MTH-1A13 : Linear Algebra I

1. Introduction: This is one half of the first pure mathematics course, **Pure Mathematics I** and it gives an introduction to Pure Mathematics (shared with Foundations and Analysis) and the basic foundations of linear algebra. This unit is a prerequisite for the second semester **Pure Mathematics II**. The course is a rigorous treatment of linear equations, matrices and determinants.

2. Hours, Credits and Assessment: The course is a 10 UCU unit of 23 lectures. Support teaching is via 6 seminars shared with **Foundations and Analysis.** Assessment is by homework (20%) at fortnightly intervals and a course test at the end of the semester (80%).

3. Overview: Introduction to Pure Mathematics: The notion that mathematical results are <u>proved</u> rigorously on the basis of carefully stated <u>definitions</u> is central to modern mathematics. We illustrate these concepts with some simple examples and discuss the various styles of proof which will be encountered (direct; by contradiction; by induction).

Much of mathematics nowadays is phrased in the language of set theory, and we give a quick, informal introduction to this, with examples of how to read the notation. Using this, we make precise the notion of a function, and introduce various properties which distinguish between various types of functions (onto; one-to-one; bijective).

Linear Algebra: We start off by considering systems of linear equations in an arbitrary number of variables (the coefficients in the equations will be taken to be real numbers, but the theory developed will be applicable to arbitrary fields). The main result is the Gauss elimination procedure, which is an algorithm for finding the solutions of such systems. This will be illustrated in examples, and we will also deduce some theoretical consequences of it (such as the fact that a system of homogeneous linear equations with more variables than equations has a non-trivial solution).

Consideration of the array of coefficients in a system of linear equations leads to the notion of a matrix, and the Gauss elimination process is most conveniently described in terms of elementary row operations on such a matrix. We treat matrices as mathematical objects in their own right, and develop the basic algebraic operations (addition and multiplication) on them. Then we describe how elementary row operations can be used to compute the inverse of an (invertible) square matrix, and relate this back to solving linear equations.

The final section of material is concerned with the determinant of a square matrix. This is a polynomial function of the entries of the matrix. It has many interesting properties and applications: in particular, the determinant is non-zero precisely when the matrix is invertible, and one can even give a formula for computing the inverse in terms of certain determinants.

4. Recommended literature and references: The following should be in the Library. There are many other books in the library which will cover the material in the course. We recommend you purchase [1] or [2] . These will also be the basic texts for Linear Algebra II next semester.

1. Linear Algebra, A. O. Morris.

2. Elementary Linear Algebra, H. Anton.

5. Contents:

Proof by induction. **(1 lecture)**

Sets. Membership, containment, power set (size of this). Venn diagrams, union and intersection, distributivity. Difference of 2 sets, complement, De Morgan's laws. Inclusion-exclusion principle and applications. **(3 lectures)**

Ordered pairs, relations, functions. Examples. Inverse of a function. 'Onto', 'one to one correspondence'. Examples (finite sets, ordered n-triples). Equivalence of bijection and inverse being a function. **(2 lectures)**

Composition of functions. Existence of right/left inverses equivalent to onto/one-to-one. **(1 lecture)**

Systems of linear equations: Equivalent systems; the augmented matrix; elementary row operations; Gaussian elimination. Examples. The theorem that a homogeneous system of linear equations with more unknowns than equations has a non-trivial solution. **(5 lectures)**

Matrix Algebra: Addition and multiplication of matrices (over R) . The inverse of a square matrix. Connection with solving linear equations. Singular matrices and non-invertibility. Method for inverting a square matrix. **(5 lectures)**

Linear independence in R^n (definition and examples; geometric interpretation in R^3). **(1 Lecture)**

Determinants: 1. The 2•2 and 3•3 cases. Properties in these simple cases. Definition of n •n determinants via expansion. **(2 lectures)**

2. Properties and equivalent ways of computing the determinant (with proofs). Examples (including Vandermonde). det(AB) = det(A)det(B) and det(A) = det(A_T). Inverting matrices by using determinants. Cramer’s rule. **(3 lectures)**

MTH-1A15 : Foundations and Analysis

1. Introduction: This is one half of the first pure mathematics course **Pure Mathematics I** and it gives the basic foundations of modern mathematics and analysis. This unit is a prerequisite for the second semester **Pure Mathematics II**. The notion of what constitutes a mathematical proof is emphasised along with standard introductory material on real numbers, sequences and series.

2. Hours, Credits and Assessment: The course is a 10 UCU unit of 22 lectures. Support teaching is via 6 seminars, shared with Linear Algebra I. Introduction to Pure Mathematics occupies 9 hours and Analysis 13 hours of the course (all hours are approximate only). Assessment is by homework (20%) at fortnightly intervals and a course test at the end of the semester (80%).

3. Overview:

Part I. Introduction to Pure Mathematics: The notion that mathematical results are <u>proved</u> rigorously on the basis of carefully stated <u>definitions</u> is central to modern mathematics. We illustrate these concepts with some simple examples and discuss the various styles of proof which will be encountered (direct; by contradiction; by induction).

Much of mathematics nowadays is phrased in the language of set theory, and we give a quick, informal introduction to this, with examples of how to read the notation. Using this, we make precise the notion of a function, and introduce various properties which distinguish between various types of functions (onto; one-to-one; bijective). The lectures conclude with a discussion of equivalence relations.

