3 Development and application of computational methods (1)

3.1 Calculation of grid coordinates and orientation

The topographic maps of most modern states are based on square kilometre grids - for instance the British "Ordnance Survey" (OS), the French IGN and the IGM maps of Italy. These grids are the result of a variety of different transformations from the surface of the ellipsoidal Earth to a plane. However,

> "all the common transformations preserve angles well enough and are of sufficiently low distortion so that linear distance measurements can be made on a map sheet of scales 1:25000 and larger." (Scollar 1989: 255).

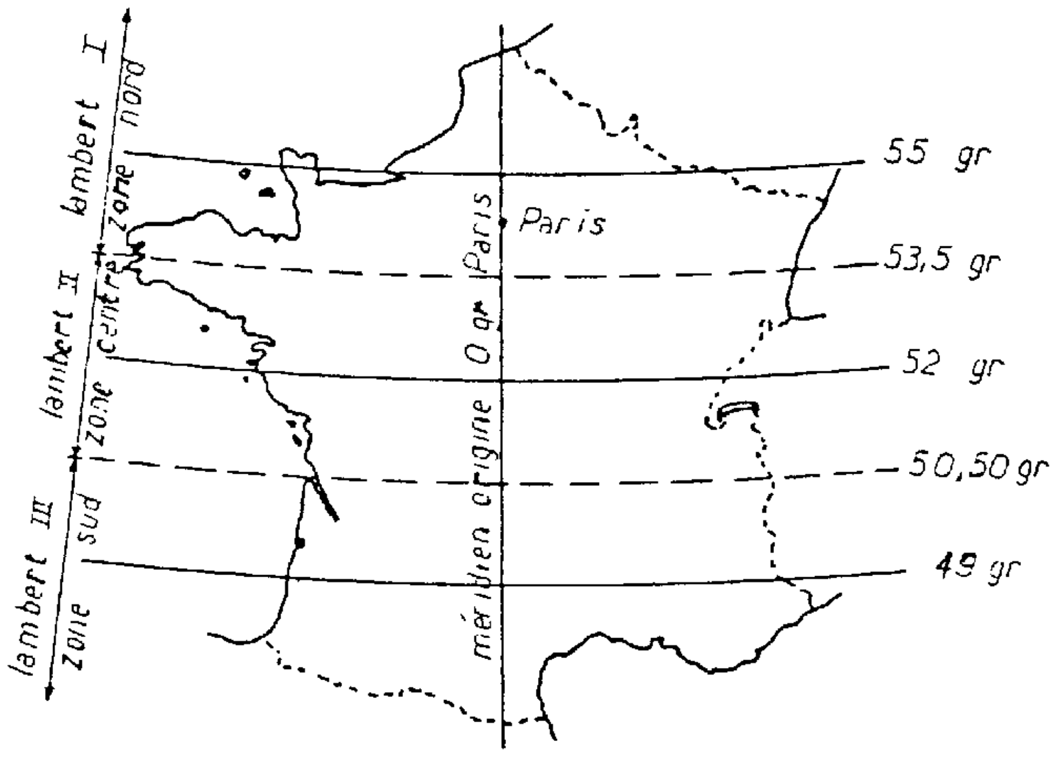

Figure 3.1 Latitude strips for the French Polyconic system, after Scollar (1989: fig. 25.7).

> "For searches or distance measurement over a small region or if the search area is within a meridian or latitude strip, the Earth can be considered flat" (Scollar 1989: 265).

Examples of small regions are England and Scotland (a meridian strip), or one of the three zones of the Lambert projection coverage of France *(figure 3.1)*.

Thus it is possible to use a Euclidean transformation - rotation, translation and scaling - to express the coordinates of an accurately surveyed Roman grid in terms of the coordinates of a modern survey.

The survey coordinates of any number of potential intersections of a hypothetical square or rectangular grid, such as a centuriation, can be calculated and printed, given the following parameters:
(a) The number of **actus* on each side of the centuries,
(b) the coordinates of one intersection[37],
(c) the position of the first intersection to be printed, in terms of displacement in centuries in each direction from the origin,
(d) the number of rows of survey coordinates to appear on the printout,
(e) the grid module, as a metric equivalent of 20 *actus*,
(f) the grid orientation[38].

The computer program CADCOORD operates in batch mode, reads a file holding these parameters and prints eight figure coordinate-pairs for the intersections of a centuriation grid, in rows of twenty, according to the following simple algorithm. The x (easting) and y (northing) for a point at i, j grid squares from an origin at (a,b), in a grid with orientation β , formed of rectangular units of k *actus* in the direction of the *kardines* and d *actus* in the direction of the *decumani*, where 20 *actus* is equal to m metres, is

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[37] Note that, although referred to as the origin, this can be any intersection - not necessarily the origin chosen by the *mensor*.

[38] Orientations are expressed in degrees as a real number, those west of north being represented as a negative number.

$x = a + m\ (i\ d\ \mathrm{Cos}\beta + j\ k\ \mathrm{Sin}\beta)/20$
$y = b + m\ (j\ k\ \mathrm{Cos}\beta - i\ d\ \mathrm{Sin}\beta)/20$

Although this calculation of the possible grid intersection coordinates is trivial in principle, the methodology has several substantial benefits. It has saved much work; centuriations can cover large areas and without computer aid the calculations would be daunting. It allows for a hypothetical grid to be located correctly, according to the hypothesis, on any map sheet. A local study at any scale, and at any distance from the origin, can use a representation of the grid which conforms precisely to its originally specified position, module and orientation. This solves a problem arising from the dimensional instability of paper maps. This was noted by Chouquer (1981: note 14) with reference to Italian IGM 1:25,000 maps. Even a single sheet may have slightly different scales along the two axes and it may be impossible to match two maps precisely at the edges, even if they have the same nominal scale. This can lead to imprecision in the use of large transparent overlays covering several joined map sheets, even if the overlays have themselves have been plotted by computer on a stable medium such as Mylar[39].

Furthermore, the calculated grid points provide a unique and accurate model. This removes one possible source of the investigator's bias; the grid cannot be adjusted from place to place in order to obtain a better fit. The methodology also guarantees that the resulting grid is truly at right angles, and has an invariant module and orientation with respect to a modern kilometre square survey grid.

3.1.1 Determining the orientation of a hypothetical centuriation

In order to calculate the kilometre survey grid coordinates of the cadastral intersections it is necessary to establish the parameters of the grid with considerable precision. A change as small as one metre in the module can lead to a large cumulative error. For potentially large cadastres the angle also needs to be specified to

[39] The plotter itself will also be a source of error (Diehl and Apiki 1988).

better than one hundredth of a degree, for even such a small shift will produce a displacement of 17m at a distance of 100km from the origin. Such an error is significant, given that it is reasonable to use survey coordinates accurate to 10m for plotting on 1:25,000 topographic maps[40],[41].

The orientation and module of a centuriation of 20 x 20 *actus* may be established in several ways[42]:

1. From two points on the same supposed *limes*.

This approach was described 35 years ago by Legendre (1957) in an article on the Tunisian cadastres. Given two points (x_1, y_1) and (x_2, y_2), the angle is $\mathrm{Tan}^{-1}((x_2-x_1)/(y_2-y_1))$. The module must then be determined independently, for example by measurement of the distance between parallel *limites*.

2. From two points at the corners of centuries.

If the distance between the two points A (x_1, y_1) and B (x_2, y_2) in the direction of the *kardines* is k centuries and in the direction of the *decumani* is d centuries, then the angle of the line AB to the cadastre is $\mathrm{Tan}^{-1}(d/k)$.

