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This version is unformatted, without page numbers, with footnotes as endnotes and footnote references in bold. On the same topic you may also like to read the preceding article

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TRIGONOMETRY IN ROMAN CADASTRES

John Peterson

Un cadastre centurié romain peut inclure d'autres traits topographiques - également romain - comme les routes. Ces traits rectilignes peuvent couper le cadastre en biais, et apparemment ils ne tiennent aucun compte de sa grille. Mais les apparences initiales trompent. Dans plusiers cas, on voit clairement des liens trigonométriques. Il est peu probable que ces rapports aient lieu par hazard. Ces faits, ainsi que l'evidence des documents contemporains, démontrent l'organisation systématique de ces rapports. S'il en est généralement ainsi, la présence de ces liens trigonométriques peuvent démontrer l'existence d'un cadastre centurié, alors qu'à première vue la grille cadastrale n'apparaît pas.

A Roman centuriated cadastre may include other Roman linear features - such as roads - which are oblique to the square grid, and appear to ignore it. But initial impressions are deceptive; there are several cases which reveal clear trigonometrical links. These relationships are unlikely to have occurred by chance and, supported by evidence from contemporary documentation, they indicate that the links were planned. If this is generally so, the presence of these trigonometrical relationships can suggest that a centuriated cadastre existed, even if its grid is not immediately apparent.

This paper describes the development, in the last few years, of a theory of oblique planning - a theory which now seems to be supported by the findings of others. It proposes that a particular sort of planning produced a limited set of angles between oblique linear features and cadastres. It then suggests that this theory is not just a modern invention; it seems to have been in the minds of the *agrimensores*; and this makes it likely that there was some written source from which they learnt it. Finally, it shows how the theory can be inverted to argue backwards - from the existence of certain relationships between linear features to the specification of cadastral grids which could have produced them.

I. A theory of oblique planning

In the investigation of the "South Norfolk A" cadastre **(1)** attention was first focussed on the possible surviving traces of *limites*. Once the positions of some of these had been suggested by two well-preserved areas of rectilinear landscape and a small fragment of Roman road, a computer program was used to calculate the coordinates of other theoretical *limites* intersections, over an area approximately 15 x 25 km, in order to see which features of the modern landscape fitted the grid.

It was noted that many of the existing junctions on a major Roman road which crosses the cadastre coincided in a non-random way with the E-W *limites*; figure 1 shows examples of this on the most northerly straight segment of the road. It was also noted that two segments of this road - the largest segment shown in figure 1 and a longer segment immediately to the South - have a common orientation. They are parallel and are both at about 30˚ to the cadastral grid. Given that multiples of 30˚ play a major part in Ulrix's theory **(2)**, it was initially supposed that further study of this case would reveal the same angle. However, after making careful measurements of the angles between the road segments and the Ordnance Survey grid, using a vernier protractor, it was clear that the angles between these segments and the cadastre were likely to be closer to 31˚, rather than 30˚.

This was a disappointing result. Nevertheless, since the angles of the road segments appear to be the same, the idea persisted that they could have both been planned in the same way, possibly in relation to the cadastre. Furthermore it was recognised that a numerical specification, in Roman mathematics, would not involve real numbers, but must be in integers or, perhaps, ratios of integers. This was why 30˚ had been attractive, because its sine is 1/2.

Since a theory based on the use of rational sines was unsatisfactory, attention then turned to rational tangents. This produced a much better result. $\tan^{-1} 3/5$ is 30.96˚, more nearly in agreement with the measured angles. Furthermore, the segment A-B of the Roman road shown in figure 1 passes through a point halfway along the side of a century (point X). This is clearly a point of special significance in the layout of the century's internal divisions, which is likely to have been marked by a *terminus*. This road segment would thus fit a model in which, in principle, oblique features could have been surveyed by joining the *termini* of the cadastre. The oblique feature is then the diagonal of rectangles with integral sides, formed by the cadastral axes; and its angle is determined by their ratio. Hence the angle of the road segment A-B in figure 1 can be expressed as 5:3, using the convention that the length along the axis nearer to north is given first.

Now, this rather simple theory seemed to fit the South Norfolk A cadastre, but it was unlikely to be believed in this context. Extreme scepticism meets any claim that a centuriated Roman cadastre could possibly be found in Britain. British scholars argue against the idea **(3)**; they also tend to deny certain realities, such as superimpositions of cadastres and other features, which are crucial to the understanding of the landscapes of Antiquity **(4)**. In such a climate of opinion, the South Norfolk system - despite its obviousness in comparison to some other genuine cadastres **(5)** - was liable to instant dismissal, along with the purported trigonometrical relationships. It was therefore necessary to discover if this oblique planning theory could be applied generally, thus demonstrating that it is not constructed - *ad hoc* - solely to explain the features of this small part of

Britain. This possibility was investigated by examining data from other parts of the Roman world.

Two publications were initially surveyed: that of J. D. Bradford (1957), and that edited by M. Clavel-Lévêque (1983) **(6)**. Many oblique features appeared **(7)**, and a number of these seemed to have an angle to the cadastre which could be expressed as a ratio of small integers. Also, in several cases the relationship seemed not to be accidental, either because the angle is consistently maintained for a large number of grid squares **(8)**, or because the alignment clearly passes through *termini*. Figures 2 and 3 lists all the oblique features together with the author's confidence in the genuineness of each apparent relationship, when it occurs.

The apparently non-random nature of these coincidences suggests that many of these relationships are the result of planning **(9)**. Furthermore it appears that in 20 *actus* grids the relationship could not be <u>any</u> ratio. This limitation suggests a way in which the orientation may have been specified.