Part II. Analysis of Sequences and Series: We begin by carefully defining the real numbers in familiar terms: the real numbers are those things that can be written as infinite decimal expansions. The first result is to check that a consequence of this is that any increasing list x1,x2,x3,... of real numbers all of which are less than some fixed number must <u>converge</u>. This basic result is the key tool to unlock properties of sequences (infinite lists of numbers) and series.

The two main things studied using these methods are <u>sequences</u> and <u>series</u>. We develop tests for convergence of sequences and rules for manipulating convergent sequences. Series are infinite sums: expressions like 1+1/2+1/3+1/4+... or 1+1/4+1/9+1/16+...

Using earlier results on sequences we develop a precise language for talking about series and tests to determine when they converge.

The section concludes with a preview of some of the topics to be covered in **Real Analysis**: limits, continuity and differentiability of functions from the reals to the reals.

4. Recommended literature and references: All of the following should be in the Library. There are many other books in the library which will cover the material in the course. We recommend you purchase [4] or [5]. These will also be the basic texts for Pure Mathematics II next semester.

<u>For Part I:</u>

1. Set Theory, S. Lipschutz, Schaum's Outline Series.

2. Guide to Abstract Algebra, C. Whitehead, Macmillan.

3. Sets and Groups, J. A. Green.

For Part II:

4. Analysis, P. E. Kopp, Arnold, 1996 .

5. Fundamentals of Mathematical Analysis, R. Haggarty.

In addition lecture notes of Part II will be available on the UEA intranet.

5. Contents:

PART I

Definitions, theorems and proofs (Easy examples). **(1 lecture)**

Proof by induction. **(1 lecture)**

Sets. Membership, containment, power set (size of this). Venn diagrams, union and intersection, distributivity. Difference of 2 sets, complement, De Morgan's laws. Inclusion-exclusion principle and applications. **(3 lectures)**

Ordered pairs, relations, functions. Examples. Inverse of a function. 'Onto', 'one to one correspondence'. Examples (finite sets, ordered n-triples). Equivalence of bijection and inverse being a function. **(2 lectures)**

Composition of functions. Existence of right/left inverses equivalent to onto/one-to-one. **(1 lecture)**

Equivalence relations and partitions. **(1 Lecture)**

PART II

Introduction: The real numbers - what they are and how can they be manipulated. Use of the modulus |.| to give exact definitions for convergence of a sequence of real numbers. Proof that a bounded increasing sequence converges. **(3 lectures)**

Sequences: Examples of convergent and non-convergent sequences. Algebra of convergent sequences. Euler's number as an example. **(3 lectures)**

Series: Convergence for series. Basic tests: comparison, limit comparison, ratio and integral. Standard examples: harmonic and geometric series; series of powers. **(4 lectures)**

Overview of Real Analysis: Limits, continuity and differentiation. The Intermediate Value Theorem and the Mean Value Theorem (without proofs). **(3 lectures)**

MTH-1A24 : Linear Algebra II

1. Introduction: This second semester course follows on from **Linear Algebra I** which is a prerequisite. This unit is a prerequisite for many second year units for mathematics students. The course looks at vectors and matrices from a more sophisticated viewpoint by introducing the concepts of vector spaces and linear transformations.

2. Hours, Credits and Assessment: The course is a 10 UCU unit of 20 lectures. Support teaching is via 6 seminars (shared with Real Analysis). Assessment is by homework (20%) at fortnightly intervals and examination (80%).

3. Overview: In **Linear Algebra I** we introduced matrix multiplication as a way of writing a system of linear equations as a single matrix equation. A more sophisticated viewpoint is to see multiplication by an m-by-n matrix as describing a function from n-dimensional to m-dimensional space. To understand better what this function is doing it is usually necessary to make a change of coordinates in the various spaces. To formalise this properly, we introduce the notion of an abstract vector space, and linear transformations between vector spaces. The key concept for vector spaces is that of a basis (--what we might informally have called a coordinate system), and we show that any two bases of a vector space have the same number of elements (the dimension of the vector space). We distinguish certain subsets of a vector space which can themselves be considered as vector spaces (subspaces), and relate the dimensions of subspaces to the dimension of the ambient space.

We then develop some general results about linear transformations. A linear transformation is determined by its effect on a basis, and, given bases, we can encode all the information needed to describe a linear transformation in a matrix. Changing the bases changes the matrix, and we describe this precisely. We also investigate how 'simple' this matrix can be made by the right choice of bases. This leads to the notions of eigenvalues and eigenvectors of (square) matrices, and the final section of the course is devoted to a study of these, how to compute them and applications.

The origins of linear algebra are in the middle of the 18th century when mathematicians began to analyse systems of (linear) equations more closely. The theory of determinants, already known to Leibniz in 1693, and much cultivated in the 19th century also provided an important stimulus. The notion of vector space is due to Grassmann (1844) and the vector space axioms in use today are almost exactly the same as those developed by Peano (1888). These more modern viewpoints on an old theory have applications everywhere in mathematics and wherever mathematics is applied. They provide unity and clarity in such diverse areas as differential equations, geometry, analysis, algebra, number theory and discrete mathematics. The axiomatisation leads one to consider vector spaces over fields other than the real numbers, for example over finite fields. This has enormously important applications in digital technologies via areas such as error correcting codes.