Thus the angle of the cadastre is

$\mathrm{Tan}^{-1}((x_2-x_1)/(y_2-y_1)) - \mathrm{Tan}^{-1}(d/k)$ and the module is

$$\frac{\sqrt[2]{(x_2-x_1)^2+(y_2-y_1)^2}}{\sqrt[2]{d^2+k^2}}.$$

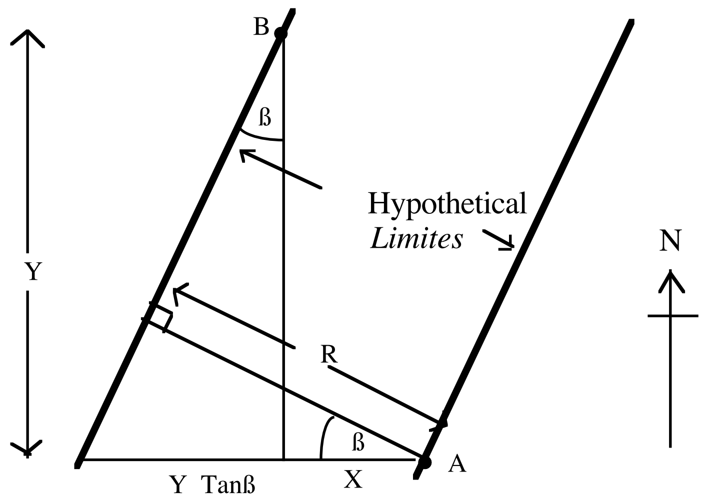
40 A precision of 10m, which is 0.4mm at this scale, approaches the limit of what the author finds possible. Compare this with the view of Dodinet, Leblanc and Vallat (1987: 330) who estimate that an average operator's eye can distinguish a separation equivalent to 7.5m.

41 If we accept that the Roman surveyors were capable of plotting a straight line 29km long within the German frontier zone with a maximum deviation of only 2m (Dilke 1987b: 32), this concern for precision is necessary.

42 These methods would require minor modification for grids thought to be composed of rectangles.

<u>3. From points on two parallel hypothetical *limites*.</u>

Two linear features are selected which are clearly defined, parallel and separated by some multiple of the grid module. Given arbitrarily chosen points, one on each of two linear features, it is possible to calculate the orientation of a grid which fits *(figure 3.2)*.



To find the angle, ß, that the line through B, lying at a perpendicular distance R from A, makes with North.

X = difference of X coordinates
Y = difference of Y coordinates

$$\text{Sec}\,ß = \frac{X + Y\,\text{Tan}\,ß}{R}$$

$$R^2(1 + \text{Tan}^2 ß) = X^2 + 2XY\text{Tan}\,ß + Y^2\,\text{Tan}^2 ß$$

$$\text{Tan}\,ß = \frac{-XY + R\sqrt{(X^2 + Y^2 - R^2)}}{Y^2 - R^2}$$

Figure 3.2 *Calculating orientation from points on two hypothetical* limites.

The result can readily be calculated on a pocket calculator, but it may be necessary to experiment with different grid module values R in order to obtain a good visual fit with the presumed *limites*. Accordingly, a routine ORIENT has been made available, written in

Modular 2 for the Apple Macintosh, which allows the calculation to be repeated using a number of alternative module values *(figure 3.3)*.

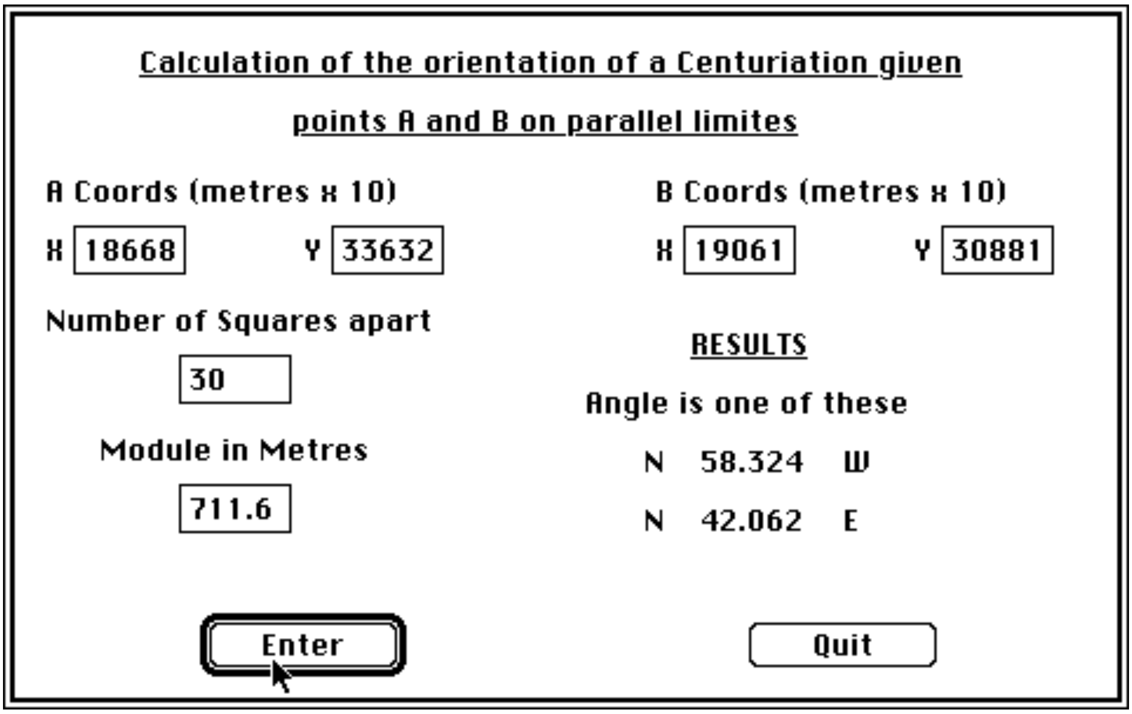


Figure 3.3. Interaction with the ORIENT program.

Finally a point of origin of the grid must be determined. If method 2 was used then this has already been done, but in the other cases it is necessary to determine an origin by inspection of the orthogonal *limites*. Once this has been done the set of parameters can be used by the CADCOORD programme to calculate the coordinates of any number of intersections. The position of these points can then be checked against the map to see if the parameter values require any slight change to achieve a better fit.

The application of this technique to three cadastres will shortly be described, but at this point it may be useful to discuss further the question of the measurement of orientation. Clearly the angle between our model of a centuriated cadastre and a modern survey grid is constant, because they are related by a Euclidean transformation in the plane. But what is the link with geographic north?

The answer lies in the fact that if a square grid is placed on the earth's surface then the direction of geographic north, the direction of the earth's north pole, is not constant. This can easily be seen if we imagine ourselves in a grid placed at the north pole, in which case north would be in all directions, depending on our position in the grid. At lower latitudes the effect is not so striking but it is still significant, For example the angle between the borders of the 1:25,000 IGN map sheets (which are oriented to geographic N-S and E-W) and the lines of the Lambert projection[43] is different for each border of each map[44]. The situation is different on other maps, such as the British OS, whose borders are formed by lines in the kilometre grid. It must be concluded that geographic north is not a sound basis for determining the orientation of centuriated cadastres. If the angle of the cadastre is given in such terms then it may appear that it varies from place to place when, in fact, it is constant with respect to the kilometre survey grid.

3.1.2 Modelling a cadastral grid, example 1: Orange B and the Cèze valley

The position of the northern part of the Orange B cadastre can be established using the results published by Valérie Bel and Jean Benoit (Bel and Benoit 1986). Two *limites* intersections can be located on 1:25,000 maps. One is at CK II DD XXXIV at a corner on the western commune boundary of Allan (Bel and Benoit 1986: figs 6 & 7), *(figure 3.4, point A)*. This has coordinates in the Lambert zone III: $x_1 = 793.03$, $y_1 = 3248.36$ (to 10m). The other is the crossroads shown in (Bel and Benoit 1986: fig 9) at CK IV DD VIII, $x_2 = 793.35$, $y_2 = 3229.88$ (using the Lambert zone III coordinates rather than the Lambert zone II étendu coordinates used in their figure), *(figure 3.4, point B)*.

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[43] These are shown at intervals of one kilometre by small crosses at the intersections. The complete grid for only one Lambert zone is shown in this way. The intersections of the grid lines for other zones with the edges of the sheet are shown in the margin.

[44] The angle between the border and the survey grid can easily be measured by using the inverse tangent, as in method 1.