The Romans represented ratios less than unity as a whole number of smaller units **(10)**. This limits the ways in which a unit can be divided. For example, the only fractions into which the *as* (12 *unciae*) could be divided, without going to units smaller than the *uncia*, were multiples of divisors of twelve. The Romans could write (and, presumably, talk) about a fraction but they specified it in mathematical notation as a whole number of parts. So *quadrans*, one <u>fourth</u> (of an *as*, twelve *unciae*) was notated as **=-** (<u>three</u> *unciae*). In this notation certain fractions could be expressed, but others, e.g. 2/5, could not, at least not in *unciae*.

The tangents of the angles of oblique features could have been defined in a similar way. The orientation of an oblique feature could have been specified by one side of a right angled triangle with its hypotenuse at the desired angle, given that the (implicit) other side was one grid distance (in this case 20 *actus*). For example, XV would indicate 15, i.e. 15:20 or 3:4.

This method of specification corresponds precisely to that used by the Egyptians to specify the slope of pyramids at so-many palms and fingers per cubit, as documented in the Rhind papyrus **(11)**; and it fits the observed data. By using a range of multiples of two and five (divided by 20), we can represent all the relationships listed in figures 2 and 3, as figure 4 demonstrates.

It is noticeable that, even in this small sample, there is a marked preference for angles with simple ratios, i.e those having a small sum of numerator and denominator when reduced to the lowest terms. This could imply that this hypothetical specification method is unnecessarily complex. Could the surveyors have worked on the ground, joining up *termini* which were quite near to each other, with no other restriction on their choice of angle?

The existence of oblique features which seem to use the ratio 11:4 (LV) argues against this idea **(12)**. Surveying this angle directly on the ground would be awkward in practice since it would involve the use of remote *termini*. Also, why does this angle appear, when a simpler angle such as 3:7 does not? It seems more likely that the angles were normally constrained to those shown in figure 4 because they were specified in writing; indeed it is difficult to see how the layout of a large cadastre and its associated main roads could have been carried out unless written instructions were produced for the survey teams on the ground. If so, and if only certain angles could have been specified, this would explain why only they are observed

This theory can be tested, since we are dealing with a constrained system whose outputs are predictable. There are some potential grid-related alignments which should not be observed in practice in a 20 *actus* grid. The simplest, and the most distinguishable, would be 3:7, 6:7, 7:9, and 7:11. The theory would be weakened if these alignments were observed **(13)**.

II. Further evidence

The theory described above was developed independently of other work; it had in fact already been suggested in 1983 **(14)** that a section of the *Via Domitia* between Béziers and St-Thibéry forms the diagonal of a series of rectangles of 3x4 squares. Such independent observation of the same phenomenon in widely separated locations is striking. Did such relationships appear because they were bound to appear, on the basis of chance, sooner or later? Or do they represent different aspects of some common reality? If so, why did they appear at about the same time?

It is possible that the relationships occurred by chance, but it is not very likely. An estimate, based on computer simulation, indicates that the chance is not better than 1 in 20 that a randomly determined grid will fit a section of straight road at one of the angles listed in figure 4, given that we allow a reasonable tolerance on the size of the grid points **(15)**. Nevertheless, a persistent search through several cadastres would inevitably reveal a road "linked" to the grid purely by accident.

But this is not so in the examples we are discussing. In both cases - South Norfolk and Béziers - the geographical areas were among the first that the investigators had worked on. It is not a question of an extensive search and selection of good examples to fit a theory. Several oblique relationships appeared in each area, at the first attempt. This makes it look more likely that the trigonometrical relationships were really planned. If so, it seems reasonable to suppose they were discovered because of improvement in techniques - for example the use of computer-ruled grids and computer-calculated coordinates - and a greater concern for the accurate measurement of landscape.

Another example, apparently perceived without any knowledge of what "should" appear, can be seen in a cadastre of Turin **(16)**. This is a clear 1:1. Further instances occur in Central-Southern Italy **(17)**. One of these, which the authors point out, is at 2:3. Another is at 1:1.

Cremona provides a final example. Some probable cases of oblique planning are shown in figure 5, based upon Tozzi's reconstruction **(18)** of the first cadastre. It seems that the northern quadrant, framed by roads at 90˚, is further divided into three sectors by roads which partially use the ratio 3:5 and 5:3. The streets of Cremona itself also appear to be at this angle to the cadastre.

III. Contemporary documentation

Despite difficulties of interpretation, there are some signs from contemporary documentation that the *agrimensores* were aware that oblique relationships occurred, or should occur, in Roman cadastres.

The stone *formae* of Orange provide a number of examples of oblique relationships of roads - these are listed in figure 3 - and closer examination of Piganiol's illustrations **(19)** reveals a number of other oblique lines carved in the stone. However, the meaning of these, and their angular relationship, is not clear.

And, although the representation of one relationship - that of "Agrippa's road" to cadastre B of Orange - is clear, it also raises some problems. The area under discussion is immediately south of the Logis de Berre, which is approached from the south by the modern D 158. There is a current consensus that this road is very likely to overlay the course of the Roman main road **(20)**, and this hypothesis is strengthened, in part, by the observation that the point in the modern landscape where the first *limes* CK and the seventeenth *limes* DD would have intersected - at x = 791,61, y = 3236,37 (21) - lies at a distance of only about 10 metres from the centre line of the modern road; this coincidence is in agreement with the representation on the *forma*, which clearly shows the road passing through this point. The problem is that the *forma* shows the road bending, onto an alignment which looks very much like 5:1 **(22)**, whereas the D 158 goes straight on.

Now, it is possible that the *forma* is correct, that the D 158 is only partly based on "Agrippa's road", that the Roman road turns more to the west. But it is also possible that the engraver drew it that way because he knew how it ought to be drawn, whether it corresponded to external reality or not. He knew - presumably as a result of theoretical studies - that road segments should pass through intersections of the grid, and that they should usually show rational oblique relationships.

