatment of geometry based on coordinates. This is the Cartesian Geometry which makes use of the algebraic properties of number systems such as the real numbers. Many elementary properties in geometry can be treated very efficiently in term of coordinates and this will occupy the largest part of the module. Towards the end we will establish the connection to linear maps and linear algebra.

4. Recommended Reading and References:

Introduction to Linear Algebra, S. Lang, Springer UTM. (This book contains most material in Parts II and III.) Occasionally cheap copies can be found on www.AbeBooks.co.uk

Euclid's Elements online:
http://aleph0.clarku.edu/~djoyce/java/elements/elements.html
(An excellent edition of the most remarkable science text in use for 23 centuries, now electronically. Part I of the module will be based on this.)

University of St Andrews History of Mathematics:
http://www-groups.dcs.st-and.ac.uk/~history/index.html

Linear Algebra, Seymour Lipschutz, Schaum Outlines Series

Geometry, D.A.Brannan, Cambridge University Press; (A comprehensive text worth buying if you enjoy learning more about real geometry.)

Euclidean and Non-euclidean Geometry, by Marvin Jay Greenberg. (An instructive and easy to read text on the axiomatic treatment of geometry, including non-euclidean geometries.)

Lecture Contents:

Part I: A glimpse at Euclid's Elements: (3 Lectures)
The first few propositions in Book 1. The need for proof: definition, axiom and rules of deduction. Proposition 47 and 48, proofs of Pythagoras' theorem and its converse.

Part II: An introduction to Cartesian Geometry: (8 Lectures)
(a) Vectors as n-tuples of real co-ordinates, addition and scalar multiplication. The geometric meaning of vector as located vector in two and three dimensions. The line through two two points.

(b) The equation of the line and the plane in parametric form. The line through two distinct points, the plane through three (suitably distinct) points. Intersections of lines and planes, and their equations.

(c) The dot-product of vectors and elementary properties of this product; length. Justify length and orthogonality geometrically in terms of Pythagoras' theorem. Orthogonal projection onto a line or a plane. Schwarz inequality, cosine of angle.

(d) The normal equation of a line in the plane and of a plane in 3-space. Cross-product in 3-space.

Part III: Maps and Matrices **(5 Lectures)**

(a) Notions of symmetry via maps which preserve vector sum and scalar multiplication. Linear maps. Basic observation that these are determined by their effect on a few suitably chosen vectors in the space. Examples: translation, reflection, rotation, projection, transvection.

(b) The use of matrices to describe linear maps such as reflections, rotations and projections. Multiplication of matrices and geometric interpretation. Rotation as product of two reflections. Trigonometric formulae as application.

(c) Determinant of 2x2 and 3x3 matrix as area and volume. Cross-product and triple-products.

(Last updated 27 November 2007)

MTH-1C32 : Mechanics and Modelling

Hours, Credits and Assessment: The course is a compulsory (MTH students), 10 UCU unit of 20 lectures taught in the Spring Semester. Support teaching is via seminars. Assessment is by coursework (20%) and a 2 hour examination (80%).

Overview: The study of classical mechanics takes as its point of departure Newton's three celebrated laws of motion. We may apply these fundamental laws to make accurate predictions about the movement of particles and solid bodies, from calculating the path of a cricket ball to computing the orbits of the planets. First determining the forces acting on a given system, we may use Newton's second law to formulate a differential equation describing its motion. We can then integrate this equation to determine a body's subsequent velocity and position. We will discuss particle trajectories under gravity with or without air resistance. Using Hooke's law, we will examine forced undamped and damped oscillations of springs and discuss the phenomena of resonance and beats. Circular motion will be discussed and the notion of conserved quantities such as energy and angular momentum will be introduced.

Recommended literature and references:

R D Gregory “Classical Mechanics” Cambridge

Lecture Contents:

Mechanics: Rectilinear motion of a particle under gravity. Inverse square law: escape velocity. Newton's laws of motion. Resisted Motion. Terminal velocity. Hooke's law and horizontal/vertical oscillations on elastic string or spring. Forced undamped vibrations: resonance. Beats. Damped motion.