4. Recommended literature and references:

A O Morris Linear Algebra, (Van Nostrand)

H Anton Elementary Linear Algebra, (Wiley)

5. Contents:

Fields: The g.c.d. of 2 integers and the Euclidean algorithm. Modular arithmetic; Fields. **(2 lectures)**

Motivation for vector spaces: Matrices as functions, examples of change of basis. **(1 lecture)**

Vector spaces: Definition of a vector space. Examples (main example Rn; function spaces). Linear combinations and subspaces. Linear independence. Definition of a basis and existence of dimension. Dimensions of subspaces and the modular law. Further examples. **(6 lectures)**

Linear Transformations: Definition and examples (the linear transformation arising from a matrix; examples in analysis; geometric examples such as rotation and projection). Kernel and image of a linear transformation. Existence and uniqueness of linear transformations with prescribed images on a basis. The rank+nullity theorem. Examples (projections, derivatives and integrals of polynomials, linear equations) and applications. **(4 lectures)**

The matrix of a linear transformation once bases are chosen. Examples. Change of basis formulas for matrices. **(2 lectures)**

Eigenvalues and eigenvectors: The characteristic polynomial. Eigenspaces. Diagonalising matrices. Connection with eigenvectors. Examples of non-diagonalisability. Proof that 'n distinct eigenvalues' implies 'A diagonalisable'. Examples. Computing powers and roots of diagonalisable matrices. Inner product and orthogonal vectors in R_n . The Gram-Schmidt process. Diagonalisability of real symmetric matrices. Examples. **(5 lectures)**

MTH-1A26 : Real Analysis

1. Introduction: This second semester course follows on from **Foundations and Analysis,** which is a prerequisite. It develops further Real Analysis (limits, continuity, differentiability, Taylor's theorem and integration). The unit is compulsory for most first-year mathematics students.

2. Hours, Credits and Assessment: The course is a 10 UCU unit of 20 lectures. Support teaching is via 6 seminars, shared with **Linear Algebra II**. Assessment is by homework (20%) at fortnightly intervals and examination (80%).

3. Overview: Analysis is that part of mathematics concerned with functions and graphs. Throughout the 17th and 18th centuries, problems in mathematics and physics were being successfully solved using ideas that we now call Differential Equations, Fourier Analysis, and Fixed Point Theorems. Despite these successes, much of the work was not accurate and many basic questions were left unanswered. For example: What are the real numbers? Which functions have Taylor expansions? Do all differential equations have solutions?

During the 19th century these problems were formulated precisely and solved. The study of analysis begins with a deep understanding of the real numbers, and then goes on to study functions and how they behave. The course will provide you with the basic tools (limits, sequences, and series) to rigorously examine functions, graphs, and differential equations.

4. Recommended literature and references:

P E Kopp Analysis, (Arnold)

S G Stirling Mathematical Analysis, (Ellis Horwood series)

M Spivak Calculus, (Benjamin)

J B Reade An Introduction to Mathematical Analysis, (Oxford Science)

J R Kirkwood An Introduction to Analysis, (PWS-Kent)

Lecture notes may be made available to students on the UEA Intranet.

5. Contents:

Introduction: The real numbers as a complete ordered field. **(1 lecture)**

Limits of functions: Neighbourhood, punctured neighbourhood. Bounded functions (above and below). Limit at a point. Standard theorems, including the sandwich theorem. Non-existence of a limit via sequences. One-sided limits and connection with full limit. Other types of limits. Examples throughout. **(5 lectures)**

Continuity: Definition and standard theorems including equivalent sequential characterisation of continuity. Results concerning function continuous on closed, bounded intervals. Intermediate-Value theorems. Applications. **(4 lectures)**

Differentiability: Definition and standard properties. Monotone functions. Local maxima, minima, stationary points. Rolle's theorem, Mean-Value theorem. Applications include connections with monotone functions, Taylor's theorem. **(4 lectures)**

Riemann integral: Partition; norm; upper and lower sums; Riemann integral. Examples of existence and non-existence of integrals. Standard theorems on integrals. Continuity of "an integral". The fundamental theorem of the Integral Calculus. Integration using primitives. **(4 lectures)**

Infinite integral: Convergence/divergence with examples. Integral test; application to series $\Sigma 1/n_{\gamma}$. Comparison test with applications. **(2 lectures)**

MTH-1A33 : Mathematical Methods A

1. Introduction: This unit together with Mathematical Methods B is compulsory at level 1 for all single subject, three and four year, first degree programmes in mathematics, and for the two joint, Computing and Mathematics, Chemistry and Mathematics, programmes, and also for the BSc Geophysical Sciences degree in the School of Environmental Sciences. It is an optional unit for other candidates who have successfully attained A-level or an equivalent standard in mathematics; the unit organiser is available to interview candidates in doubt. A brief indication of formulae and topics that candidates are expected to have met and understood beforehand is provided on a sheet which is handed out. The unit is the first of a succession of level 1 and 2 units on mathematical methods (**Mathematical Methods A, B and C, Advanced Calculus I and II**) that prepare particularly for subsequent units in applied mathematics and for optional level 3 methods units.

2. Timetable Hours, Credits, Assessments: This unit is of 10 UCU and is taught in the first half of the Autumn Semester by means of 22 hours of lectures, four per week, supported by 3 hours of seminars. Assessment is by homework (20%) at fortnightly intervals and a course test at the end of the semester (80%).

3. Overview: Vector algebra is designed to describe entities that have an intrinsic direction in three-dim space as well as a magnitude (such as line segments, displacements, velocities, forces etc). The origins of the *integral calculus* can be traced back to ancient times, in the calculation by limiting processes of the areas of plane figures enclosed by curved boundaries. The process of *differentiation* arose in modern times in the seventeenth century, to calculate the slope of a plane curve, and to define rates of change, such as the variable velocity of a falling particle. The inter-relation between differentiation and integration was discovered by Newton and Leibniz, to lead on to the role of the calculus as the language and principal tool of mathematical science. *Complex numbers* were introduced in the sixteenth century to help construct solutions of cubic equations in particular, and algebraic equations in general. When functions of a complex variable are developed more fully in units Advanced Calculus I and II, they are found to illuminate the study of real functions and to facilitate certain integration processes.

