MTH4001Y : Sets, Numbers and Probability

1. Introduction: This module is compulsory at level 1 for all first degree programmes in mathematics. The module consists of two parts:

Part A: **Sets, Numbers and Proofs (semester 1)**
Part B: **Probability (semester 2)**

2. Hours, Credits and Assessment: This 20-credit module of 40 lectures is taught in semesters 1 and 2. It is compulsory for first-year MTH-students. Support teaching is via tutorials in semester 1 and seminars in semester 2. 20% of the marks for the module come from 3 pieces of coursework in semester 1; 20% comes from 1 piece of coursework in semester 2; the remaining 60% comes from a 2 hour exam which covers all parts of the module.

3. Overview:
Part A: The unit provides a thorough introduction to some systems of numbers commonly found in Mathematics: natural numbers, integers, rational numbers, modular arithmetic. It also introduces common set theoretic notation and terminology and a precise language in which to talk about functions. There is emphasis on precise definitions of concepts and careful proofs of results. Styles of mathematical proofs discussed include: proof by induction, direct proofs, proof by contradiction, contrapositive statements, equivalent statements and the role of examples and counterexamples.

Part B: The term **probability** refers to the study of randomness and uncertainty. In any situation in which one of a number of possible outcomes may occur, the theory of probability provides methods for quantifying the chances or likelihood associated with the various outcomes. The study of probability as a branch of mathematics goes back over 300 years and it is now a fundamental prerequisite for the study of statistics.

4. Recommended Reading:

Part A: Printed notes are available, but the book:

Kevin Houston, 'How to think like a Mathematician', Cambridge University Press

Will be a very useful addition to these notes.

Also useful, but no need to buy these:

Geoff Smith, Introductory Mathematics: Algebra and Analysis, Springer Undergraduate Mathematics Series, Springer-Verlag, London, 1998.

M. Liebeck, A Concise Introduction to Pure Mathematics.

S. Lipschutz, Set Theory and related topics, Schaum's Outline Series, 1998.

Part B:

Ross, S "A First Course in Probability" (Pearson Prentice Hall)

W Mendenhall, D D Wackerly and R L Scheaffer "Mathematical Statistics with Applications" (PWS Kent.)

Y A Rozanov "Probability Theory: A Concise Course" (Dover)

5. Lecture Contents

Part A : (number of lectures is approximate)

Basic set-theoretic notation. Mathematical induction. **(4 lectures)**

Euclidean algorithm. Greatest common divisors. Prime numbers, the Fundamental Theorem of Arithmetic. **(3 lectures)**

Rational and irrational numbers: irrationality of root 2. **(1 lecture)**

Modular arithmetic and applications **(3 lectures)**

Definitions, Theorems and styles of proof. **(1 lecture)**

More on sets. Venn diagrams, union and intersection, distributivity. Difference of 2 sets, complement, De Morgan's laws. Inclusion-exclusion principle and applications. Power set. Ordered pairs. **(3 lectures)**

Equivalence relations; modular arithmetic revisited. **(2 lectures**)

Injectivity and surjectivity of functions. Countability (classical examples of countable and uncountable sets) **(3 lectures)**

Part B: (18 lectures)

Introduction, probability model for an experiment. Sample space, events, Kolmogorov's axioms of probability, basic properties proved from the axioms. Equally likely events, combinatorics.

Conditional probability, independence, Bayes' Theorem.

Random variables and distributions. Expectation, variance, standard deviation, the law of the unconscious statistician.

Binomial, geometric and hypergeometric random variables. Poisson process, Poisson random variable.

Markov chains: definition, transition matrix, classification of states. Long term behaviour of an absorbing chain. .

Continuous random variables, expectation and variance of a continuous random variable.

Uniform and normal random variables, use of the tables.

Exponential random variable. Reliability: series and parallel systems, more general systems, minimal path sets, minimal cut sets.

MTHA4002Y: Linear Algebra

1. Introduction: This year-long module is compulsory at Level 1 for all MTH Students.

2. Timetable Hours, Credits, Assessments: A 20 UCU module of 20 hours of lectures in the Autumn Term and 20 hours of lectures in the Spring Term. Support teaching via tutorials and problem classes. Assessment is by coursework (40%) and one two-hour examination (60%) in May/June.