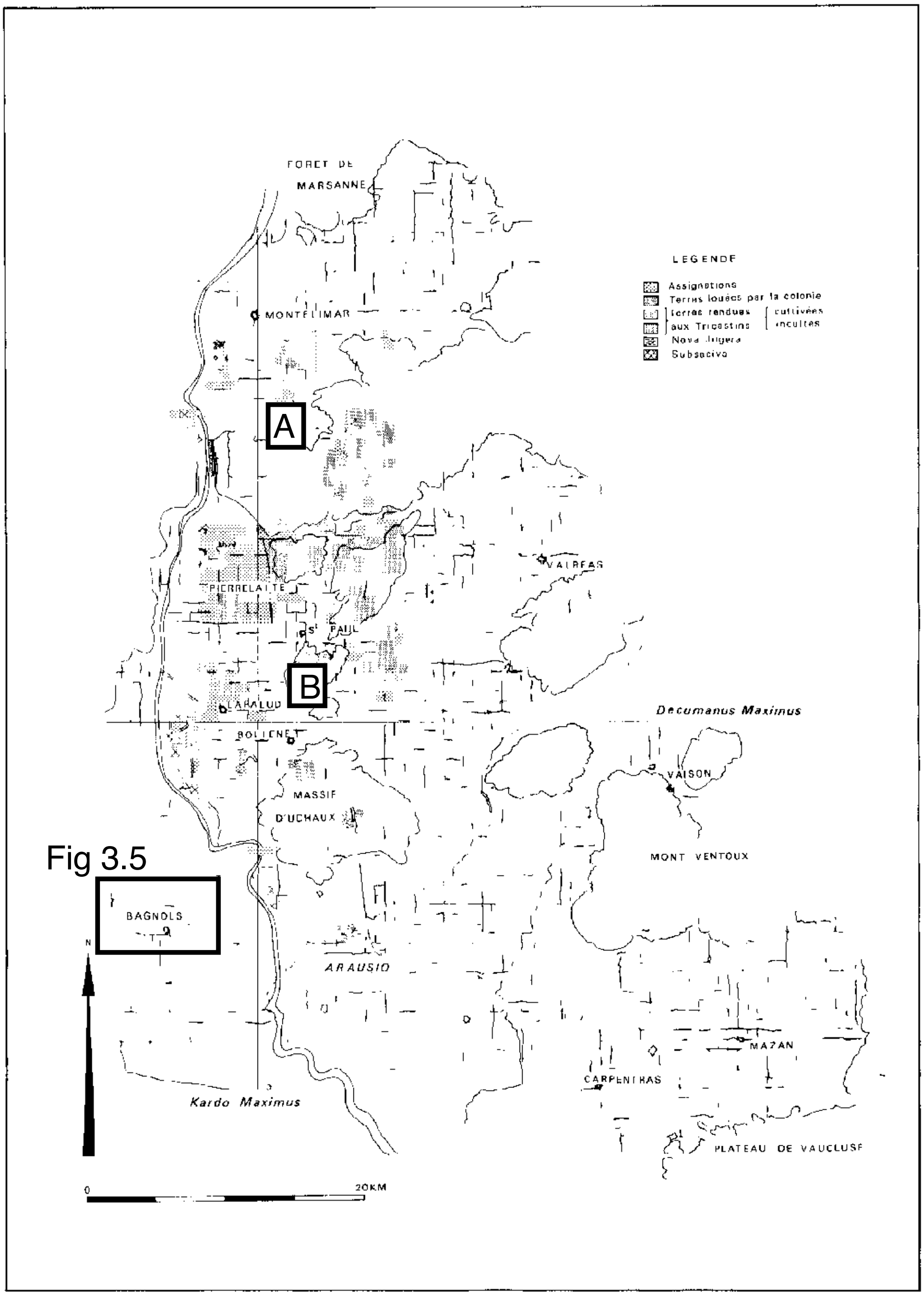


Figure 3.4 *Location of the Orange B cadastre and the Cèze valley, after Clavel-Lévêque* (1983c), *and with acknowledgements to G. Chouquer.*

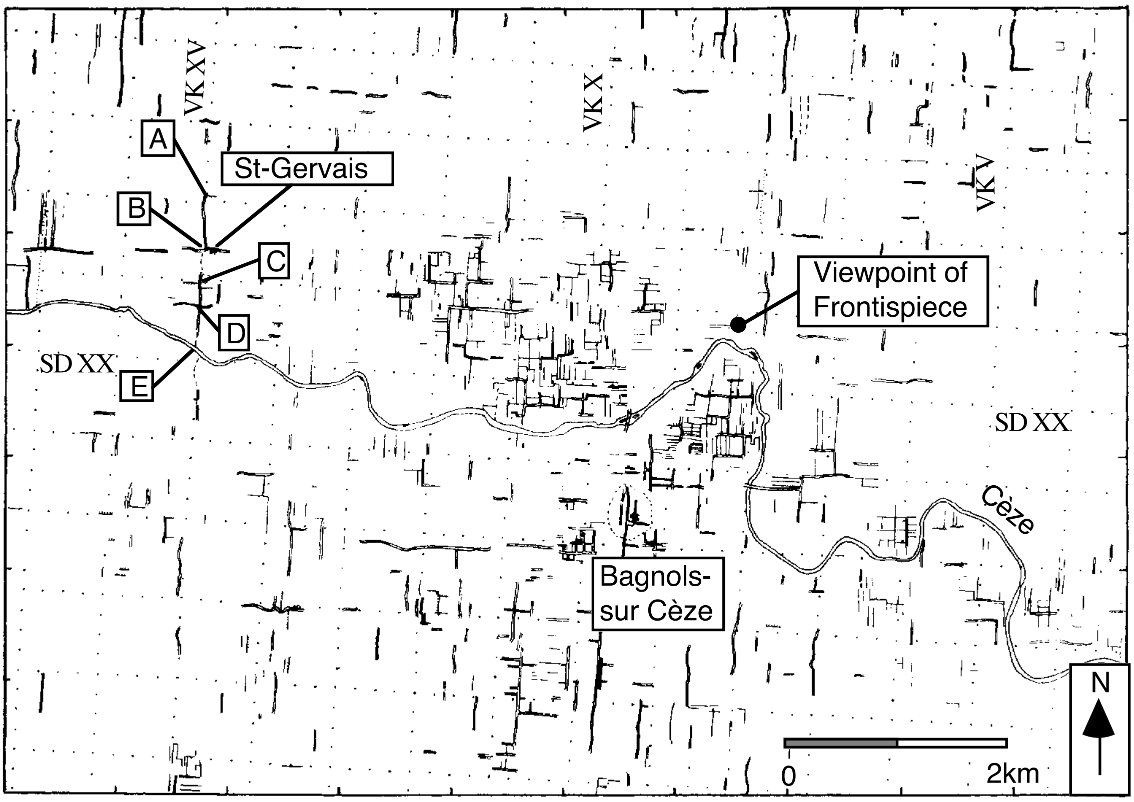
Using method 2 described above, the distance between these two points is obtained by taking the square root of the sum of the squares of the differences of x and y, which are, in tens of metres, 32 and 1848 respectively. That is

$\sqrt[2]{(x_2-x_1)^2+(y_2-y_1)^2}$ = 18.483km (± 5m).

This, being equal to the diagonal of a rectangle of 2 by 26 centuries, gives a module of 708.8m (±0.4m). The angle to Lambert north of the line joining the two points is (-1)Tan^{-1}(32/1848). This must be added to Tan^{-1}(1/13) to give the angle of the cadastre, which is 3.407 (±0.016)° (east of Lambert north).

The position of the origin of the cadastral grid (KM DM) was calculated to lie at x = 790.18, y = 32224.39 and this was used with the other parameters, module = 708.8, angle = 3.407, to calculate coordinates for 6000 hypothetical intersection points.

At and near Bagnols-sur-Cèze there are many traces which conform to the orientation of this grid, *(figure 3.5 and frontispiece).* There



From 1:25,000 sheet 2940 est © ign 1984

Figure 3.5 Possible traces of Orange B cadastre , Cèze valley.

are however not so many examples of possible survival of its major divisions or plausible subdivisions. Nevertheless, two can be cited: (a) to the north-east of the town where the fields ignore the course of the river and appear to be divided in an east-west direction by lanes at intervals of approximately 5 *actus*, corresponding to quarter divisions of a century, and (b) in Bagnols itself, where the north-south road passing to the west of the central square, the former Grand Rue, lies on a mid-century line.