4. Recommended Reading: The notes taken in lectures from the blackboard are intended to be complete and self-contained in themselves, and do not as a rule follow closely the order, content or style of any particular printed textbook. The subject is a basic, long established one that can be found in a large number of books for sale or in the library.

A particular book that is close in scope and style to the lecture course is "Guide to Mathematical Methods" by John Gilbert (published by MacMillan).

A larger range of methods is covered fairly briefly in "Mathematical Methods for Science Students" by G Stephenson (published by Longmans).

It is helpful to refer to a formula book such as "Mathematical Formulae for Engineering and Science Students" by S Barnett and T M Cromin (published by Longmans), or "Mathematical Handbook of Formulas and Tables" by M R Spiegel (published by Schaum-McGraw-Hill).

Each of these books are also relevant to Mathematical Methods B & C.

5. Lecture Contents: This unit is preceded by a brief induction course (4 hours) covering study skills, unit structure and a summary of simple mathematical properties. This induction course also serves an introduction to Pure Maths I. The lectures revise briefly the A-level calculus and proceed to more advanced topics in the calculus and algebra of complex numbers. A detailed list with section headings and approximate lecture hours is as follows:

A. Vectors in 2 and 3 dimensions: Vectors: Definition as having magnitude and direction; illustrated by directed line segments. Addition - parallelogram and triangle laws. Multiplication by scalar. Single letter notation. Components: as arrays and unit base vector i, j, k notation. Magnitude in terms of components; direction cosines. The parallelogram rule. Angle at 0 between two vectors in R2 and R3.

Geometrical applications avoiding components. Vector equation of a straight line. Scalar product - geometrical definition and component form. Projection of a vector. Equation of a plane. Vector product - geometrical definition and component form. Area of triangle. Distributive law.

Scalar triple product and volume of parallelepiped. Projecting area. Vector triple products. **(6 lectures)**

B. Complex Numbers: Solution of quadratic equations. Definition and basic algebraic operation: sum, product, quotient, conjugate, modulus.

Geometrical interpretation: Argand diagram, parallelogram rule of addition, triangle inequality, parallelogram equality, argument. Exponential (polar) form of complex number. Argument of product and quotient, roots. De Moivre's theorem; trigonometric identities. **(4 lectures)**

C. Calculus 1: Differentiation of a Function of One Variable: Motivated by gradient of a curve, maxima and minima, rate of change. Definition. Rules and examples for differentiation of a sum, product and quotient of functions, inverse of a function, parametric and implicit relations; nth derivative of a product (Leibniz formula, by induction). **(4 lectures)**

Integration of a Function of One Variable: Indefinite integral as inverse of differentiation. Definite integral, area under curve, and properties: sums, scalar multiplication, sequence of limits, reversal. Methods: change of variable, parts, reduction formulae, partial fractions for rational integrand, use of complex exponential. Curve length. Volume and Surface of Revolution. **(6 lectures)**

MTH-1A35 : Mathematical Methods B

1. Introduction: Combined with Mathematical Methods A (MTH-1A33) this unit forms Mathematical Methods I (MTH-1A31) which is compulsory at level 1 for all single subject, three and four year, first degree programmes in mathematics, and for the two joint, Computing and Mathematics, Chemistry and Mathematics, programmes, and also for the BSc Geophysical Sciences degree in the School of Environmental Sciences. It is an optional unit for other candidates who have completed Mathematical Methods A. This unit is part of a succession of level 1 and 2 units on mathematical methods (**Mathematical Methods A, B and C, Advanced Calculus I and II**) that prepare particularly for subsequent units in applied mathematics and for optional level 3 methods units.

2. Timetable Hours, Credits, Assessments: This unit is of 10 UCU and is taught in the second half of the Autumn Semester by means of 20 hours of lectures, four per week, supported by 3 hours of seminars. Assessment is by homework (20%) at fortnightly intervals and a course test at the end of the semester (80%).

3. Overview: The mathematical expression of physical laws often involve relations between rates of change of quantities. The study of *differential equations* thus provides a way of describing the behaviour of physical systems. This analysis leads naturally to the study of Newtonian mechanics in Mathematical Methods C.

In the modelling of many physical systems, such as the fluid flows considered in the level 2 **Hydrodynamics** units functions of more than one variable are required. Analysis of such systems involve *partial differentiation* of functions.

4. Recommended Reading: The notes taken in lectures from the blackboard are intended to be complete and self-contained in themselves, and do not as a rule follow closely the order, content or style of any particular printed textbook. The subject is a basic, long established one that can be found in a large number of books for sale or in the library.

A particular book that is close in scope and style to the lecture course is "Guide to Mathematical Methods" by John Gilbert (published by MacMillan).

A larger range of methods is covered fairly briefly in "Mathematical Methods for Science Students" by G Stephenson (published by Longmans).

It is helpful to refer to a formula book such as "Mathematical Formulae for Engineering and Science Students" by S Barnett and T M Cromin (published by Longmans).

Each of these books are also relevant to Mathematical Methods A & C.

5. Lecture Contents:

Hyperbolic Functions: Definition. Differentiation. Integration **(1 lecture)**

Power Series: Informal, illustrative introduction to geometric and other power series; interval of convergence. Differentiation, integration, addition, multiplication and division of power series.