3. Overview: Linear Algebra plays a key role in pure mathematics and its applications. In this course we provide a thorough introduction and develop this theory from first principles. In the first semester the main topic include matrices over the real numbers, linear equations and the geometric context of linear algebra, leading up to the definition of abstract vector spaces. In the second semester this theory is developed further: Linear spaces, subspaces and linear maps, followed by the metric theory of inner product spaces over the reals. Our emphasis is on showing how abstract concepts become natural and useful in applications.

4. Recommended Reading:

'Introduction to Linear Algebra', Serge Lang, Springer Undergraduate Texts in Mathematics, (First and second semester). This text is widely available electronically. A classical text, recommended as a companion text for the course.

'Linear Algebra, Concepts and Methods' Martin Anthony and Michele Harvey, Cambridge University Press. Recommended as a contemporary companion for the course.

'Linear Algebra', Serge Lang, Springer Undergraduate Texts in Mathematics, (First and second semester)

Watch Gilbert Strang's lectures at MIT. Excellent material on linear algebra can be found at http://ocw.mit.edu/courses/mathematics/18-06sc-linear-algebra-fall-2011/

5. Lecture Contents: (Tentative Schedule)

Autumn Semester:
Matrices: Addition, Scalar Multiplication, Multiplication (3 Lectures)

Systems of Linear Equations: Gauss elimination, elementary matrices, rank of matrix, inverse of a matrix; Determinants, practical examples using Maple. (8 Lectures)

Vectors in R^n : linear independence, spanning set, basis, subspaces, basic geometry in R^3 (4 Lectures)

Eigenvalues and Eigenvectors (3 Lectures)

Definition of abstract vector pace and examples. (2 Lectures)

Spring Semester:

R^n as a vector space (1 Lecture)

Subspace, linear independence, spanning sets, dimension (4 Lectures)

Definition of linear transformation with examples; kernel, image and Rank-Nullity theorem. (5 lectures)

Change of basis, characteristic polynomial and diagonalisation (4 Lectures)

Eigenvalues and Eigenvectors (4 Lectures)

Real inner product spaces, orthogogonal basis and Gram-Schmidt theorem (2 Lectures)

MTHA4003Y : Real Analysis

Hours, Credits and Assessment: The module is a compulsory (for MTH students), 20 UCU module of 40 lectures, split 20 in the Autumn Semester and 20 in the Spring Semester. Support teaching is via tutorials and workshops in the Autumn Semester and via seminars in the Spring Semester. Assessment is by 3 Autumn courseworks (7%, 7%, 6%), two Spring courseworks (10%, 10%) and a Summer exam (60%, 2 hours).

Overview: Analysis is that part of mathematics concerned with functions and graphs. Throughout the 17th and 18th centuries, problems in mathematics and physics were being successfully solved using ideas that we now call Differential Equations, Fourier Analysis, and Fixed Point Theorems. Despite these successes, much of the work was not accurate and many basic questions were left unanswered. For example: What are the real numbers? Which functions have Taylor expansions? Do all differential equations have solutions?

During the 19th century these problems were formulated precisely and solved. The study of analysis begins with a deep understanding of the real numbers, and then goes on to study functions and how they behave. These developments resulted in calculus becoming a rigorous subject. The module will provide you with the basic tools to rigorously examine functions, graphs, and differential equations.

Part I. Real Numbers, Sequences and Series: The study of analysis begins with a deep understanding of the real numbers **R**. We will see the basic properties (axioms) for **R** including the completeness property: any set of real numbers all of which are less than some fixed number has a *least upper bound*. This basic property is the key tool to unlock properties of *sequences* (infinite lists $x_1, x_2, x_3, \ldots$ of numbers) and *series* (infinite sums, like 1+1/2+1/3+1/4+... or 1+1/4+1/9+1/16+...). We give an exact definition of what we mean by *convergence* of a sequence (that is, the notion of tending towards a limit) and develop tests for convergence of sequences and rules for manipulating convergent sequences. Using these results we develop a precise language for talking about series and tests to determine when they converge.

Part II. Functions: We now go on to study functions and how they behave. In particular, we look at the notions of limits, continuity, differentiability and integration, which are the basic tools needed to examine rigorously functions, graphs, and differential equations.