At St-Gervais the traces are generally much more sparse. This is because the existing fields follow the contour lines of the valley side and generally do not conform to the orientation of Orange B. There is however one grand exception: an alignment of features A-E, which lies close to the predicted line of a *quintarius* VK XV (15 to the west of the *kardo maximus*). The section AB is a sunken lane which descends the steeper part of the valley side, oblique to the line of steepest descent. The point B is the location of the small triangular 'place' where a weekly market is currently held. Its northern edge is the main road through the village, and its eastern edge, which lies on the theoretical line of the *limes quintarius* is bordered by a house with a round tower at the corner, known locally as the 'chateau'. From this point a drain runs near the line of the *limes* to join another road near point C. For about 50 metres the drain and road run side by side. They then merge into a road which runs to the river at point E. This road has a gently falling gradient. Its surface is at times raised above the level of the adjoining fields by one to two metres. It has well-maintained side walls.

During a visit it was possible to see this road-drain system in action. On the afternoon of 30 July 1991 there were thunderstorms accompanied by sudden bursts of rain. The uphill section, AB, captured the water flowing in a sudden torrent from a gully running straight down the hillside. This water poured into the 'place', where it was joined by even more water collected in the village main street. Then the combined flow entered the water channel BC, and flowed onto the lower drainage road to empty into the river at E. It was observed that each time it rained this road, which had initially been dry, was converted after a lapse of about five minutes into a steadily flowing stream about 5cm deep.

Clearly this system performs an important economic function. It prevents the fields from being flooded and eroded by sudden downpours. The effects of such meteorological events, in areas where the drainage systems could not cope, could be seen next day (31 July 1991). Following a night of storms in the area of Chateauneuf-du-Pape, vineyards were inundated or swept away, and an instance was noted in which a roadside ditch had been completely filled. Stones were visible on the surface in one place, and sand in another.

The lower section of the system is marked by two wayside crosses. One is at D where the drainage road intersects a lane running at right angles along the edge of a terrace. The other is at C, at the point where the existing road and ditch would be cut by the theoretical line of another *limes*, SD XIX. This *limes* is marked at this point by only a few features. A hedgerow is on its line to the east of the road; to the west there is a field entrance with a house on the south side and the cross on the north. This cross is the smaller of the two which mark the line of this road.

These features aligned on the hypothetical *quintarius* at St-Gervais are, together with features on the same line on the south side of the Cèze, exceptional traits in an area which is not generally organised according to the orientation of Orange B. They fulfil the prediction of the computer-calculated model in a situation where there is a relatively low chance that any given arbitrary prediction will be fulfilled. This gives reason to believe that this road-drain system correctly represents the course of *quintarius* VK XV. Furthermore the position of the small cross at C, difficult to explain in a modern context, lends support to the idea that the predicted position of an orthogonal *limes* is also correct.

It is quite possible that the St-Gervais road-drain system has a wider significance since such a structure, if present in antiquity, would provide another example of the sort of construction found elsewhere in Languedoc. As Monteil, Poupet and Sauvage (1990) say, "Il est aussi tentant de mettre en relation cette technique de construction des voies, reflétant une parfait maîtrise du drainage, avec les procédés de mise en place des grands cadastres par les arpenteurs romains".

Furthermore the Orange B cadastre may extend further up the Cèze valley. There is another seasonal watercourse, parallel to the *limes* running through St-Gervais and two centuries further west *(figure 3.5)*, which is also very near a section of the present commune boundary. There also appear to be other traces (not shown) even further up the valley.

This simple example shows the value of the computational approach in projecting a cadastral grid from an area where it is well known to a new area. It can easily be checked how well the features with an alignment similar to that of the cadastre fit within the model cadastral framework. At St-Gervais they fit very well, which leads to the suggestion that the Cèze valley was included within the same survey as the northern part of Orange B.

Another interesting result is the finding that the projected grid does not fit precisely onto Chouquer's (1983d) reconstruction of the cadastre near Orange. This suggests that the cadastral survey in this area may be slightly discordant to the survey at a greater distance. This clearly needs further investigation, because it may reveal phases in the development and extension of the cadastre.

3.1.3 Modelling a cadastral grid, example 2: South Limburg

It has been proposed by Edelman and Eeuwens (1959) that a centuriated cadastre can be seen in the South Limburg district of the Netherlands. Although this claim is long-standing, it has attracted little attention, presumably because it relates to a remote area of the Empire where, as we have seen, many archaeologists would doubt that such structures could exist. Thus the situation is quite different to that in the previous example. In that case we were considering a possible extension of a well-known, physically attested and generally accepted system. In South Limburg the very existence of the system is in doubt.

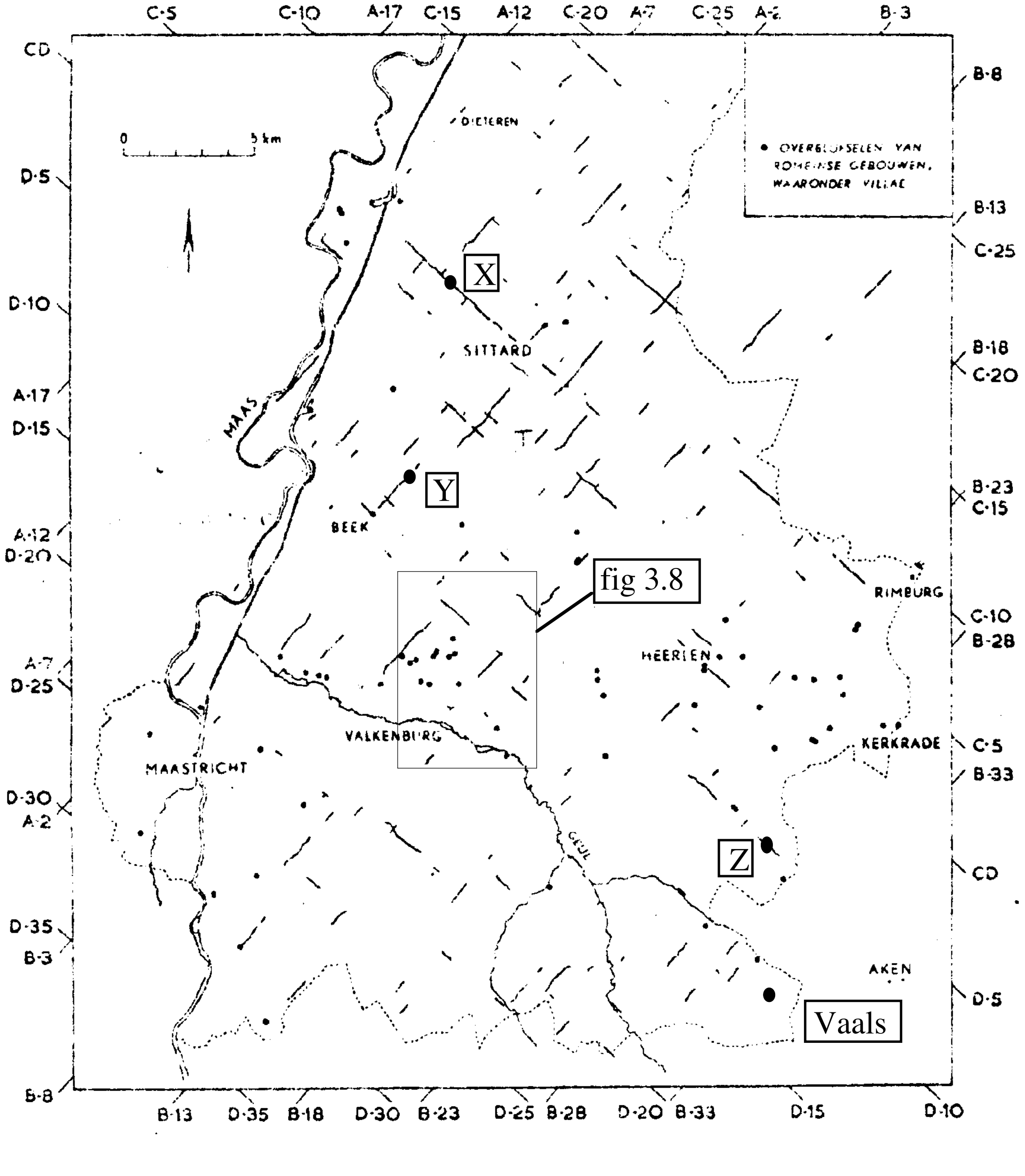


Figure 3.6 *Presumed remnants of cadastral traces in the province of Limburg, after Edelman and Eeuwens (1959: fig. 2).*

Edelman and Eeuwens' (1959) summary figure *(figure 3.6)* shows features on topographic maps corresponding to their proposed system of *limites*. The most prominent of these passes through the point labelled 'X', at. x = 186680, y = 336320, the location of the St-Salviuskerk of Limbricht which, according to them (1959: 53), stands "precies aan een hoekpunt" (precisely at a corner). Other proposed *limites* are those passing through points 'Y' at Neerbeek (x = 185070, y = 329270) and 'Z' at Bocholtz (x = 198920, y = 314740). Once these points have been identified on the 1:25,000 maps, the ORIENT program can be used with a range of module values in order to see if there could be a common module and angle determined by the two pairs of points X &Y and X & Z.