Of course, this is a heavy load of speculation to put on one example; the engraver may have just made a mistake and hit upon 5:1 by accident. However, there are the other rational oblique relationships, through grid points, in the *forma* of cadastre A, and there are also illustrations which may show such relationships in Lachman's representation of the manuscript vignettes of the *corpus agrimensorum*: for example La. Fig. 23 (possibly 2:5) and La. Fig. 135 (apparently 1:1) **(23)**. This clearly provides some support for the existence, during the Roman period, of a theory of oblique planning, which would perhaps have been recorded in another part of the *corpus agrimensorum*, or in a hypothetical road surveying "training manual", now lost.

IV. Applications to research

There is now enough evidence to make us believe, with considerable confidence, that many of the relationships between oblique linear features and Roman cadastres were planned. If so, the proposed planning mechanism would produce only a limited range of angles between, for example, roads and grids, or roads and other roads. This "spectrum" of angles could thus be a symptom of such planning in the context of a known or suspected cadastre.

Two examples illustrate this. Firstly, Rita Compatangelo **(24)** has demonstrated the presence, in the southern tip of Italy, of a centuriation whose angle is N 36˚ 50' E (or 36.83˚ if we adopt the convention that angles to the east of north are positive). She also describes (p. 132) the presence of a number of roads whose angle is 55˚. The difference between these two angle is 18.17˚, which is only a quarter of a degree away from the angle whose tangent is 1/3, which is 18.43˚. This raises some questions: are the roads parallel because they were planned at the same angle to the cadastre, as in the example from South Norfolk? Or are they part of another cadastre which has a 1:3 planned relationship to Compatangelo's stone cadastre?

Another example arises in Gallia Belgica, in the *civitas Tungrorum*. G. Raepsaet demonstrates the existence of an area, 25 by 50 km, of commonly oriented ancient land parcelling in the *pagus Condrustis*, about 50 km south of Tongres **(25)**. Prudently, he comes to no certain conclusion about the origin of the system. He puts several arguments for, and against, the idea that it is a centuriation. One of these concerns the relationship of main Roman roads, as follows.

«La relation du cadastre avec les routes antiques est peut-être plus significative. La route présumée ancienne de Dinant vers Cologne sert d'appui, en plusiers points, à un parcellaire quadrillé. *Mais* l'Arlon-Tongres, dont le tracé et l'ancienneté sont reconnus, traverse de biais le parcellaire dans les environs de Clavier-Vervoz.» **(26)** [Emphasis added]

So, in favour of the centuriation hypothesis, one road shares its orientation; but, against the hypothesis (in Raepsaet's eyes), the other road is oblique.

However, the oblique road could also have some planned relationship to the cadastre. The angle of the Dinant-Cologne road is given as 58˚ and 59˚, and these are also stated to be the angles of some of the cadastral traces in the area of Clavier. The angle of the oblique Arlons-Tongres road is given as 347˚ (-13˚). The angle between the roads is thus 71˚ or 72˚, around the angle, 71.57˚, whose tangent is 3. There is thus no "but" about it. The angle to the cadastre of the oblique road (3:1) supports the idea of a centuriation just as strongly as the angle of the parallel road (0:1).

There are also cases where a centuriated cadastre may be suspected, but no definite "privileged" orientation of the countryside is immediately apparent. In such cases the observation that the angles between Roman roads are those produced by rational planning can be used to generate hypotheses about the orientation and position of cadastres which might have caused such angles to appear.

Two British examples (see figures 6 and 7) demonstrate the application of this technique. In both cases, once the rational relationship between the roads (labelled A and B) has been observed, a trial orientation for the cadastre (shown by the arrow) can be adopted which best fits the existing road network and the natural topography. This leads to the observation of one or more other Roman roads (C, D) which also fit the grid obliquely.

Both these cases have a simple solution; the cadastre is positioned so that road A coincides with one of its *limites*, and Road B goes through grid points. But this is not the only sort of solution that can be attempted, since it may also be possible to put a grid into simultaneous rational relationship with two roads. For example, if we observe that two roads are at 1:1 then a grid can be positioned so that it is at 1:3 to one of them and 1:2 to the other. This is so because, in general,

$$\mathrm{Tan}(a + b) = \frac{\mathrm{Tan}(a) + \mathrm{Tan}(b)}{1 - \mathrm{Tan}(a).\mathrm{Tan}(b)}$$

which can be satisfied by setting Tan(a) = 1/2 and Tan(b) = 1/3, since this gives Tan(a + b) = 1.

This problem can be restated as follows: given a ratio c, find two other ratios a and b that satisfy the equation

$$\frac{a + b}{1 - ab} = c$$

or

$$c - (a + b) = abc$$

This is a Diophantine equation **(27)**, expressable in words as "find three rational numbers such that the difference between one and the sum of the other two is equal to their product". Were the mathematicians of the Alexandrian school aware that such problems could arise in this context? Was there originally some practical impetus for this area of study **(28)**?

Whatever its implications for the history (or even the archaeology) of Mathematics, the theory of oblique planning has considerable potential in the study of the remains of Roman land planning. It could be called a "key" to the Roman landscape. Powerful hypotheses can be constructed from very simple observations of a limited number of angles between Roman features. These angles may betray the presence of Roman land planning, even in the absence of obvious physical traces. They are, like a spectrum, a means of grasping a reality which lies, at least for now, beyond our direct experience.

In this context, the words of Francis Haverfield, one of this century's most distinguished Romanists, appear particularly prophetic.