Recommended literature and references: There are many books in the library that will cover the material in the module; the classmarks for Analysis are QA300 and QA303. The following are particularly suitable:

Howie, *Real Analysis*, Springer 2001. An excellent book which covers the module material (and a bit more) in detail. If you want to buy a book this is the top recommendation.
Reade, *An Introduction to Mathematical Analysis*, OUP 1986. Another good book covering the module material. Probably out of print now, but there are copies in the library.
Smith, *Introductory Mathematics: Algebra and Analysis*, Springer 1998. This is a good book for the ``Sets and Numbers" part of module 1A1Y, but chapters 6 and 7 are a gentle introduction to some of the material for this module.
Stirling, *Mathematical Analysis and Proof*, Woodhead 2nd ed 2009. Contains material for 1A1Y as well as this module. A nice feature is that the book includes ``rough working" for many proofs as well as the final polished proof.
Protter and Morrey, *A first course in real analysis*. The material is covered in a different order, but it should be useful for revision or for the second semester.

Lecture Contents:

The Real Numbers: The real numbers as a field. The order and absolute value. Upper bounds and quantifiers. Least upper bounds and completeness. **(6 lectures)**

Sequences: Sequences and their convergence or divergence. Boundedness of convergent sequences, and bounding away from 0. The algebra of convergent sequences. Geometric sequences. The sandwich principle. A bounded increasing sequence converges. Euler's number e as an example. **(8 lectures)**

Series: Convergence for series. Standard examples: harmonic and geometric series. Algebra of convergent series. Basic tests for convergence: comparison test, limit comparison test, ratio test and integral test. The alternating series test. Absolute convergence. **(6 lectures)**

Limits of functions: Neighbourhood, punctured neighbourhood. Bounded functions (above and below). Limit at a point. Standard theorems, including the sandwich theorem. One-sided limits and connection with full limit. Other types of limits. Examples throughout. **(5 lectures)**

Continuity: Definition and standard theorems including equivalent sequential characterisation of continuity. Results concerning function continuous on closed, bounded intervals. Intermediate-Value theorems. **(5 lectures)**

Differentiability: Definition and standard properties. Monotone functions. Local maxima, minima, stationary points. Rolle's theorem, Mean-Value theorem. Applications include connections with monotone functions, Taylor's theorem. **(5 lectures)**

Riemann integral: Partition; norm; upper and lower sums; Riemann integral. Examples of existence and non-existence of integrals. Standard theorems on integrals. Continuity of "an integral". The fundamental theorem of the Integral Calculus. Integration using primitives. Infinite integrals. **(5 lectures)**

MTHA4004Y: Problem Solving, Mechanics & Modelling

1 Introduction

This module is divided into two distinct parts. The autumn semester is about how to approach mathematical problems and how to write mathematics. It aims to promote accurate writing, reading and thinking about mathematics, and to improve students' confidence and abilities to tackle unfamiliar problems and writing proofs. The spring semester is an introduction to Mechanics and Modelling.

2 Timetable Hours, Credits, Assessment

This is a 20-credit module, half in the autumn semester and half in the spring semester. The autumn semester consists of 6 hours of lectures, 16 hours of seminars, and 6 hours of workshops; this is supported by 12 hours of small group peer-guided sessions. The spring semester consists of 20 hours of lectures, supported by 3 seminars and 3 problem classes. The autumn semester is assessed by coursework only: a written assignment (20%), an oral assessment (20%), and an online course-test (10%). The the spring semester is assessed by coursework (20%) and a summer examination (30%).

3 Overview

3.1 Problem Solving (Autumn semester)

The aims are to develop an ability to approach and solve problems independently; a logical writing style adopting appropriate mathematical conventions; oral communication skills, and mathematical confidence. Students will be expected to take active parts in seminars where a number of mathematical problems will be discussed in groups. The aim is not to teach a set part of the mathematics curriculum, but to help students to understand what it means to engage with a mathematical problem and to develop a number of helpful strategies for problem solving.

3.2 Mechanics and Modelling (Spring semester)

Classical mechanics is built upon Newton's three celebrated laws of motion, and concerns the movement of particles, clusters of particles, and solid bodies. It can be used to determine the trajectory of a golf ball or to calculate the motion of the planets. Central to the subject is Newton's second law of motion which states that a mass will accelerate at a rate proportional to the force imposed upon it. This enables us to write down a differential equation describing a particle's acceleration at any given time. Depending on the complexity of the situation, this differential equation can be integrated using elementary methods to determine the particle's velocity and position. We will discuss both particle kinematics and particle dynamics, including resisted motion at low and high speeds. Using Hooke's law, we will examine oscillations of springs and pendulums, and discuss the important phenomenon of resonance. In the latter part of the course we will see how to describe circular motions using polar coordinates, and introduce the notion of a conserved quantity such as energy or linear momentum. The course assumes no prior knowledge of mechanics or physics.