Taylor's and Maclaurin series, in particular for exponential, logarithmic, binomial, inverse tangent, circular and hyperbolic functions. L'Hôpital's rule for limit of indeterminate quotient. Statement of Taylor's theorem: the remainder term. **(4 lectures)**

Ordinary Differential equations: First-order types: separable, homogeneous, quasi-homogeneous, linear, Bernoulli. Second-order (and higher order) linear type with constant and variable coefficients: reduced equation, complementary function, particular integral (by trial substitution), Euler type, reduction of order, factorisation of operator (Riccati type). Second-order non-linear cases with absent variable, first integrals. **(8 lectures)**

Partial Differentiation: Real function of two or more real variables. Equation of a surface; contour plots. Definition of first partial derivative; geometric interpretation for two variables. Product and quotient rule. Higher order derivatives; equality of mixed derivatives. Directional derivative and total derivative. Rate of change along a curve.

Change of variables. Chain rule. Taylor's theorem for function of two variables. Stationary points; classification. **(5 lectures)**

MTH-1A44: Mathematical Methods C

1. Introduction: This unit is compulsory at level 1 for all single subject, three and four year, first degree programmes in mathematics, and for the two joint, Computing and Mathematics, Chemistry and Mathematics, programmes, and also for the BSc Geophysical Sciences degree in the School of Environmental Sciences. It is an optional unit for other candidates who will be expected normally to have already obtained credit in the pre-requisite unit of the Autumn semester, **Mathematical Methods I**. The first half of this unit consists of Multiple Integration in preparation for level 2 & 3 units in applied mathematics. The second half of the unit concerns the study of the motion of a particle using the mathematical methods developed in Mathematical Methods I.

2. Timetable Hours, Credits, Assessments: The unit is of 10 UCU and is taught in the first half of the Spring Semester by means of 20 hours of lectures, four per week, supported by 3 hours of seminars. Assessment is by homework (20%) at fortnightly intervals and a $1\frac{1}{2}$ hour examination near the end of the semester (80%).

3. Overview: Multiple Integrals extend the integral calculus to calculate quantities such as the volume under an open surface or within a closed one, the area of a surface, the total "mass" of material in a region at a given density. Mathematical calculus was applied firstly in the 17th and 18th centuries, to describe and predict the motion of falling, projected and oscillating particles, and of planetary and comet orbits. This is the astoundingly successful classical Newtonian model, which set the example for further mathematical applications to the whole range of physical phenomena in the real world.

4. Recommended Reading:

For the methods component of the unit the following is recommended

John Gilbert "Guide to Mathematical Methods" (MacMillan)

For the mechanics component of the unit the following is recommended

C D Collinson & T Roper "Particle Mechanics" (Arnold)

5. Lecture Contents:

1. Multiple Integrals:

Line Integrals. Double integrals interpreted as sum over region of plane: Limits which vary. Examples. Change of variables, Jacobians. Double integrals in polar coordinates e.g. exp $(-x_2)$. Green's Theorem in the plane. Triple integrals for calculating volume or mass - cylindrical and spherical coordinates. **(8 lectures)**

2. Mechanics:

Rectilinear motion of a particle under gravity. Inverse square law: escape velocity, black hole. Newton's laws of motion. Resisted Motion. Terminal velocity. Hooke's law and horizontal/vertical oscillations on elastic string or spring. Forced undamped vibrations: resonance. Damped motion. **(5 lectures)**

Planar motion of a particle: Polar components of velocity and acceleration, angular velocity, angular momentum, moment of force as a vector. Projectile in constant gravity (parabola), and with air resistance directly proportional to particle's velocity. Central forces: Conservation of energy and angular momentum. Trajectories under inverse square law: ellipse, parabola, hyperbola. Orbital period, Kepler's Laws. **(7 lectures)**

MTH-1A46 : Mathematical Modelling

1. Introduction: This unit will show how the mathematical techniques presented in **Mathematical Methods A, B** and **C** can be used to describe and predict behaviour in a number of different applications; for example population dynamics.

2. Timetable Hours, Credits, Assessments: The unit is of 10 UCU and is taught in the second half of the Spring Semester by means of 18 hours of lectures, four per week, supported by 3 hours of seminars. Assessment is by homework (20%) at fortnightly intervals and a $1\frac{1}{2}$ hour examination at the end of the semester (80%).

3. Overview:

For problems that involve the complex interactions between humans, other animal life and plant life, it is often possible to construct simplified mathematical models. Such models must reflect common observations and have a predictive capability. This unit will concentrate on two areas. The first, from ecology, namely population dynamics, which leads to the study of systems of two first-order differential systems. Simple results for non-linear oscillations and chaos.

4. Recommended Reading:

R. Habermann "Mathematical Models" (Prentice-Hall)

D. Acheson "From Calculus to Chaos" (Oxford)

5. Lecture Contents:

1. Population Dynamics

Introduction. Single species, limited food supply. Stability of equilibrium. Two species models. Classification of equilibria: centre, saddle, node and spiral.