«I venture to suggest to antiquaries who have a taste for playing with instruments that they should measure the relations to the north point of any really straight pieces of Roman road which interest them, and note the deflection of each roadway from the north. I suspect that curious coincidences might be discovered, which would throw light on the roads and centuriation of Britain, and might also help to explain the process by which the Roman roads were laid out so straight - a process which I think has not yet been fully solved, but of which I cannot here treat in detail.»**(29)**

Now, after 70 years, and with the development of computer aids, we have the "instruments" which allow us to do as he suggested.

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Notes

1. The author's investigation of a possible Roman cadastre in South Norfolk started in late 1986; some initial results were published in J. W. M. Peterson «Roman cadastres in Britain: I - South Norfolk A» *Dialogues d'Histoire Ancienne* 14, 1988, p. 167-199. At the same time a contrary view was being presented, see T. M. Williamson «Parish Boundaries and early fields: continuity and discontinuity» *Journal of Historical Geography* 12 (3), 1986, p. 241-248. According to this theory the southern part of the system is a coaxial field system which - because of the oblique relationship - predates the main Roman road and which could thus be prehistoric. See also T. M. Williamson «Early Co-axial Field Systems on the East Anglian Boulder Clays» *Proceedings of the Prehistoric Society* 53, 1987, p. 419-431.

2. F. Ulrix «Recherches sur la méthode de traçage des routes romaines» *Latomus,* XXII (2), 1963, pp. 157-181.

3. Sometimes these arguments are based on "facts" which go against the evidence. For example, it is hard to believe the claim of S. E. Cleary in *Extra-mural areas of Romano-British towns*, BAR 169, Oxford, 1987. p. 58 that «Huge areas of formal limitatio seem to have passed out of fashion at the beginning of our era,..». He uses this "fact" to support his claim that the *colonia* of Camulodunum was «probably» never provided with a centuriated cadastre. This is contradicted by the existence of the large Saône Plain and Béziers A systems, both initiated in the second half of the first century, see p. 286 of G. Chouquer and H. de Klijn «Le Finage antique et médiéval» *Gallia* 46, 1989, p. 261-299, and M. Clavel-Lévêque, this volume.

4. See Anthony King, *Roman Gaul and Germany*, London, British Museum Publications, 1989, p. 99. He doubts the existence of the

three superimposed Bézier cadastres on the grounds that it «.. seems to fly in the face of evidence of continuity from most areas through long periods of time once a system was established». This argument, which is based on the assumption that re-cadastration implies discontinuity, is weak. If it were true, there would have been discontinuity of land use in England during the 19 century parliamentary enclosures. In any case, evidence of superimpositions has been available for many years from Africa; see, for example, R. Chevallier and J. Soyer «Cadastres Romains d'Algérie» *Bull. de la Société Française de photogrammétrie* 5, 1962, p. 43-48, who mention (p. 45) centuriations «parfois superposées», and O. A. W. Dilke, *The Roman Land Surveyors*, Newton Abbot, David & Charles, 1971, p. 155.

5. Nimes A was cited in Peterson op. cit. (note 1) fig 9. There are other examples, such as the three grids of Béziers, which are hard to see in places.

6. J. D. Bradford, *Ancient Landscapes*, London, Bell, 1957, and M. Clavel-Lévêque, *Cadastres et Espace Rural . Approches et réalités antiques* (table ronde de Besançon mai 1980), Paris, CNRS, 1983.

7. All observable straight features in Bradford's op. cit. (note 6) illustrations were included, even if possibly modern, in order to avoid accusations that data was selected to fit the theory.

8. See, for example, Bradford's op. cit. (note 6) Plate 38 which shows a road at 1:1 to the centuriation near Cesena.

9. This statement is not meant to imply that all oblique features with a rational relationship must postdate the grid. The oblique road cited in note 8 appears to be a counter-example, see pp 217-221 of M. Clavel-Lévêque «Pratiques impérialistes et implantations cadastrales» *KTEMA* 8, 1983, p. 185-251.

10. See D. E. Smith, *History of Mathematics*, New York, 1951, p. 208.

11. See O. A. W. Dilke, *Mathematics and Measurement . - Reading the past*, London, 1987, p. 9.

12. The angle occurs here twice, in figure 3. It also occurs in South Norfolk, see Peterson op. cit. (note 1) p. 173, and north of London, see J. W. M. Peterson «Roman cadastres in Britain 2» *Dialogues d'Histoire Ancienne* 16, 1990, (in press).

13. But note that the theory would not be falsified in the Popperian sense, as was claimed on p. 139 of my article, J. W. M. Peterson «Information systems and the interpretation of Roman cadastres» in S. P. Q. Rahtz, (ed.), *Computer and Quantitative Methods in Archaeology: CAA 88* BAR S446, Oxford, 1988, pp. 133-149. This system of oblique planning cannot be regarded as positivistic, since there is a possibility that such a rational relationship, even through *termini*, will be found by chance, see note 15.

14. See G. Chouquer, M. Clavel-Lévêque, M. Dodinet, F. Favory and J.-L. Fiches «Cadastres et Voie Domitienne» *Dialogues d'Histoire Ancienne* 9, 1983, pp. 87-122. But even this was not the first suggestion of oblique relationships, since on p. 257 of their paper «Reciproca rotazione di tracciata delle "Centuriato" romane» *L'Universo* 57, 1977, pp. 249-269, L. Morra and R. Nelva proposed that the rotation of one cadastre with respect to another might be limited to angles such as Tan^{-1} 1/2, Tan^{-1} 1/3, ..., etc. However this proposition is not as general as the theory discussed here.

15.Peterson op. cit. (note 13) describes a computer simulation, "throwing" lines representing oblique features onto a square grid, using a point radius of 2.8% of the grid size (which corresponds to 0.4 mm for a representation of a 20 actus grid on 1:50,000 maps). This indicates that about 3.5% of these random lines will conform to the oblique planning hypothesis.