4 Recommended Literature and References

4.1 Problem solving (Autumn semester)

There is no set textbook for the part of the module on problem solving. However, the following will provide some useful background reading:

- HOUSTON, K. *How to Think Like a Mathematician*, CUP (2009).
- TAO, T. *Solving Mathematical Problems: A Personal Perspective*, OUP (2006).
- VIVALDI, F. *Mathematical Writing*, Springer (2014).
  (Available free online at `www.maths.qmul.ac.uk/~fv/books/mw/mwbook.pdf`.)

4.2 Mechanics & Modelling (Spring Semester)

The following is the recommended textbook:

- GREGORY, R.D. *Classical Mechanics*, CUP (2006).

Other books that students may find useful include:

- SMITH, P & SMITH, RC *Mechanics*, Wiley (1990).
- CHESTER, W *Mechanics*, Springer (1979).
- GOLDSTEIN, H. *Classical Mechanics*, Addison Wesley (2001). (Hard)
- COLLINSON, C.D. & ROPER, T. *Particle Mechanics*, Butterwirth–Heinmann (1995).

5 Mechanics and Modelling Lecture Contents

- **Mechanics**

  Rectilinear motion of a particle under gravity. Inverse square law: escape velocity. Newton's laws of motion. Resisted Motion. Terminal velocity. Hooke's law and horizontal/vertical oscillations on elastic string or spring. Forced undamped vibrations: resonance. Beats. Damped motion. **[10 lectures]**

- **Planar motion of a particle**

  Polar components of velocity and acceleration, angular velocity, angular momentum, moment of force as a vector. Projectile in constant gravity (parabola), and with air resistance directly proportional to particle's velocity. Conservative forces. Central forces. Conservation of energy and angular momentum. **[10 lectures]**

MTHA4005Y: Calculus and Multivariable Calculus (Year-long)

1. Introduction: This module is compulsory at level 1 for all single-subject, three and four year, first degree programmes in mathematics. It is an optional module for other candidates who have successfully attained a good A-level or an equivalent standard in mathematics. The module is the first of a succession of level 1 and level 2 modules on Calculus that prepare for subsequent modules in applied mathematics.

2. Hours, Credits and Assessment: This module is of 40 UCU spread over Autumn and Spring semesters. It is taught in the Autumn semester by means of 40 hours of lectures, four per week, supported by 5 hours of tutorials and a 5 hours of problem classes. Assessment in the Autumn semester is by 5 submissions of course work. There are also 40 lectures in the Spring semester supported by 5 hours of seminars and 5 hours of problem classes. Assessment is by submission of course work (40%) and an end-of-year exam (60%).

3. Overview: The origins of integral calculus can be traced back to ancient times, in the calculation by limiting processes of the areas of plane figures enclosed by curved boundaries. The process of differentiation arose in modern times in the seventeenth century, to calculate the slope of a plane curve, and to define rates of change, such as the variable velocity of a falling particle. The inter-relation between differentiation and integration was discovered by Newton and Leibniz, to prepare for the role of the calculus as the language and principal tool of mathematical science. Newton's development of Calculus made the theory and use of Differential Equations possible.

4. Recommended literature and references: The notes taken in lectures from the blackboard are intended to be complete and self-contained in themselves, and do not as a rule follow closely the order, content or style of a particular printed textbook. Summary lecture notes will appear on Portal at the end of each section, together with brief solutions of coursework questions.

It is strongly recommended that students consult books in addition to consulting their lecture notes and course material. The subject is a basic, long established one that is well covered in a large number of books for sale or in the library. A particular book which is close in scope and style to the lecture course is:

J. Gilbert and C. Jordan *Guide to Mathematical Methods*, Palgrave Macmillan.

Other books which also cover the module include:
Adams: *Calculus: A complete course*
Smith & Minton: *Calculus: Concepts & Connections*

The recommended book for the second semester is:

E. Kreyszig: *Advanced Engineering Mathematics*.

This book is a good buy as it is extremely useful also for all the Year 2 methods.