	Angles determined by points	
Module (m)	**X and Y**	**X and Z**
707	42.127	42.170
708	42.172	42.143
709	42.218	42.117
710	42.263	42.090

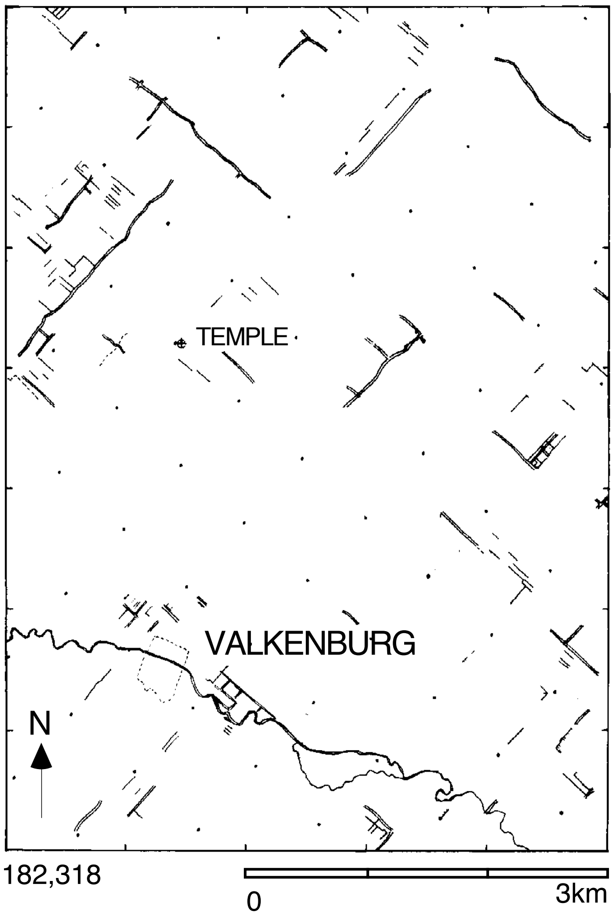
Figure 3.7 ORIENT calculations for South Limburg.

These calculations *(figure 3.7)* indicate that a common angle is to be found for a module value between 707m and 708m, and further calculation shows that a module of 707.6m gives an angle of 42.154° in both cases.

However, when these parameters were used to calculate the coordinates of the intersections of a centuriation, it was clear that there were significant differences between this model and the traces identified by Edelman and Eeuwens in the south east of the area, near Vaals. Furthermore it appeared that these were the only major traces with the right orientation and at the appropriate intervals. It was clear that the module needed to be altered in order to obtain a good fit.

Accordingly a new theoretical intersection point was chosen in this latter area, at x = 190610, y = 308810. This point was used with the original intersection point X, as in the preceding example, to produce a module of 711.61m and an orientation of 42.064°. A grid produced by these parameters, using point X as an origin, gives a good fit to the proposed traces over the whole area. As an example consider the area north east of Valkenburg *(Figure 3.8)*.[45]

There are traces at the calculated orientation, and those near theoretical *limites* correspond to those identified in Edelman and Eeuwens's reconstruction. Furthermore, even if we ignored their suggested



From 1:25,000 sheet 69B Maastricht © Topografische Dienst, Netherland 1989

Figure 3.8 Possible cadastral traces near Valkenburg.

[45] The module identified may seem unusually large, but since we can confidently propose a module almost as large in the Saône plain system, discussed in the next section, it is not implausible and gives a value for the foot of 0.2965m, compared to a widely accepted value of 0.296m. It is admittedly larger than the previously proposed value of 710m by 0.23%, but it is clear that Edelman and Eeuwens were using a conventionally accepted value. They give no indication that they employed the kilometre grid coordinates for calculations; thus it seems unlikely that they would have been able to achieve a high degree of accuracy.

positioning of the *limites*, we could hardly do better in fitting a grid to the traces.

This example shows the need to modify an initial hypothesis if it does not correspond to the findings on the ground. One should not expect that a first attempt using the method of "points on two parallel *limites*" will necessarily give a good fit, particularly when the traces are ill-defined and the points are relatively close.

For this cadastre a large amount of data is available on the location of sites of the Roman period. This gives us the opportunity to study objectively the cadastre itself, and its relation to the sites, by means of statistical techniques which will be described below *(3.2)*. However, if such tests are to be objective we must establish the position of the cadastre independently using only the evidence from topographic traces.

This was what was done in this case. In trying to find a grid which is a good fit to that proposed in 1959, it was necessary to try different methods of obtaining the grid parameters, with the results shown. However the change of method was brought about by the poor fit to the cadastral traces, as proposed, and in the absence of any knowledge of site location. The location of the temple *(figure 3.8)*, which appears to be typically situated at the corner of a century, was unknown to the author until after the most likely position of the grid, as perceived by Edelman and Eeuwens, had been established.

3.1.4 Modelling a cadastral grid, example 3: The Saône plain

In the Saône plain - between Chalon-sur-Saône, Nuits-St-Georges and Dole - many topographic traces have been observed which conform to an angle of approximately N 32° E. Gérard Chouquer (1983c: fig. 3; 1980) has proposed the reconstruction of a large centuriation *(figure 3.9)* which would form a major extension of the system initially postulated half a century ago by André Déléage (1940) in the area to the north east of Chalon.