16. See C. Cerato Pontrandolfo «Lo sviluppo delle rete viaria» in G. C. Cresci Marrone and E. C. Culasso Gastaldi (eds.) *Torino Romano*, Padova, Editoriale Programma, 1988, pp. 185-193. The road is described thus: «Ricalcato su una strada antica è quasi certamente anche il tratto della statale nr. 460 che unisce Rivarola a Salassa ..., attraversando *in diagonale* le maglie del reticolo centuriato ...» [p. 189, emphasis added.]

17. See G. Chouquer, M. Clavel-Lévêque, F. Favory, and J. P. Vallat, *Structures agraires en Italie centro-méridionale*, Rome, l'Ecole Francaise de Rome, 1987. Firstly p. 229 (fig. 78), relationship of *Atella II* and *ager Campanus II*, at 3:2. Secondly the 1:1 relationship of the *Via Praenestina* to the *ager Collatinus*, best illustrated on p. 287 (fig. 104).

18. P. Tozzi, *Storia padana antica. Il territorio fra Adda e Mincio*, Milano 1972.

19. A. Piganiol, *Les documents cadastraux de la colonie romaine d'Orange (Gallia, Supplement XVI)*, Paris, CNRS, 1962.

20. See, for example, F. Salviat «Le Cadastre B d'Orange : la route au sud de Montélimar» *Rev. Arch. de Narbonnaise* 18, 1985, pp. 277-287.

21. This coordinate was calculated by establishing the position of the cadastre so that it fitted the traces discovered by aerial photography and excavation, see V. Bel and J. Benoit «Les limites du cadastre B d'Orange» *Rev. Arch. de Narbonnaise* 19, 1986, pp. 79-99.

22. See O. A. W. Dilke «Archaeological and Epigraphic Evidence of Roman Land Surveys» in H. Temporini (ed.), *Aufstieg und Niedergang der Römischen Welt, Vol 2, Part 1*, Berlin, de Gruyter, 1974, p 564-592. Dilke observes (p. 576) that the bend shown on the forma does not occur in the modern road. He also draws attention to the faint line on the engraving which continues the line of the road; this passes through the centre of the representation of century DDXX CKI, thus strengthening the impression that the road is deliberately depicted as 5:1.

23. I am most grateful to M. J C Rieux for drawing these figures to my attention. However, perhaps too much weight should be put on them, since they are not the original drawings. Clearly, a more detailed study of the manuscript figures is needed before we can be confident that the theoretical relationships are deliberately depicted in this way.

24. R. Compatangelo, *Un cadastre de pierre: le Salento romain*, Paris, Les Belles Lettres, 1989.

25. G. Raepsaet «Quelques aspects de la division du sol en pays tongre» in D. Haupt and H. G. Horn (eds), *Studien zu den Militärgrezen Roms 2. Vorträge des 10 Internationalen Limeskongress in der Germania Inferior*, Koln, Rheinland Verlag - Bonn, Habelt, 1977, p 147-157.

26. G. Raepsaet, op. cit. (note 26) p. 155.

27. See, for examples, O. Ore, *Number Theory and its History,* 1948, pp. 165-208.

28. See L. Hogben, *Mathematics in the Making*, London, 1960. Hogben presents the growth of Mathematics as a response to material needs and intellectual climate. His remarks (p. 123) are notable: «Such so-called Diophantine equations have subsequently had an understandable fascination for pure mathematicians; but they have had no pay-off in the domain of measurement». The discovery of multiple oblique relationships which are related by such equations would suggest that this last statement is no longer true.