5. Lecture Contents:

AUTUMN SEMESTER

Basic Concepts: Radians, trigonometric identities, exponents, hyperbolic functions, parametric curves. **(1 lecture)**

Complex Numbers:

Definition of complex i . Argand diagram. Operations on complex numbers. Complex exponential form. Rotations and stretching in complex plane. Relation of hyperbolic functions to trigonometric functions. **(5 lectures)**

Vectors:

Basic properties of vectors, notation, algebra, vector geometry, scalar product, vector product, triple products, equations of lines and planes. **(5 lectures)**

Differentiation of a function of one variable:

Definition, motivated by rate of change and gradient of a curve. Maxima and minima, curve sketching. Standard elementary functions: exponential and logarithm, trigonometric, hyperbolic. Rules and examples for differentiation of a sum, product and quotient of functions, a function of a function, the inverse of a function. Parametric and implicit relations. nth derivative of a product by Leibniz formula. **(7 lectures)**

Power Series:

Geometric and general power series, examples, reference to radius of convergence. Properties: addition, multiplication and differentiation of power series. Taylor and Maclaurin series of a function, applications. Indeterminate quotients, l'Hopital's rule. **(3 lectures)**

Integration of a function of one variable:

Indefinite integral as inverse of differentiation. Definite integral: area under curve, and properties, reversal of limits, infinite upper limit etc. Methods: change of variable, integration by parts. Length of plane curves. Volume of a solid of revolution. Reduction formulae, partial fractions for rational integrand. **(8 lectures)**

Ordinary Differential Equations (ODEs):

First-order differential equations of the following type: separable, homogeneous, quasihomogeneous, linear, numerical solutions using Maple. Second-order (and higher order) linear differential equations with constant coefficients: complementary function, particular integral (by trial substitution). Euler type differential equations. Reduction of order. **(12 lectures)**

SPRING SEMESTER

Partial Differentiation:

Real function of two or more real variables. Equation of a surface; contour plots. Definition of first partial derivative; geometric interpretation for two variables. Product and quotient rule. Higher order derivatives; equality of mixed derivatives. Directional derivative and total derivative. Rate of change along a curve. Change of variables. Chain rule. Taylor's theorem for function of two variables. Stationary points; classification. **(10 lectures)**

Multiple Integrals:

Line Integrals. Double integrals interpreted as sum over region of plane: Limits which vary. Examples. Change of variables, Jacobians. Double integrals in polar coordinates e.g. $\exp(-x^2)$. Green's Theorem in the plane. Triple integrals for calculating volume or mass - cylindrical and spherical coordinates. Surface integrals. **(10 lectures)**

Vector Calculus:

Scalar fields, visualization and level surfaces. Directional derivative and rate of change along a curved path. Gradient. Grad as a differential operator. Interpretation of grad. Applications: two-variable Taylor series, surface normal.

Vector fields, visualization and field lines. Flux illustrated as a volume flow and defined by integration over a curved surface with vector surface element. Divergence as a differential operator, defined in terms of components. Interpretation as a flux per unit volume. Properties of the div operator. The Laplacian. Application to gradient fields: Laplace's and Poisson's equations.

Curl as a differential operator, illustrated for particular fields. Irrotational and solenoidal fields. Operators **u**.grad and div.grad as applied to vector fields. Vector operator identities.

Integral theory. The divergence theorem. Application to express div, grad, curl as limits of integrals, and to derive a differential equation from an integral formulation. Stokes' theorem. Path independent line integrals. **(12 lectures)**

Dynamical systems:

Nonlinear differential equations. Phase space as a means to analyse the behavior of differential equations. Elementary oscillations (linear and nonlinear). Equilibrium points and stability. Limit cycles. Nonlinear oscillations and chaos. **(8 lectures)**

MTHB4006Y: Calculus and Probability (Year long)

1. Introduction: This module is an optional module on calculus, complex numbers and vectors for students not following a mathematics degree programme provided that they have successfully attained a good A-level in mathematics, or an equivalent standard. The module is the first of a succession of level 1 and level 2 modules on Calculus that prepare for subsequent modules in applied mathematics. The module finishes with an introduction to Probability.