Predator-Prey Models: Unlimited food supply. Average population. Competing species. Linearisation about equilibrium points. Examples. **(9 lectures)**

2. Epidemics

Introduction. Kermack-McKendrick model for infected, immune and susceptible individuals. Affects of vaccination. Spread of immunity. threshold theorem. Incubation model. **(5 lectures)**

3. Non-linear Oscillations and Chaos

Limit cycles. Lorenz equations. Chaotic mixing. Period doubling. Implications for weather and climate forecasts. **(4 lectures)**

MTH-1A63 : IT for Mathematicians

1. Introduction: There are no prerequisites for this unit, which is a compulsory unit for the majority of Year One students on mathematics programmes. The unit consists of an introduction to using computers, obtaining electronic information, mathematical typesetting and mathematical problem solving using computers - Lectures are supplemented by extensive practical sessions.

2. Timetable Hours, Credits, Assessments: This unit is of 10 UCU and is taught in the Autumn Semester by means of 10 hours of lectures, (week 2 to 6) supported by 12 hours of practical classes, (week 2 to 7). The Assessment is 100% coursework and is composed of 4 pieces of coursework:

Use of windows, e-mail and the network (10%)

Use of the UNIX system (15%)

Mathematical typesetting with LateX (35%)

Mathematical problem solving with Maple (40%)

The first two pieces of work will be produced in supervised limited time. The last two pieces of work are to be produced in free time.

All pieces of coursework have to be submitted electronically. Feedback is given to students by providing solutions to coursework exercises through the Web and details on the way marking schemes are applied.

A strict anti-plagiarism/collusion policy is applied as prescribed by University regulations. Students are reminded of regulations and made aware of that policy at the start of the Unit.

3. Overview: No prior knowledge of computing is assumed. The emphasis is on using computers for information, retrieval and problem solving in Mathematics, in association with other units making up the degree program.

The main aims of the unit are to:

Show how a computer can help mathematicians.

Introduce students to the computing facilities available at UEA.

Provide a brief introduction to computer hardware and software concepts.

Introduce the LaTeX scientific typesetting package.

Introduce the mathematical software package, Maple.

By the end of the unit, all students will feel comfortable using a computer to:

Communicate electronically.

Transfer files and information.

Search for relevant information.

Prepare good technical documents.

Solve smallish mathematical problems, using packages.

4. Recommended Reading:

There is no recommended reading list for this unit. The lecturer will provide detailed notes on the Web. These notes include:

Electronic versions of all transparencies used in the lectures.

Additional documents produced by the lecturer on Unix and Emacs

"The Not So Short Introduction to LaTeX2e" by Tobias Oetiker Hubert Partl, Irene yna and Elisabeth Schlegl (Free book under the terms of the GNU General Public License as published by the Free Software Foundation).

html version of the Maple worksheets produced by the lecturer.

5. Lecture contents:

Windows, WWW, Email, Networks: **1 Lecture, 2 Practicals**

The Unix operating system: **1 Lecture, 2 Practicals**

Editing text files with Emacs: **1 Lecture, 2 Practicals**

Introduction to mathematical typesetting using LATEX: **3 Lectures, 2 Practicals**

Introduction to the MAPLE mathematical software: **4 Lectures, 2 Practicals**

MTH-1A65 : Symbolic Algebra

In preparation - please contact the webmaster for details.

This page maintained by: webmaster@mth.uea.ac.uk

Last updated:

MTH-1A66 : Programming for Mathematicians

1. Introduction: This unit gives an introduction to computer systems and to programming using Java. Lectures are shared with SYSC1S22 but seminar sheets and coursework are tailored for mathematicians. This unit is an alternative pre-requisite for a number of second level SYS units, including SYSC2B24, SYSC2A23, SYSC2A25 and SYSC2E21.

2. Hours, Credits and Assessment: This unit is delivered as a programme of 33 lectures, supported by seminars and labs. Exercise sheets and lab sheets are set every week throughout the course. Each seminar group has 10 one hour seminars and ten two hours labs during the semester. *Students are expected to attend all seminars and labs.* Extra lab sessions will be arranged if necessary.

The unit is assessed by coursework, contributing to 50% of the unit mark, and by University examination, contributing to 50% of the unit mark.

The coursework consists of three small programming exercises, worth a total of 20% of the unit mark, and a programming project, worth 30% of the unit mark. The small exercises are intended to test students' mastery of the basics of Java, the project is meant to test the students understanding of software development, data abstraction and algorithm design.

Coursework has a deadline which is 1600 hours Friday on the deadline date. Work will be accepted up to five working days after the required submission date, but in this case **it will be penalised by the loss of 20% of the marks awarded.**

Work completed on time is deposited in the appropriate box outside Room S1.50. Work completed late must be **handed** to the School secretary. The work will be date stamped with either the given submission date or the date of submission if later.

Students who have problems with submitting coursework by the due date should report them to the person setting the coursework. Where medical or other problems exist, extensions to coursework deadlines may be given by the Unit Organiser. Proper documentation must be obtained in such cases, for example, a medical certificate, and forwarded to the School Clerk at the time of the difficulty.

3. Overview: To understand the principles of how programs run on computers and how 3G languages like Java convert text into executable code.

* To learn the fundamental building blocks of the Java language, to include:

* statements, arithmetic expressions, logical expressions;

* basic control structures of sequential control, selection and repetition;

* classes and methods;

* scope and the flow of control;

* arrays and vectors;

* inheritance, encapsulation, method overloading and overriding, static methods and classes;

* To introduce the principles behind software engineering through a structured approach to problem solving and an understanding of object oriented programming.

* To introduce the issues behind the development of algorithms using of simple sorting algorithms (selection sort, insertion sort and bubble sort)

* To introduce the concept of modelling real world problems in a program using simple abstract data types (stacks, queues and lists)

Transferable Skills:

* To gain further experience in a broad range of IT skills.