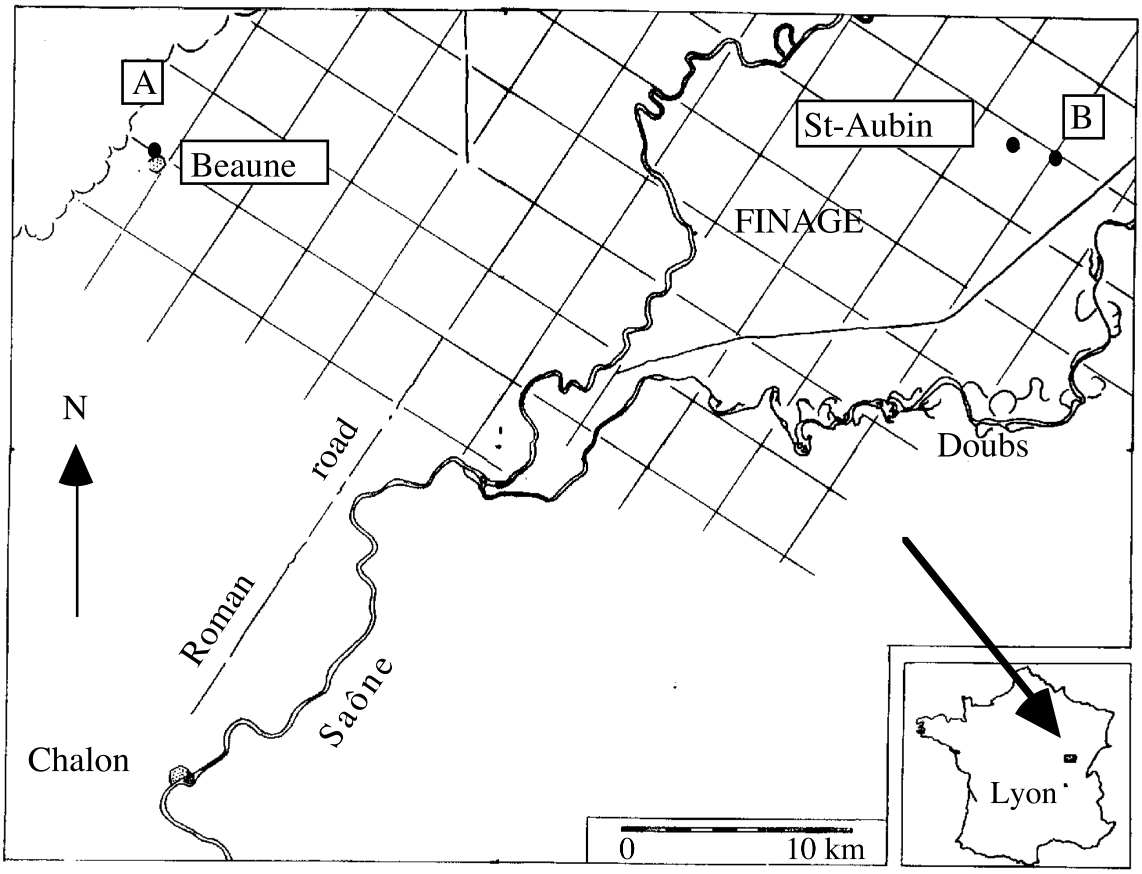


Figure 3.9 *General layout of presumed* quintarii *of the Saône plain cadastre.*

Two widely separated fragments were shown on aerial photographs published in the issue of *Photo-interprétation* (15 5, 1983) devoted to Roman rural cadastres. One of the two photographs showed Beaune and its suburbs (Chouquer 1983a) and the other showed an area of countryside near St Aubin in the Finage (Chouquer and Daubigny 1983). At Beaune all the cadastral traces are visible as features in the modern landscape, whereas at St Aubin the principle feature - which according to Chouquer is a *limes *quintarius* - shows up in this particular photograph in the form of a soil-mark. Both these fragments could clearly form part of a centuriation. But could they really form part of the same grid, and what is the connection, if any, with the cadastre postulated by Déléage?

Despite the large distances between these two fragments, the available publications clearly identify two points of intersection of the grid, one in each area. Given the coordinates of these two points we can calculate the coordinates of any number of other intersections of the centuriation which would, in theory, pass through them. We can then see how well this theoretical centuriation models Chouquer's proposed survey grid. In effect we can conduct an experiment which will test his theory, and which may also give us information about the degree of accuracy which the *agrimensores* could achieve.

This experiment was planned in five steps:
(i) identification of two grid intersections, one in each area,
(ii) calculation of the module and orientation of a theoretical grid which fits these points, using method 2,
(iii) verification that the calculated coordinates of the grid accord with existing reconstructions and with the topography in each area,
(iv) estimation of the difference between the theoretical grid and the Chalon-Dijon Roman road,
(v) explanation of these differences[46].

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[46] These differences may be small, in which case they could be the product of a plausible lack of precision in roman surveying. Indeed, one hardly expects an antique survey to be as accurate as a modern one - maybe there will be a slight difference in module from place to place or a very slight variation from a right angle between *kardines* and *decumani*. Alternatively the differences could be so large that one could say that the road did not fit the model, in which case one would have to conclude that the proposed centuriation is improbable or even impossible.

A general impression of the structure of the theoretical centuriation can be obtained from the 1:100,000 map (IGN serie verte 37, dijon-tournus). A map at such a small scale would not normally be used for topographic studies, but in this case the published earlier hypotheses indicate the supposed position of the *limites*. It is supposed that the Chalon-Dijon Roman road forms part of the grid, which is also in conformity with the two published fragments at Beaune and in the Finage. Thus we can draw the *quintarii* on the map *(figure 3.9)* in accordance with the layout proposed in the Finage (Chouquer and Favory 1980: fig. 41). From this we can fix the position of intersection of the *limites* to about 100m and estimate the number of squares of the cadastre, in two orthogonal directions, which lie between any two supposed *limites* intersections.

Step (i). Identification of two intersections

For the purpose of establishing the outlines of the cadastre the 1:100,000 map suffices, but to obtain reasonably precise coordinates for the two initial fixed points we must use the 1:25,000 maps. The two chosen points were marked on the appropriate sheets, having the following coordinates[47] in the "Lambert zone II étendu" kilometre grid:

- point A: Faubourg St-Martin, Beaune
  $x_1 = 789.99$, $y_1 = 2228.40$

- point B: east of St Aubin (Corvée l'Allemand)
  $x_2 = 829.01$, $y_2 = 2230.02$

These points were chosen because they were determined by the most clear crossing of possible *limites* in the two aerial photographs *(figures 3.9, 3.10 and 3.11)*.

[47] Expressed as kilometres to an accuracy of 10m.

<u>Step (ii).</u> Calculation of parameters

The preliminary study of the 1:100,000 map had indicated that the two points were separated by 30 squares in the direction of the Chalon-Dijon road and by 46 squares in the orthogonal direction. These figures give us a distance AB of

$\sqrt[2]{d^2+k^2} = 54.918 \times 20$ *actus*.

In the same way, using the Lambert coordinates, the distance is

$\sqrt[2]{(x_2-x_1)^2+(y_2-y_1)^2} = 39.054\text{km}$

The ratio of these two figures, $\frac{39054}{54.918}$, gives us 711.13m for 20 *actus*.

The angle of the line AB is $\mathrm{Tan}^{-1}((x_2-x_1)/(y_2-y_1))$ with respect to Lambert north. Thus in this case, the angle is $\mathrm{Tan}^{-1}(3902/162)$ or 87.6226°. To the cadastre, the angle of the line AB is $\mathrm{Tan}^{-1}(d/k)$, which is $\mathrm{Tan}^{-1}(46/30) = 56.8886°$. Thus the angle we are looking for is the difference between these two angles. So the angle of the cadastre (or rather of a square grid which could be a model for the cadastre), as determined by the two points A and B, is N 30.734° E.

This angle is clearly different from the previously published figures of 32° - 32.5°, This is partially explained by the difference between Lambert north and geographic north. In the area of the Saône plain Lambert north is further to the east. At the eastern margin of IGN 1:25,000 map sheet 3025 est (near Beaune) the difference is about 1.8°, Whereas at the eastern margin of sheet 3225 ouest, which includes St Aubin, it is about 2.2°. Clearly the difference is large and it increases significantly as one moves eastwards across the Saône plain.

At Beaune the angle of the cadastre to geographic north (30.73° + 1.8°) is the same as the figure (32.5°) given by Chouquer in 1983[48] However at St Aubin the same angle - obtained by calculation - is 30.73° + 2.2°, nearly 33°. It must be observed that there is a very large discrepancy between this figure and that given by

[48] The figures are the same, if one uses the degree of precision used by Chouquer.

Chouquer and de Klijn (1989: 278). They proposed a reduction of the angle at St Aubin to 31°45'.

Nevertheless, later in the same article (1989: 286), Chouquer and de Klijn retain - for the Finage - the figure of 32.5°, not 31°45'. Thus we seem to have the option of accepting the proposed reduction in the angle, or else retaining the original proposal for the location of this cadastre in the Finage (Jeannin and Chouquer 1978), for the comparisons to be made in the next step.

Step (iii). Verification of the fit at Beaune and St-Aubin

At the stage we compare, as objectively as possible, the computer-calculated model of the *limites* with the reconstructions proposed earlier, and with existing topographic features. The aim is to see if the two fragments, as previously published, could actually fit within a single square grid system. Thus, in order not to create an interpretation of the published results which may be unconsciously improved to fit the grid, the previously established position of *limites* was drawn on the 1:25,000 maps before the coordinates of the intersections were calculated.

A transparent grid of squares of 20 *actus* at a scale of 1:25,000[49] was superimposed on the maps in the areas of Beaune and St Aubin. The transparent grid was placed on each map in the position which seemed to accord best with the published reconstructions as well as with the present day topography. The aim was to establish, before obtaining the results of the calculation, the position of a linear feature in each area which would define the orientation of the cadastre unambiguously.