29. F. Haverfield «Centuriation in Roman Essex» *Trans. Essex Arch. Soc.* XV, 1921, p 115-125, conclusion.

Figures

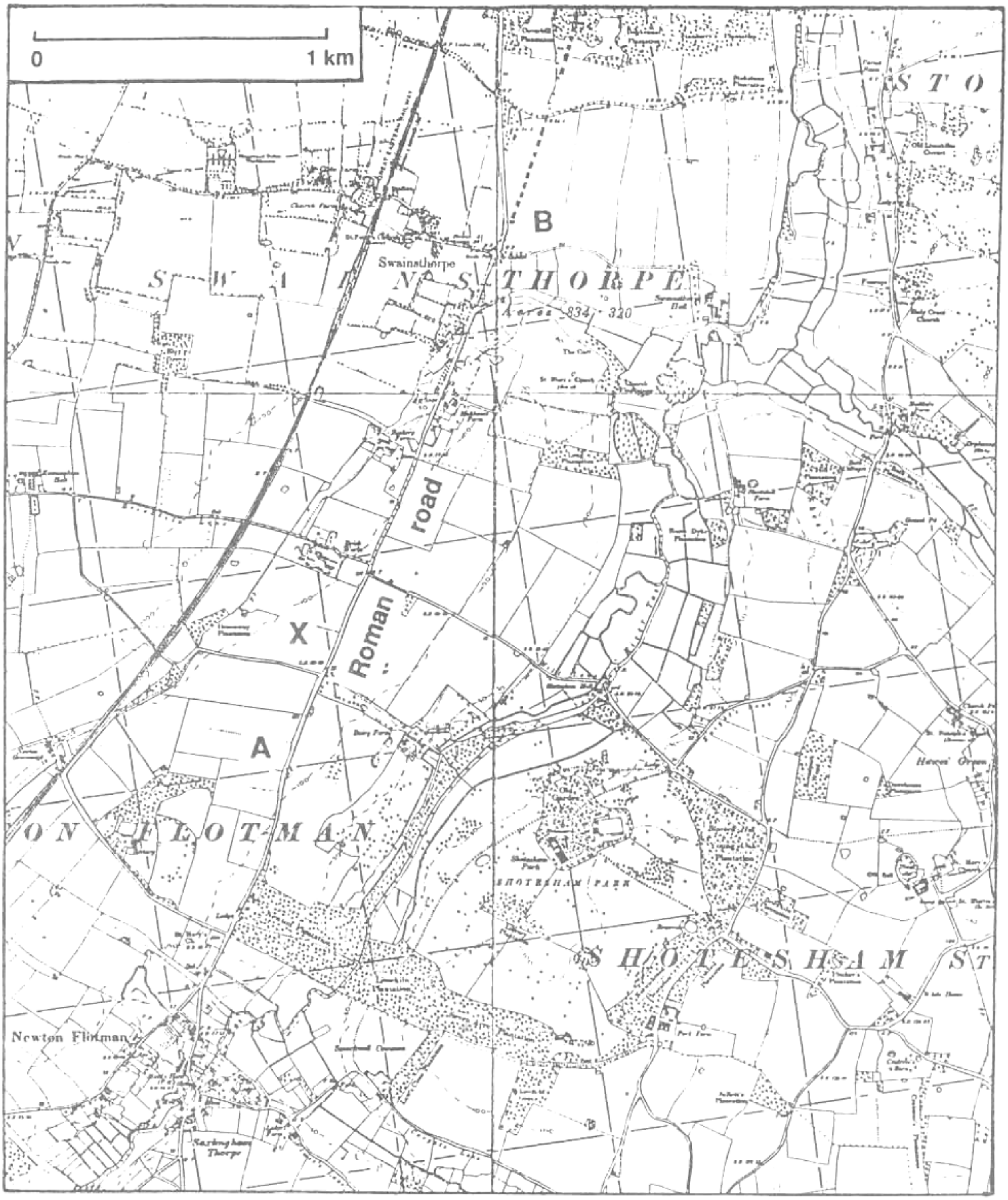

Figure 1. Oblique relationship in the South Norfolk A cadastre
(Based upon 6" Ordnance Survey maps 1871-1907)

Bradford Illustration	Feature	Ratio	Through termini?	Confidence	Notes
Fig 13, Pl 38	Road to NE	1:1	No	Very high	But road postdates grid
Pl 39	Road, N corner	1:1	-	High	15 actus grid,
	Road, N corner	2:3	-	Moderate	see note below
Pl 42	?Path N of 'G'	2:3	No	Moderate	
	Road to top left	3:1	Yes	High	Cont. by road in fig. 16
	Road lower right	1:1	No	Moderate	
Pl 43	Road lower left	6:5	Possibly	Low	
	Long diagonal road	1:1	No	Very high	Cont. by road in fig. 16
Pl 44	Diocletian's palace	3:2	No	High	
	Road to Salona	3:4	Yes	Very high	
	Road W of harbour	1:2	No	High	
	Streets E of Palace	1:2	No	Moderate	
	Roads E corner	5:2	Yes	Moderate	
Fig 16	Road SE corner	---	No	--------------	Not 1:1
Excluding	Road SE corner	9:4	Yes	Moderate	
Pl 42, Pl 43	Road NW corner	1:3	No	Moderate	
Pl 49a	Diagonal road	---	No	--------------	Alignment arbitrary

Note: in Pl 39 the position of the cardines in the 15 actus grid is not totally clear. However, there is a position for the cardines in which both alignments of this road pass through the corners of grid squares, and the road bends at one of these. The oblique road on the E side fits at 3:5 through these grid square corners, but the portion shown is too short for certainty, The road in the SW corner appears not to be grid-related.

Figure 2. Orientation of linear features in Bradford illustrations

Clavel-Lévêque Illustration	Feature	Ratio	Through termini?	Confidence	Notes
p 133, fig 30	Road, N side	4:3	Yes	Moderate	
p 140, fig 3	2 Field Systems	1:1	Yes	High	
p 202, fig 3	Straight Roads to Elche	-	-	-	Relationship unclear
P 203, fig 4	Road to SW, lower part	2:3	?	Low	
P 204, fig 5	Road to NE, upper left	2:3	Yes	Low	
	Road to NW, upper centre	?	?		
	Road in SW corner	-	-	-	?Not related
p 205, fig 6	Sect of Roman road, lwr rt	3:1	Yes	High	
	" " " , top right	11:4	Yes	Low	
p 268, figs 8 10,11	Voie domitienne	1:3	Yes	Very high	
	Road to NE from Nimes	4:11	Yes	Moderate	
p 280, fig 2	Road N of Arausio	10:1	Yes	Moderate	
	Arausio, streets	-	-	-	?Arbitrary
	Road DD18 & 19, CK1	?5:1	Yes	-	
p 285, fig 4	Road NE from Ernaginum	1:1	Yes	High	
p 286, fig 6	Road NW from Ernaginum	3:2	Yes	High	
	Voie domitienne	1.10	Yes	Moderate	
	SD8VK2 - DD4CK1	2:1	Some	Moderate	See note below
	Avignon to DD13CK4	3:1	Yes	High	
	Via Aurelia	-	No		Not related
p 288, fig 7	Via Aurelia	-	No		Not related
p 313, fig 1	Via Appia	?	?		Relationship unclear

Note: Road passes through 2 out of 3 corners. It appears to have been made to bypass Ernaginum

Figure 3. Orientation of linear features in Clavel-Lévêque illustrations

1:10	(2:20)	II	9:10	(18:20)	XVIII
1:5	(4:20)	IV	1:1	(20:20)	XX
1:4	(5:20)	V	6:5	(24:20)	XXIV
3:10	(6:20)	VI	3:2	(30:20)	XXX
2:5	(8:20)	VIII	7:4	(35:20)	XXXV
1:2	(10:20)	X	9:4	(45:20)	XLV
3:5	(12:20)	XII	11:4	(55:20)	LV
7:10	(14:20)	XIV	3:1	(60:20)	LX
3:4	(15:20)	XV	7:2	(70:20)	LXX
4:5	(16:20)	XVI	9:2	(90:20)	XC