Planar motion of a particle: Polar components of velocity and acceleration, angular velocity, angular momentum, moment of force as a vector. Projectile in constant gravity (parabola), and with air resistance directly proportional to particle's velocity. Conservative forces. Central forces. Conservation of energy and angular momentum. **(20 lectures)**

MTH-1C33 : IT for Mathematicians

1. Introduction: There are no prerequisites for this unit, which is a compulsory unit for the majority of Year One students on mathematics programmes. The unit consists of an introduction to using computers, obtaining electronic information, mathematical typesetting and mathematical problem solving using computers.

2. Timetable Hours, Credits, Assessments: This unit is of 10 UCU and is taught in the Autumn Semester by means of 18 hours of practical classes, (weeks 2 to 10). The Assessment is 100% coursework and the mark is composed of:

Attendance and satisfactory progress in Lab Classes	10%
Mathematical problem solving using MAPLE	30%
Latex report	20%
Personal web page	20%
Programming concepts using MAPLE	20%

Feedback is given to students by providing solutions to coursework exercises through the Web and details on the way marking schemes are applied.

3. Overview: No prior knowledge of computing is assumed. The emphasis is on using computers for information, retrieval and problem solving in Mathematics, in association with other units making up the degree program.

The main aims of the unit are to:

Show how a computer can help mathematicians.
Introduce students to the computing facilities available at UEA.
Introduce HTML and produce a basic web site.
Introduce the LaTeX scientific typesetting package.
Introduce the mathematical software package, Maple.

By the end of the unit, all students will feel comfortable using a computer to:

Communicate electronically.
Transfer files and information.
Search for relevant information.
Write a personal web page.
Prepare good technical documents.
Solve smallish mathematical problems, using packages.

4. Recommended Reading:

There is no recommended reading list for this unit. The lecturer will provide detailed notes with links to additional material on the Web.

5. Lab Class contents:

Windows, WWW, Email, Networks **(1 hour)**

HTML **(3 hours)**

Introduction to the MAPLE mathematical software **(4 hours)**

Introduction to mathematical typesetting using LATEX **(4 hours)**

Programming Concepts using MAPLE **(6 hours)**

MTH-1C34 : Probability

1. Introduction: This unit provides an introduction to Probability Theory. It is largely self-contained and it is suitable for non-mathematics students with the proper elementary mathematics background. Some knowledge of matrix algebra (covered in MTH-1C17) is required.

2 Timetable Hours, Credits, Assessments: This course is a 10 UCU unit of 15 lectures with 3 seminars. Assessment is by coursework (20%), consisting of three homework sheets, and examination (80%).

3. Overview: The term **probability** refers to the study of randomness and uncertainty. In any situation in which one of a number of possible outcomes may occur, the theory of probability provides methods for quantifying the chances or likelihood associated with the various outcomes. The study of probability as a branch of mathematics goes back over 300 years and it is now fundamental prerequisite for the study of statistics.

4. Recommended Reading:
Ross, S "A First Course in Probability" (Pearson Prentice Hall)

W Mendenhall, D D Wackerly and R L Scheaffer "Mathematical Statistics with Applications" (PWS Kent.)

Y A Rozanov "Probability Theory: A Concise Course" (Dover)

5. Lecture Contents:

Introduction, probability model for an experiment, axioms of probability, basic properties. Equally likely events, combinatorial analysis. **(3 lectures)**

Conditional probability, independence, Bayes' Theorem. **(1 lecture)**

Random variables and distributions. **(1 lectures)**

Binomial, geometric and hypergeometric distributions. **(2 lectures)**

Poisson process, Poisson distribution. **(2 lectures)**

Markov chains: absorbing states, probability of absorption, absorption times. **(2 lectures)**

Continuous random variables, expectations. **(1 lecture)**

Uniform distribution, normal distribution, use of the tables. **(1 lecture)**

Exponential distribution. Reliability: series and parallel systems, minimal path sets, minimal cut sets. **(2 lectures)**

MTH-1C36 : Discrete Mathematics – Computational Number Theory

1. Introduction: Discrete Mathematics underpins many branches of mathematics, computing and science more generally. This introduction to computational number theory has no formal prerequisites and is suitable for students from outside the School. Some applications to public key cryptography will be given towards the end of the course.