2. Hours, Credits and Assessment: This module is of 40 UCU and is taught in the Autumn semester by means of 40 hours of lectures, 4 per week, supported by 5 hours of seminars and a 6 hours of problem classes. In the Spring semester there are 20 lectures, 4 per week, with 3 seminars and 2 problem classes on Calculus. Assessment of the Calculus part is solely by submission of coursework. The module finishes with 18 lectures on Probability. There is one piece of coursework and a University exam on Probability.

3. Overview: The origins of integral calculus can be traced back to ancient times, in the calculation by limiting processes of the areas of plane figures enclosed by curved boundaries. The process of differentiation arose in modern times in the seventeenth century, to calculate the slope of a plane curve, and to define rates of change, such as the variable velocity of a falling particle. The inter-relation between differentiation and integration was discovered by Newton and Leibniz, to prepare for the role of the calculus as the language and principal tool of mathematical science. Newton's development of Calculus made the theory and use of Differential Equations possible.

Probability: The term **probability** refers to the study of randomness and uncertainty. In any situation in which one of a number of possible outcomes may occur, the theory of probability provides methods for quantifying the chances or likelihood associated with the various outcomes. The study of probability as a branch of mathematics goes back over 300 years and it is now a fundamental prerequisite for the study of statistics.

4. Recommended literature and references: The notes taken in lectures from the blackboard are intended to be complete and self-contained in themselves, and do not as a rule follow closely the order, content or style of a particular printed textbook. Summaries lecture notes will appear on Portal at the end of each section, together with brief solutions of coursework questions.

It is strongly recommended that students consult books in addition to consulting their lecture notes and course material. The subject is a basic, long established one that is well covered in a large number of books for sale or in the library. A particular book which is close in scope and style to the lecture course and which also covers the spring semester module in Multivariable Calculus is: .

J. Gilbert and C. Jordan *Guide to Mathematical Methods* , Palgrave Macmillan

Other books which also cover the module include:
Adams: *Calculus: A complete course*
Smith & Minton: *Calculus: Concepts & Connections*

Probability
Ross, S "A First Course in Probability" (Pearson Prentice Hall)
W Mendenhall, D D Wackerly and R L Scheaffer "Mathematical Statistics with Applications" (PWS Kent.)
Y A Rozanov "Probability Theory: A Concise Course" (Dover)

5. Lecture Contents:

AUTUMN SEMESTER

Basic Concepts: Radians, trigonometric identities, exponents, hyperbolic functions, parametric curves. **(1 lecture)**

Complex Numbers:

Definition of complex i . Argand diagram. Operations on complex numbers. Complex exponential form. Rotations and stretching in complex plane. Relation of hyperbolic functions to trigonometric functions. **(5 lectures)**

Vectors:

Basic properties of vectors, notation, algebra, vector geometry, scalar product, vector product, triple products, equations of lines and planes. **(5 lectures)**

Differentiation of a function of one variable:

Definition, motivated by rate of change and gradient of a curve. Maxima and minima, curve sketching. Standard elementary functions: exponential and logarithm, trigonometric, hyperbolic. Rules and examples for differentiation of a sum, product and quotient of functions, a function of a function, the inverse of a function. Parametric and implicit relations. nth derivative of a product by Leibniz formula. **(7 lectures)**

Power Series:

Geometric and general power series, examples, reference to radius of convergence. Properties: addition, multiplication and differentiation of power series. Taylor and Maclaurin series of a function, applications. Indeterminate quotients, l'Hopital's rule. **(3 lectures)**

Integration of a function of one variable:

Indefinite integral as inverse of differentiation. Definite integral: area under curve, and properties, reversal of limits, infinite upper limit etc. Methods: change of variable, integration by parts. Length of plane curves. Volume of a solid of revolution. Reduction formulae, partial fractions for rational integrand. **(8 lectures)**

Ordinary Differential Equations (ODEs):

First-order differential equations of the following type: separable, homogeneous, quasihomogeneous, linear, numerical solutions using Maple. Second-order (and higher order) linear differential equations with constant coefficients: complementary function, particular integral (by trial substitution). Euler type differential equations. Reduction of order. **(12 lectures)** **(12 lectures)**

SPRING SEMESTER

Partial Differentiation:

Real function of two or more real variables. Equation of a surface; contour plots. Definition of first partial derivative; geometric interpretation for two variables. Product and quotient rule. Higher order derivatives; equality of mixed derivatives. Directional derivative and total derivative. Rate of change along a curve. Change of variables. Chain rule. Taylor's theorem for function of two variables. Stationary points; classification. **(10 lectures)**

Multiple Integrals:

Line Integrals. Double integrals interpreted as sum over region of plane: Limits which vary. Examples. Change of variables, Jacobians. Double integrals in polar coordinates e.g. $\exp(-x^2)$. Green's Theorem in the plane. Triple integrals for calculating volume or mass - cylindrical and spherical coordinates. Surface integrals. **(10 lectures)**

Probability

Introduction, probability model for an experiment. Sample space, events, Kolmogorov's axioms of probability, basic properties proved from the axioms. Equally likely events, combinatorics.

