# MTH-0B91 : Basic Mathematics I

**1. Introduction:** Taught by lectures and seminars to bring students who have passed GCSE up to 'A'-level standard. The unit is part of UEA "Science with a Foundation Year" degree programme, for students who are either better prepared or more interested in pursuing mathematics. This unit is a pre-requisite for **Basic Mathematics II**, which forms the second half of a year of mathematics study which, for the most successful students, can lead to starting a mathematics degree. The unit covers laws of arithmetic, standard form, algebra, logarithms, quadratics, coordinates, trigonometry and differential calculus (including stationary points and applications to simple extremum problems).

**2. Timetable Hours, Credits, Assessments:** This is a 15 UCU unit taught in 24 hours of lectures and supported by 8 hours of seminars. The assessment is by 100% coursework (coursetest and marked homework). The timing of coursework deadlines are coordinated where possible not to clash with deadlines in other schools. Feedback is in the form of carefully marked coursework and advice on study skills, mathematical presentation etc. to aid students unaccustomed to university demands or who have not studied for some time. Set exercises issued for each coursework and practice worksheets (with solutions) for each seminar.

**3. Overview:** The content of **Basic Mathematics I** has been chosen in coordination with the organisers of Science with a Foundation Year. The aim is to give a good grounding to students who are studying natural sciences and who may want to specialise in mathematics after the end of their preliminary year (having taken **Basic Mathematics II** as well). The unit begins with arithmetic and the representation of numbers in standard form. From there the lectures and exercises progress to gradually more advanced topics, each part building on the new knowledge laid down in preceding lectures. We study algebraic manipulation, variables, unknowns and constants, techniques of solving equations. We solve quadratic equations, and draw and interpret graphs on coordinate axes. The material takes about half the semester and is preparation for differential calculus. The differential calculus, begun by Newton and Leibniz in the seventeenth century, concerns the rate of change of variable quantities. Differentiation helps you calculate for example the instantaneous speed of a falling body. We also show how to find maximum and minimum values of a varying quantity. (The other half of calculus, integration, is studied in **Basic Mathematics II** and is about finding the area of a region bounded by curves.) Calculus is essential to anyone wanting to study higher mathematics. **Basic Mathematics I** contains enough advanced material for students to learn to appreciate the need for rigour (and to be suspicious of "common sense"). Students learn from marked work how to make clear a mathematical argument. Although topics from 'A'-level are missed out (eg statistics) the included material gives a marked 'added value' to a student successfully working at the topics.

**4. Recommended Reading:**

(i) L Bostock, S Chandler "Core Maths for A-level", Stanley Thornes (Publishers) Ltd.
(ii) J K Backhouse, S P T Houldsworth, P J F Horril "Pure Mathematics. Book I" 4th edition, Longmans.
(iii) J Olive "Maths: a Student's Survival Guide", Cambridge.
(iv) (background reading) L Hogben "Mathematics for the Million", Merlin.

The above are in UEA library. For differential calculus each student is issued with an 8-page condensed set of typed lecture notes, which are greatly expanded in the lectures.

**5. Lecture Contents:**

**1. Preparations**

**1.1)** Introduction to studying mathematics**:** rigour, clarity of expression, accuracy and the use of good English. Standard form. Arithmetic and combining powers. Signs, brackets, algebra of powers. **(2 lectures)**

Logarithms - base 10, any base. Examples of the binomial theorem. **(1 lecture)**

**1.2)** Trigonometry - degrees and radians for angles, Phythagoras's Theorem. The right-angled triangle notation for sides and vertices and angles. The definition of the sin cos and tan functions, examples of trigonometric problems. Triangles without a right-angle: the sine and cosine rules. **(4 lectures)**

**1.3)** Constants, variables, unknowns - algebraic manipulation - laws and tips for solving an equation. **(1 lecture)**

**1.4)** Quadratic equations - the formula, factorization roots, examples of problems which lead to quadratics. **(1 lecture)**

**1.5)** The expression of a root of a quadratic as a surd. Surds in general and their combination, including quotients. **(1 lecture)**

**1.6)** Coordinates, sketching curves after plotting points. Appropriate labelling of axes and care with negative ordinates and abscissae. **(1 lecture)**

**2) Differential Calculus:**

**2.1)** Functions and graphs in x, y plane, revisited. Straight line: its equation and slope. The idea of rate of change. Instantaneous rate of change. **(2 lectures)**