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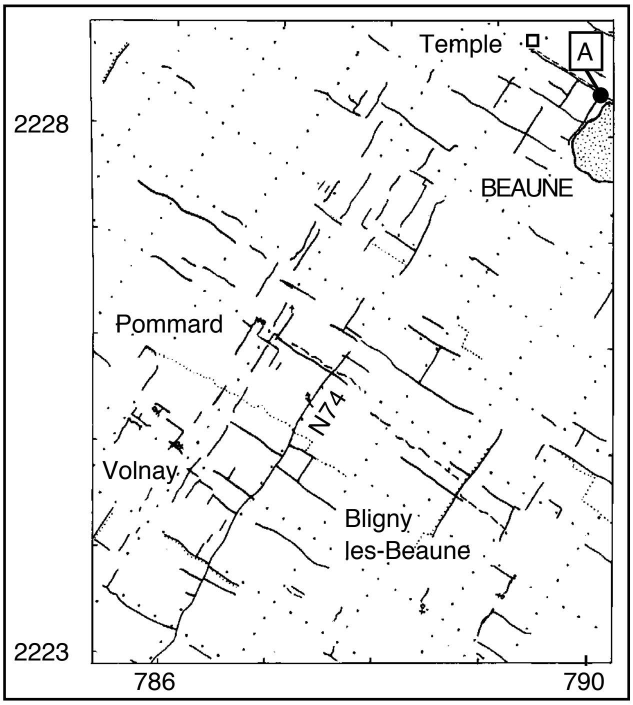
49 The grid was one that had previously been drawn by a plotting program on an A4 sheet of transparent acetate (overhead projector foil). The squares were 28.4mm. This is the equivalent of 710m, but the difference of 1.1m from the true value is not significant for the individual small areas considered here.



From 1:25,000 sheet 3024 est © ign 1989

Figure 3.10 Saône plain cadastre near Beaune.

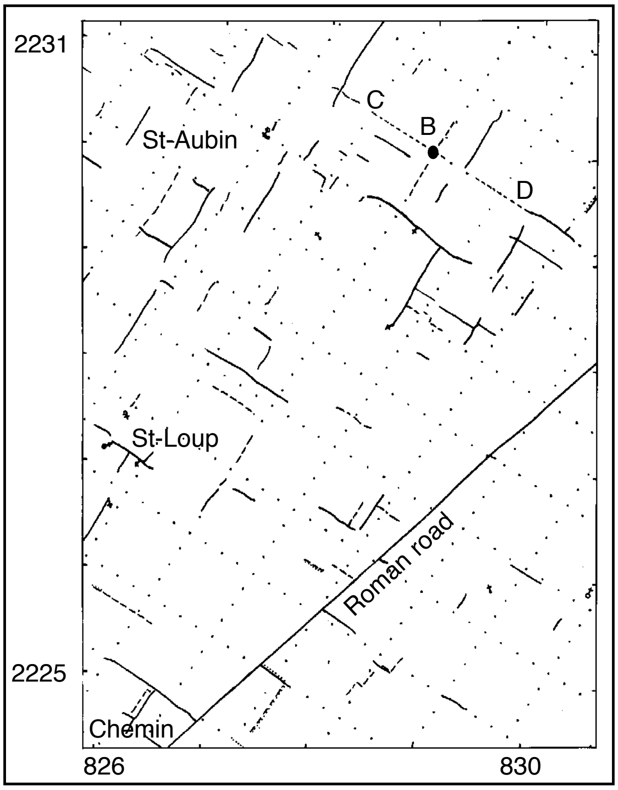
At Beaune *(figure 3.10)*, following the clear guidance given by Chouquer and Favory (1980: fig. 47), it is possible to see that a cadastral axis is very probably represented by the approximate line of the N74 immediately to the south-east of Pommard.

At St Aubin *(figure 3.11)*, in contrast to Beaune, it is difficult to find a obvious linear feature in the modern-day countryside which could define the orientation of the cadastre without ambiguity; but it is possible to draw a line CD on the map which corresponds to the line of the *quintarius* showing as a soil mark in the aerial photograph.

Then, using the parameters:
- module = 711.13m,
- angle = 30.734°,
- point of origin (point A): $x = 8999$, $y = 2840$,

the Lambert coordinates of more than 5,000 potential intersections of the cadastre were calculated.

It is now possible to make some comparisons.



From 1:25,000 sheet 3124 ouest © ign 1980

Figure 3.11 Saône plain cadastre at St-Aubin.

Straight away it can be seen that there is a very satisfactory coincidence between the model grid and the reconstruction near Beaune proposed by Chouquer and Favory (1980: fig. 47).

Furthermore the coincidence with the existing countryside at Pommard and at Volnay *(figure 3.10)* is striking; there are numerous roads, lanes, paths and field boundaries at the orientation of the model grid. Two long sections of the commune boundary of Volnay seem to have perpetuated the major *limites*[51] The churches of Pommard and Volnay are symmetrically positioned about the intervening commune boundary (they are on the next *limes* in both cases) and they both have the orientation of the cadastre, rather than pointing east-west. Finally, in several places one can see a subdivision of the cadastre by parallel boundaries at a spacing of 5 *actus*. This is a well known structure; for example it is visible in the Po valley at Cesena (Bradford 1957: plate 38), and at Imola *(figure 3.12)*.

Figure 3.12 Part of cadastre of Imola, showing subdivisions at 5 actus.[50]

In the Finage we also see *(figure 3.11)* that the computer-calculated grid is very close to the reconstruction proposed by Jeannin and Chouquer in 1978, and the line of the *quintarius*, CD, is a perfect fit to the theoretical grid.

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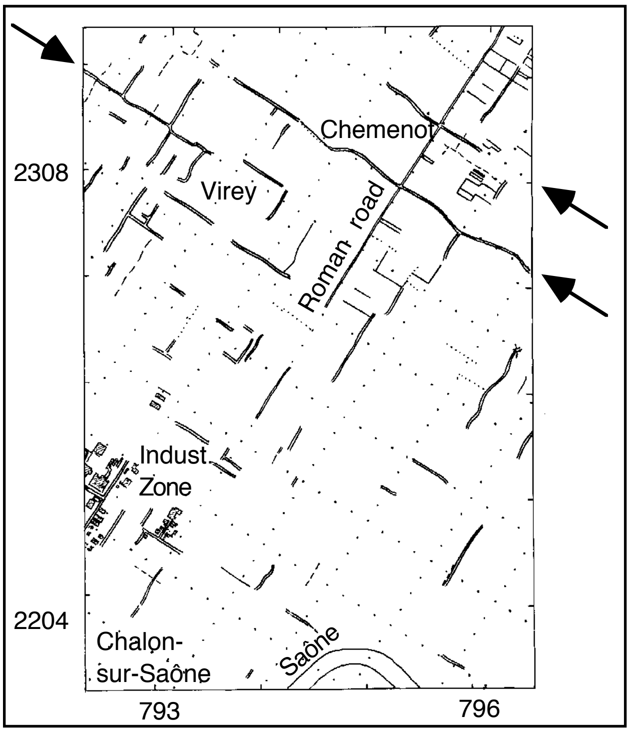
50 Illustration selectively traced from Farinelli (1976: fig.94).

51 These are shown by lines of points at an intervals of 5 *actus*.

Thus if we ignore the recent suggestion of Chouquer and de Klijn that the angle of the system in the Finage should be reduced by more than 1°, it is entirely possible that the fragments at Beaune and in the Finage are parts of the same centuriation. If so, we must accept that the module is in excess of 711m[52] As in the case of the Limburg cadastre, this is larger than the module of 710m which is conventionally attributed to centuriations of the first century AD[53].



From 1:25,000 sheet 3025 est © ign 1983

Figure 3.13 Saône plain cadastre, north east of Chalon-sur-Sâone.

Step (iv) Fit of the model to the Chalon-Dijon Roman road

The model predicted that certain points of intersection of the grid should fall on the Chalon-Dijon Roman road. Accordingly they were plotted on IGN 1:25,000 sheet 3025 est[54]. As can be seen *(figure 3.13)*, there is, at this scale, no detectable difference

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[52]This is larger than the previously published figure of "légèrement plus que 710m" (Chouquer 1983c: 117)), by about one metre. A similar variation can be seen in the module calculated above for the northern part of Orange B, which was 708.8m rather than the published value of 708m (Chouquer 1983d: 291)). These sort of errors, of about one part in one thousand, are normal for computer plotters.

[53] The date of this cadastre is given as about 70 AD (Chouquer and de Klijn 1989: 286).

[54] Rather frustratingly, this could not be done immediately. The coordinates were printed on 29 October 1990. The map only became available in February 1991, through the kindness of Gérard Chouquer.

between the line of the theoretical *limes* and the line of the road as represented on the map.

Also, if we take the road as a *kardo*, there are very prominent orthogonal traces which also fit the model. The cadastre is evidently present, as shown by the rectilinear pattern of communication and the alignment of the large group of industrial buildings on the northern outskirts of Chalon. Furthermore the three largest traces at right angles to the road (identified by the arrows) all conform to the expected positions of *decumani*.