Conditional probability, independence, Bayes' Theorem.

Random variables and distributions. Expectation, variance, standard deviation, the law of the unconscious statistician.

Binomial, geometric and hypergeometric random variables. Poisson process, Poisson random variable.

Markov chains: definition, transition matrix, classification of states. Long term behaviour of an absorbing chain.

Continuous random variables, expectation and variance of a continuous random variable.

Uniform and normal random variables, use of the tables.

Exponential random variable. Reliability: series and parallel systems, more general systems, minimal path sets, minimal cut sets. **(18 lectures)**

MTHB4007B : Probability and Mechanics

1. Introduction: The module consists of two parts both in the Spring semester: Probability and Mechanics.

2. Hours, Credits and Assessment: This 20-credit module taught in semester 2. Support teaching is via seminars and problems classes. Assessment is by coursework and exam. The exam covers all parts of the module.

3. Overview: The term **probability** refers to the study of randomness and uncertainty. In any situation in which one of a number of possible outcomes may occur, the theory of probability provides methods for quantifying the chances or likelihood associated with the various outcomes. The study of probability as a branch of mathematics goes back over 300 years and it is now a fundamental prerequisite for the study of statistics. **Classical mechanics** is built upon Newton's three celebrated laws of motion, and concerns the movement of particles, clusters of particles, and solid bodies. It can be used to determine the trajectory of a golf ball or to calculate the motion of the planets. Central to the subject is Newton's second law of motion which states that a mass will accelerate at a rate proportional to the force imposed upon it. This enables us to write down a differential equation describing a particle's acceleration at any given time. Depending on the complexity of the situation, this differential equation can be integrated using elementary methods to determine the particle's velocity and position. We will discuss both particle kinematics and particle dynamics, including resisted motion at low and high speeds. Using Hooke's law, we will examine oscillations of springs and pendulums, and discuss the important phenomenon of resonance. In the latter part of the course we will see how to describe circular motions using polar coordinates, and introduce the notion of a conserved quantity such as energy or linear momentum. The course assumes no prior knowledge of mechanics or physics.

4. Recommended Reading:

Ross, S "A First Course in Probability" (Pearson Prentice Hall)

W Mendenhall, D D Wackerly and R L Scheaffer "Mathematical Statistics with Applications" (PWS Kent.)

Y A Rozanov "Probability Theory: A Concise Course" (Dover)

R D Gregory, *Classical Mechanics,* Cambridge University Press.

5. Lecture Contents

Probability

Introduction, probability model for an experiment. Sample space, events, Kolmogorov's axioms of probability, basic properties proved from the axioms. Equally likely events, combinatorics.

Conditional probability, independence, Bayes' Theorem.

Random variables and distributions. Expectation, variance, standard deviation, the law of the unconscious statistician.

Binomial, geometric and hypergeometric random variables. Poisson process, Poisson random variable.

Markov chains: definition, transition matrix, classification of states. Long term behaviour of an absorbing chain. .

Continuous random variables, expectation and variance of a continuous random variable.

Uniform and normal random variables, use of the tables.

Exponential random variable. Reliability: series and parallel systems, more general systems, minimal path sets, minimal cut sets. **(18 lectures)**

Mechanics: Rectilinear motion of a particle under gravity. Inverse square law: escape velocity. Newton's laws of motion. Resisted Motion. Terminal velocity. Hooke's law and horizontal/vertical oscillations on elastic string or spring. Forced undamped vibrations: resonance. Beats. Damped motion. Polar components of velocity and acceleration, angular velocity, angular momentum, moment of force as a vector. Projectile in constant gravity (parabola), and with air resistance directly proportional to particle's velocity. Conservative forces. Central forces. Conservation of energy and angular momentum. **(20 lectures)**