* To encourage a logical and structured approach to problem solving.

* To improve report writing skills through the completion of analysis and design documentation for the courseware project.

* To provide a background suitable for a variety of second level SYS courses.

Learning Outcomes:

On completion of this unit students should be able to:

* Understand the syntax of the Java language.

* Write programs in Java to solve specific problems.

* Design Java classes to be used in solving problems in a wide domain.

* Grasp the principles of object oriented software engineering;

* Appreciate the importance of algorithm design and write well structured algorithms.

* Understand the principles of data abstraction and be able to implement basic abstract data types

4. Recommended literature and references: The are numerous introductory books on Java. We recommend the following as an accompaniment to the lecture notes

J. Lewis and W. Loftus "Java Software Solutions: Foundations of Program Design", 2nd Edition, Addison-Wesley 2000

http://duke.csc.villanova.edu/jss/

Other texts and useful links can be found on

http://www.sys.uea.ac.uk/Units/sys-1s22/

5. Lecture Content

Introduction (5 lectures)

Computers and programs

Problem Solving and Algorithms

Basics of Data Storage and Manipulation

Arithmetic and logical expressions

Simple and compound statements

Control Structures (5 lectures)

Selection

Repetition

Classes and Methods (6 lectures)

Flow of control and scope

References and memory

Arrays and Vectors (4 lectures)

Inheritance and object oriented programming (3 lectures)

Introduction to software development methods (2 lectures)

Introduction to algorithms: Sorting (3 lectures)

Introduction to modelling and data abstraction (3 lectures)

Introduction to applets and GUIs (2 lectures)

MTH-1B81 : Mathematics for Scientists I

1. Introduction: This unit is designed for candidates from outside the School of Mathematics who have reached the A-level or equivalent standard in mathematics and who are candidates for degrees in scientific or other subjects in other Schools.

2. Timetabled Hours, Credits, Assessment: The unit is of 10 UCU and is taught in the Autumn Semester by means of 20 hours of lectures supported by 5 hours of seminars in which candidates are given help with and answers to set homework questions. Assessment is normally by means of set homework and a one-hour course test in week 12. There is no formal university examination.

3. Overview: The course revises briefly the A-level calculus and proceeds to more advanced topics in the calculus and in the algebras of complex numbers and vectors. It is the first of a sequence of three level 1 and 2 units (**Mathematics for Scientists I, II** and **III**) that introduce the analytical techniques necessary for the mathematical modelling of physical phenomena, such as the fluid flows of **Mathematics for Scientists IV**, for example.

4. Recommended reading: The notes taken in lectures from the blackboard are intended to be complete and self-contained in themselves, for the purposes of the Assessment. The lecture topics are basic, long established ones that can be found in a large number of books for sale or in the library. The particular book "Guide to Mathematical Methods" by John Gilbert (published by MacMillan) can be recommended as close in scope and style to the lecture contents in six of its chapters, while the remaining three chapters are relevant to much of the succeeding unit, Mathematics for Scientists II. It is helpful also to refer to a formula book such as "Mathematical Formulae for Engineering and Science Students" by S Barnett and T M Cronin (Longmans).

5. Contents:

A. Differentiation of a Function of One Variable : Definition; application to gradient of a curve, maxima and minima, rate of change. Differentiation of exponential, log, circular, hyperbolic functions and their inverses. Rules for product, quotient, function of a function, parametric relation. **(3 lectures)**

B. Integration of a Function of One Variable : Indefinite (inverse of differentiation) and definite (as limit of sum, area under curve) integration and their relationship. Properties (reversal of limits etc). Methods: change of variable, parts, partial fractions. Arc length of plane curve. Volume of a solid of revolution. **(4 lectures)**

C. Power Series Expansions : Geometric series. Maclaurin series for exponential, logarithmic, binomial, circular and hyperbolic functions. Taylor's series. L'Hôpital's rule for limit of indeterminate quotient. **(3 lectures)**

D. Complex Numbers : Solution of quadratic equations to motivate. Definition and operations (equality, sum, product, quotient, conjugate, modulus and argument). Argand diagram. polar form. De Moivre's theorem; application to trigonometric formulae. Square root of a complex number. **(4 lectures)**

E. Partial Differentiation : Definition. Total derivative, errors. Chain rules. **(3 lectures)**

F. Vector Algebra : Addition law, components. Scalar and vector products. Relative velocity problems. **(3 lectures)**

MTH-1B82 : Mathematics for Scientists II

1. Introduction: This is a level 1 Spring Semester unit which follows on **from Mathematics for Scientists I** which is a prerequisite. The unit aims to introduce ideas and techniques which are particularly relevant to the physical sciences. It includes: ordinary differential equations, line and multiple integrals, matrices and linear equations, and partial differential equations.

2. Hours, Credits and Assessment: The unit is 10 UCU consisting of 20 lectures plus 4 support classes which are primarily devoted to exercises issued in the lectures. It consists of 4 modules on the topics listed above. Assessment is 100% Coursework. There is a unit test of 1 hour.

3. Overview: Line and multiple integrals are concerned with summation of a continuously distributed variable. Thus they include volumes and surface area for curved as well as flat surfaces; but they also cover quantities with non-uniform concentration, material flux and work following a curved path. There is a calculus for manipulation of these integrals.

Matrices are linked to arrays of coefficients of linear algebraic equations. These can be illustrated geometrically but have many applications. Although exercises involve small arrays of numbers some concepts extend readily to very large arrays. Rules for matrix manipulation are developed; determinants are also covered.