2. Hours, Credits and Assessment: The course is a 10 UCU unit of 15 lectures, supported by seminars and lab classes. Assessment is by coursework (20%) and examination (80%).

3. Overview: It seems obvious that a positive integer can be factorised into a product of prime (or irreducible) integers. But how easy a computational task is it a) to verify that a given integer is prime? or b) to factorise a given integer into primes? For example, how long would it take you to verify that 4057 and 5471 are primes? And how long would it take you to factorise 22195847 into primes had you not seen its factors in advance? These numbers are tiny compared to what a modern computer can handle.

We tend to think that a modern computer can do almost any computation in no time but this is a false impression. Modern research shows that problem a) above is easier than b). There are several practical and elegant solutions to a). Algorithms can be found which will verify primality for an integer with thousands of digits. Finding a method of quickly factorising 'large' integers is widely regarded as one of the greatest challenges facing computational number theory. A lot hangs on this problem. Modern cryptographic techniques, especially those used in Internet security (for example, when you use a credit card) rely upon the apparent difficulty of factorising large integers.

In the course we will look at these issues; the mathematics that goes into them as well as appropriate definitions of 'large' as above. We will do some laboratory work but no programming skills are required. The aim will be to understand something of what a modern computer package such as Maple is doing when it verifies the primality of an integer or when it succeeds (or fails) to factorise an integer.

4. Recommended Reading: The aim will be to use electronic media as much as possible.

5. Lecture Contents:

A brief history of prime. **(2 lectures)**

Euclid's division algorithm, gcd's. Modular arithmetic and primitive roots. **(2 lectures)**

Fermat's Little Theorem, Wilson's Theorem, quadratic residues, the Legendre symbol. **(2 lectures)**

Primality testing using Fermat - why it fails, Carmichael numbers and their properties. **(2 lectures)**

Laboratory session. **(2 lectures)**

Quadratic reciprocity law, the Jacobi symbol and the Solovay-Strassen primality test. **(2 lectures)**

Factorisation, Public Key cryptography. **(3 lectures)**

MTH-1E01 : Astrophysics

1. Introduction: This is an optional unit at level 1 for students within the Natural Sciences degree programme and for others as a free choice. The unit will use mathematics at A-level or equivalent standard.

2. Hours, Credits and Assessment: This unit is 10 UCU and is taught in the Autumn semester by means of 15 lectures supported by 3 hours of seminars. Assessment is by written problems (35%), a poster presentation (15%) and a course test (50%) in the final week of the semester.

3. Overview: This 10-credit unit will give an overview of astrophysics through lectures and workshops. Assessment will involve some coursework and a course test. The unit will involve using some mathematics at AS/A2 level. Topics covered will include some history of astrophysics, radiation, matter, gravitation, astrophysical measurements, spectroscopy, stars and some aspects of cosmology.

4. Recommended literature and references: The notes taken in lectures are intended to be complete and self-contained in themselves and do not as a rule follow closely the order, content or style of a particular textbook. Some background reading for an A-level physics textbook may be necessary. A book which is recommended as background reading for the whole unit is 'Cosmology-A very short introduction' by Peter Coles, published by Oxford, as a paperback, with ISBN 0-19-285416-X.

5. Lecture Contents:

Unit introduction & definitions
Overview of the universe
Newton & Einstein
Radiation & matter
Optics
Astrophysical measurement
Spectra
Observational properties of stars
Star formation
Energy generation in stars
Poster workshop (optional)
Star Life Cycle 1
Star Life Cycle 2
Cosmology I
Cosmology II