Figure 4. Representations of angles of oblique features as ratios of multiples of two and five to twenty

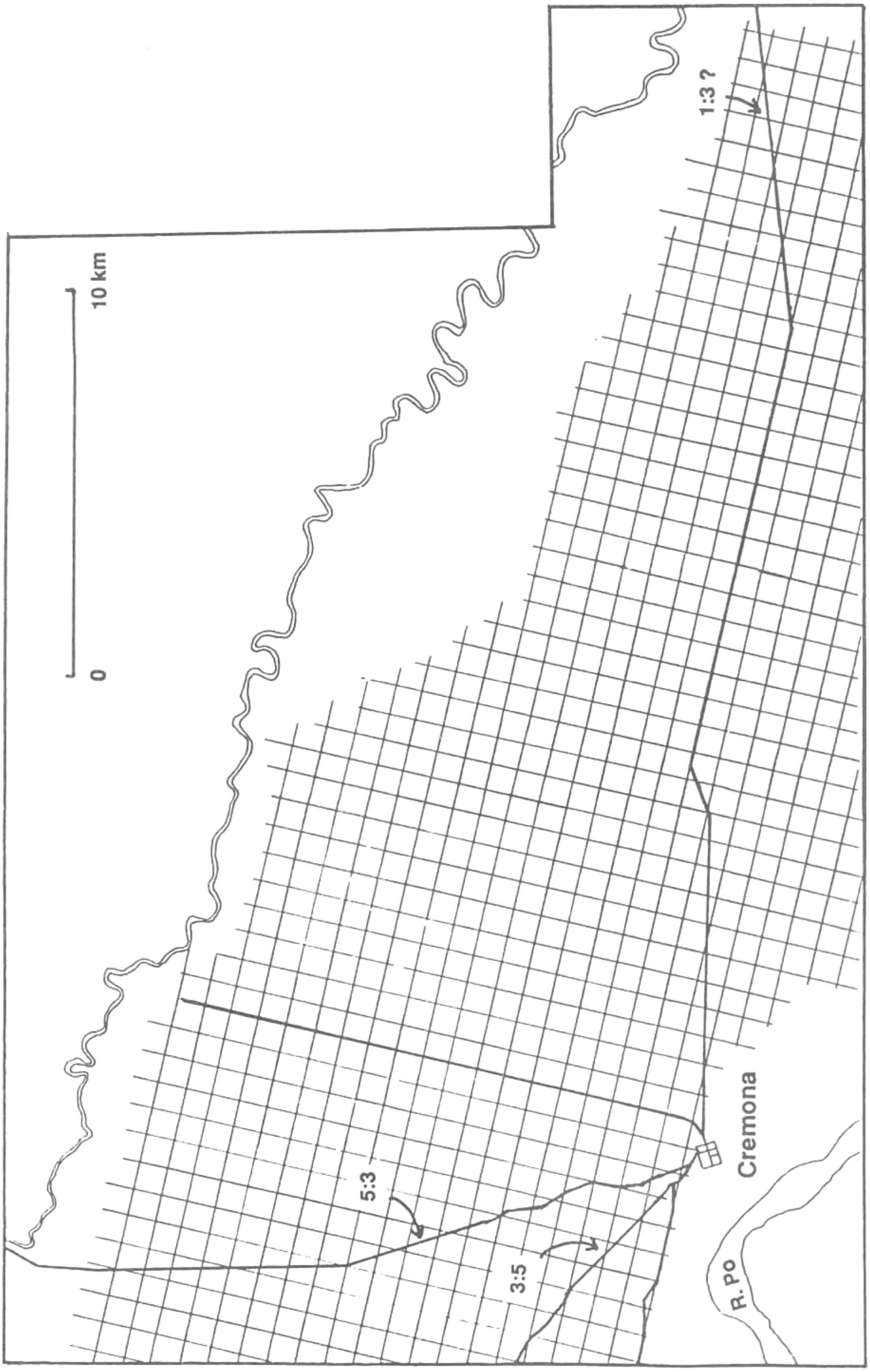