Differential equations are equations concerning rates of change and ordinary differential equations involve only one independent variable. For equations involving only first derivatives both linear and non-linear equations are covered. For equations involving higher derivatives only linear equations are covered; linearity allows synthesis of solutions and this greatly simplifies procedures.

Partial differential equations involve two or more independent variables, typically space and time. The main focus is on reducing the solution of linear partial differential equations to the solution of ordinary differential equations and then synthesising solutions. Applications include diffusion and wave processes.

4. Recommended literature and references: The first text covers the material, the second text develops topics to a higher level and is used in subsequent units:

G Stephenson " Mathematical Methods for Science Students" (Longman)

E Kreyzsig "Advanced Engineering Mathematics" (Wiley)

5. Contents:

Ordinary Differential Equations

First order. Separation of variables. Reductions to separable form. Homogeneous type. Linear, integrating factor. Second order, constant coefficients. Homogeneous equation, exponential solutions, real and complex conjugate roots of auxiliary equation. Repeated roots. Non-homogeneous equation, particular integral construction. **(5 lectures)**

Further Integration

Double integrals. Evaluation by repeated integration. Polar co-ordinates. Evaluation of volume. Triple integrals. Line integrals. Length integrals. Area of surface of revolution. Curved surface area by projection. **(5 lectures)**

Linear Algebraic Equations

Solution by elimination. Determinants and Cramer's rule. Non-homogeneous and homogeneous systems. Determinant expansion rules. Matrices: addition, multiplication by a scalar, matrix multiplication, transpose. Identity matrix and powers of square matrices. Non-singular and singular matrices. Inverse. Characteristic equation, eigenvalues, eigenvectors. **(5 lectures)**

Introduction to partial differential equations

Heat conduction as motivation. Linearity. Integration of advection (1st order) and wave equation (2nd order) to illustrate the arbitrary functions in general solutions; initial and boundary value problems are required for determined solutions. **(1 lectures)**

Separation of variables

Application to the diffusion equation and the wave equation, with trigonometric values prescribed initially and various homogeneous boundary conditions. Illustration of waves on a stretched string. The relation between standing waves and progressive waves. Application to the potential equation, with trigonometric values prescribed, exploiting superposition. Orthogonality technique for non-trigonometric prescribed values presented as a 'drill'. **(4 lectures)**

MTH-1S54 : Probability

1. Introduction: This unit provides an introduction to Probability Theory. It is largely self-contained and it is suitable for non-mathematics students with the proper elementary mathematics background.

2 Timetable Hours, Credits, Assessments: This course is a 10 UCU unit of 16 lectures with 3 seminars. Assessment is by coursework (20%), consisting of three homework sheets, and examination (80%).

3. Overview: The term **probability** refers to the study of randomness and uncertainty. In any situation in which one of a number of possible outcomes may occur, the theory of probability provides methods for quantifying the chances or likelihood associated with the various outcomes. The study of probability as a branch of mathematics goes back over 300 years and it is now fundamental prerequisite for the study of statistics.

4. Recommended Reading:

W Mendenhall, D D Wackerly and R L Scheaffer "Mathematical Statistics with Applications" PWS Kent.

5. Lecture Contents:

Introduction, Probability model for an experiment, set theoretic model, discrete case. **(1 lecture)**

Probability, Axioms of probability. **(1 lecture)**

Basic properties, equally likely events. **(1 lecture)**

Combinatorial analysis. **(2 lectures)**

Conditional probability, Independence, Bayes' Theorem. **(1 lecture)**

Random variables and distributions. **(2 lectures)**

Binomial, Geometric and Hypergeometric distributions. **(2 lectures)**

Poisson distribution, Poisson process. **(2 lectures)**

Continuous random variables, expectations. **(1 lecture)**

Uniform distribution, Normal distribution, use of the tables. **(2 lectures)**

Gamma distribution, Exponential distribution, hazard rate functions. **(1 lecture)**

MTH-1S56 : Introduction to Statistics

1. Introduction: This course introduces the main concepts of statistics and assumes prior knowledge of probability but not of statistics.

2 Timetable Hours, Credits, Assessments The course is a 10 UCU unit of 16 lectures + one lab hour a week . Assessment is via two pieces of coursework (20%), both requiring the students to use a computer, and examination (80%). Students are given an extensive collection of sample questions with worked solutions on each module.

3. Overview: The aim of Statistics is to allow inferences to be made from samples about whole populations (inferential statistics), and for data to be summarized numerically and graphically (descriptive statistics). A knowledge of probability is therefore a prerequisite to an understanding of statistics. In this course we start with a few lectures of descriptive statistics and introduce the main ideas of statistical inference, (estimation and testing). We will make considerable use of statistical software.

4. Recommended Reading: Students are asked to buy *one* of the following texts: *Mathematical Statistics with Applications* by Wackerly, Mendenhall and Scheaffer (Duxbury), *Applied Statistics and Probability for Engineers* by Montgomery and Runger (Wiley). These will also be useful in later statistical courses.

5. Lecture Contents:

Samples and populations, random variables. **(1 lecture)**

Random samples and random numbers. **(1 lecture)**

Simulation. **(1 lecture)**

Moments of random variables, Chebyshev. **(2 lectures)**

Sums of independent random variables. **(1 lecture)**

Law of large numbers and the Central limit theorem (no proof). **(2 lectures)**

Sampling distributions. **(2 lectures)**

Estimation and confidence limits. **(2 lectures)**

Permutation tests eg sign tests and Mann Whitney. **(3 lectures)**