**2.2)** Chord, tangent and limiting process described graphically. Coordinates and small coordinate increments. The slope of a line joining two points on a curve - tables showing limit of process. Differentiation from first principles of $x^2$ . **(2 lectures)**

**2.3)** Notation. Sum, product, chain, quotient rules of differentiation. Numerous examples. **(2 lectures)**

**2.4)** Stationary points and how to discriminate between them. Second and higher derivatives. **(2 lectures)**

**2.5)** Differentiation of sin, cos, exp, the importance of the number e . Natural logarithm and its derivative. **(1 lecture)**

**2.6)** Applications: e.g. maximising area with fixed perimeter, and maximising volume with fixed surface area. **(2 lectures)**

# MTH-0B92 : Basic Mathematics II

**1. Introduction:** This unit, together with **Basic Mathematics I**, covers all that is commonly taught in a good traditional A level in Mathematics. Thus it is suitable for anyone thinking of studying Maths to degree level or for somebody wishing to revise A level material. The material covered consists of integration, further trigonometry, complex numbers and vectors.

**2. Timetable Hours, Credits, Assessments:** The unit is a 20 UCU unit of 24 lectures and 8 seminars, the lectures being divided as 12 for integration and further trigonometry and 12 for complex numbers and vectors. Assessment is by coursework (20%) and a 2 hour exam (80%). The lectures are enhanced by many worked examples, the seminars are the best forum for discussion and help.

**3. Overview:**

**Part I - integration and further trigonometry:** One of the most profound and important theorems in mathematics says that the area under a curve can be calculated by first finding the anti-derivative of the function which defines it. This process, called integration is a fascinating area of study. We will develop several techniques for finding integrals, all the while keeping our geometric intuition that we are finding areas under curves. Alongside this, we will review trigonometry, for its own sake but also to enlarge the class of functions we can integrate.

**Part II - complex numbers and vectors:** There is no real number that is equal to the square root of minus one, so we have to invent such a number! We write it as $i = \sqrt{-1}$ and insist that numbers involving $i$ (called complex numbers) behave exactly like real numbers except that $i \times i$ may be replaced by $-1$ wherever it occurs. We now find that **all** quadratic equations have two roots and explore many other properties of complex numbers, e.g. Euler's formula $\exp(i\pi) + 1 = 0$ connecting the five most important numbers in mathematics!

Vectors are used to describe quantities which have direction as well as magnitude. Three examples are e.g. position of a point relative to the origin, velocity and force. We add vectors according to the "parallelogram law" and introduce the "scalar", or "dot", product of two vectors. We can find the angle between two vectors, including whether they are parallel or orthogonal. We explore vector equations of a straight line in 3D and how to find the point of intersections of two lines.

**4. Recommended Reading:**

(i) L Bostock & S Chandler "Core Maths for A Level" (Stanley Thomas)
(ii) J Olive "Maths: a Student's Survival Guide" (Cambridge)

**5. Lecture Contents:**

**Part I**

Integration as area, using $y = x^2$ from $x = 0$ to $x = 1$. **(1 lecture)**

Integration as anti-derivative, review of differentiation and 'look-see' integration. **(1 lecture)**

Integration by substitution, examples, inverse of 'chain-rule'. **(2 lectures)**

Integration by parts as inverse of product rule, examples. **(2 lectures)**

Trigonometry, angles and triangles, Pythagoras' Theorem, basic trigonometry functions. **(1 lecture)**

Differentiation of trigonometry functions and use of these in integration by substitution. **(2 lectures)**

Addition formulae for sine and cosine and use in integration. **(1 lecture)**

Integration of rational functions. **(2 lectures)**

**Part II**

**Complex Numbers**

Definition of $i = \sqrt{(-1)}$. Complex numbers, addition, subtraction and equality. **(1 lecture)**

Multiplication, complex conjugate and division. The Argand diagram. **(1 lecture)**

Quadratic equations. Square roots. **(1 lecture)**

Cube roots of unity. **(1 lecture)**

The modulus and argument (polar) form. **(1 lecture)**

De Moivre's Theorem. **(1 lecture)**

**Vectors**

Definition, examples, vector addition. **(1 lecture)**

Scalar multiplication, unit vectors, geometric properties. Cartesian components, extension to 3D. **(1 lecture)**

The scalar (or dot) product, an orthonormal triad, geometric interpretation. **(1 lecture)**

Component form of scalar product, finding the angle between vectors. **(1 lecture)**

Position vectors. Vector equation of a straight line. **(1 lecture)**

Finding point of intersection of two straight lines. **(1 lecture)**