Figure 5. Relationships at Cremona

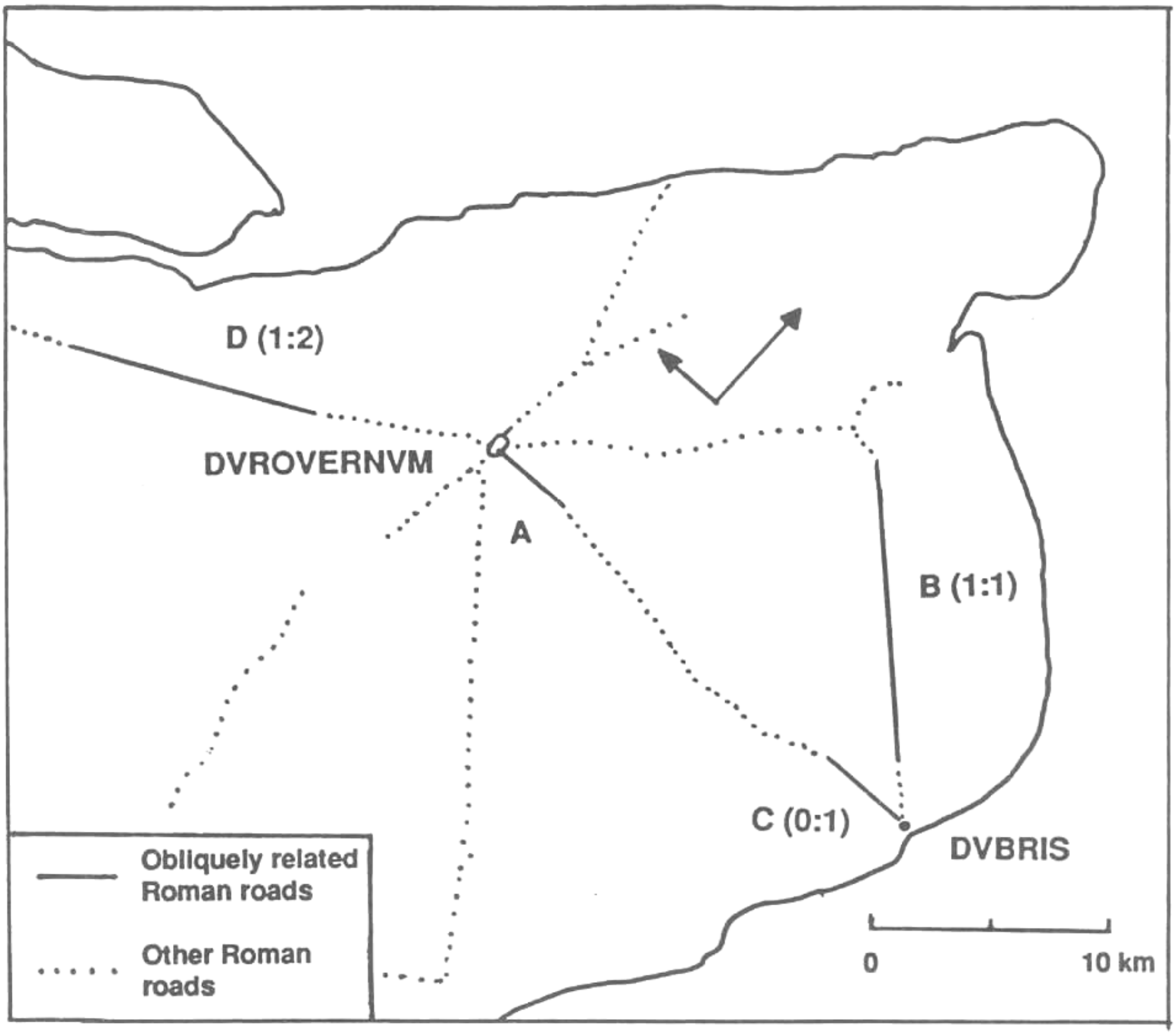

Figure 6. Britain, East Kent

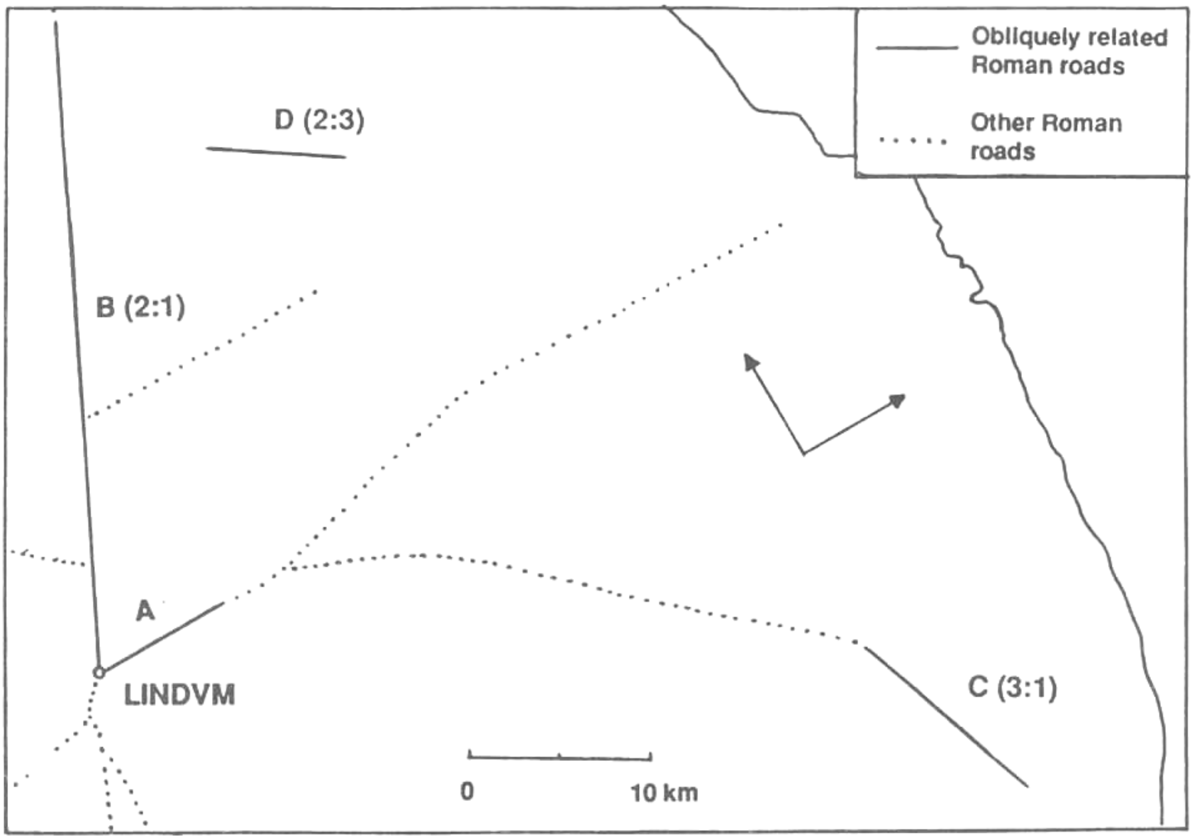

Figure 7. Britain, Lindsey