Step (v) Explanation

Contrary to expectation, there is no detectable difference between the computed model and the previously proposed survey, as inferred from supposed traces on the ground. The model, as determined by two theoretical intersections A and B, 39km apart, fits a linear feature (the Roman road) and three orthogonal linear features (the three *decumani*) to an accuracy of better than 10m. At its furthest point this linear feature is 25km from its point of intersection with the line AB. If the centuriation existed, the accuracy of linear measurement is thus better than one part in 2,500. and the accuracy of angular measurement is better than $\mathrm{Tan}^{-1}0.0004 = 0.023°$, that is 1.4' (or ±0.7').

Thus there is no discrepancy of measurement which needs to be explained. This fact suggests that the earlier proposal, regarding the existence of a survey in centuries, is strongly supported. It also suggests that the survey, if it was conducted, was astonishingly accurate. We must ask how this could have been achieved.

Three practical studies have been conducted using replicas of Roman surveying instruments. Adam (1982) reports exercises in surveying with the *groma* at Vaison-la-Romaine, and claims that the accuracy of setting out over a short distance of about 50m was very comparable to that obtainable with modern instruments, and that the error in surveying existing structures was 0 to 1.5%. Dilke (1987b: 31) describes a trial measurement of 2 *iugera* with university and school students, using replicas of the *groma* and *decempeda*, and claims that "the resulting error was minimal". Another experiment by Schlögl (1991) produced angular errors in the range 5' to 42', although the larger error was attributable to the presence of a bed of *urticae* (nettles). Schlögl regards this as "erstaunlich genau" (astonishingly exact), but none of these experiments is particularly revealing. The *mensores* would have had to have done much better to achieve the degree of accuracy observable in the Saône plain.

Distances along a straight line could probably have been measured accurately; a matched set of *decempeda* would allow this, if they were kept straight and if care were taken over levelling on slopes. It is the accuracy of the 90° angle that is most surprising; one part in 2,500 is good, even by modern engineering standards[55].

A simple way of ensuring squareness of lines between corresponding points on two parallels is to make the diagonals equal; this was the method used until this century in the setting up of the frames of steam railway locomotives (Chapman 1936: 194). It is also the method suggested by Adam (1982: 1016). However, when one considers the difficulty associated with simultaneously measuring the two diagonals of even one century (a distance of about 1km), and then of making successive adjustments in the right direction, it seems unlikely that the *agrimensores* did it this way[56]. Another method that might be suggested would involve the use of a 3:4:5 triangle, well known in antiquity, but this has the same defects.

There may have been, perhaps, another method of achieving accuracy.

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[55] The most popular grade of engineers squares (Moore and Wright series 400, to British Standard Specification 939) is made to an accuracy of one part in 12,000. (Information from James Neill catalogue, 1980s)

[56] In fact Adam suggests an approximation of 3,400 feet as the diagonal of a century of side 2,400 feet. This is an error of 5.9 feet, or one part in 575, which is less accurate than the result actually achieved in the case of the Saône plain system.

The metal parts of a *groma* were found at Pompei in the ruins of what appeared to be an instrument maker's workshop. The instrument *(figure 3.14)* has four plumb bobs for the four sighting lines which were suspended from the ends of the cross arms. These plumb-bobs are of two sorts, round and pointed[57], and it is not known how they are to be reconstructed. Are matching bobs to be placed at opposite ends of the same arm or are they adjacent?[58]

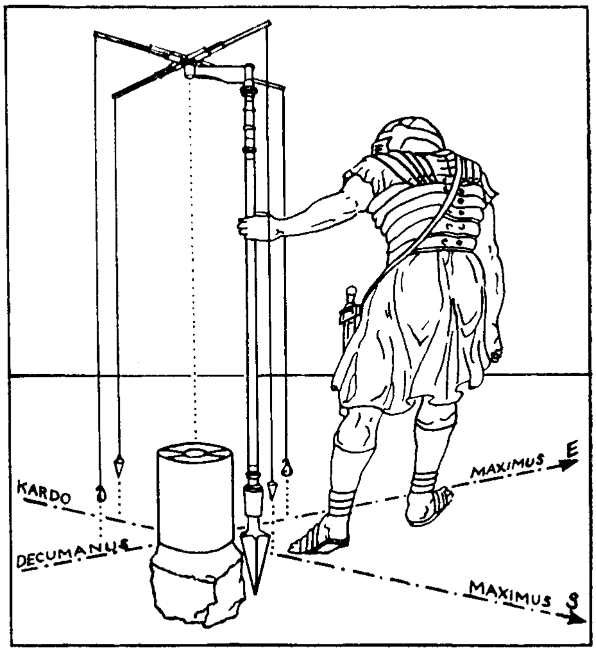

Figure 3.14 The Groma.

The physical function of the plumb-bobs is to keep the lines straight, but we can also consider whether or not the difference in their shape has some additional symbolic function. Does this difference have semantic value, and in particular is it used to discriminate between different parts of the instrument?

If the aim is to identify the two arms of the *groma*, each arm could have had two similar bobs. Given the Roman predilection for bilateral symmetry, this is a possible arrangement, but we may consider the alternative arrangement, in which similar bobs are adjacent.

It is virtually impossible to construct by hand an instrument which will measure right angles to an accuracy of one part in 2,500. Thus if the *groma* was used to set out accurate right angles there must have been some way of compensating for its inevitable error. One way of doing this may have been to survey the right angle twice

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57 Dilke (1985: 89)) says "two different pairs of plumb-bobs were found".

58 Bradford (1957: 151), following Frigerio, *Antichi strumenti technici*, depicts similar plumb-bobs at the end of each arm *(figure 3.14)*.

from the same point, but using adjacent right angles of the *groma(figure 3.15)*. An assistant with a ranging pole could mark the two indicated positions A and B and then place another mark at their mid point to obtain a more accurate right angle.

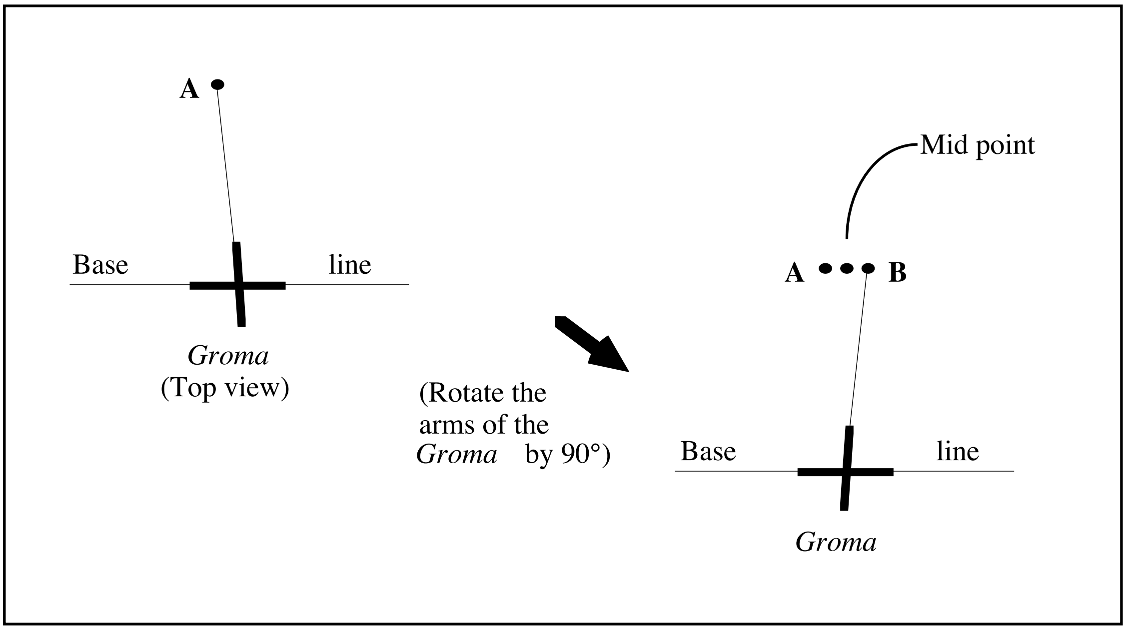

Figure 3.15 Possible means of improving the accuracy of the Groma.

In order to perform an operation of this sort, in which accuracy is achieved by arranging for the angular errors to cancel each other out, the two pairs of opposite angles between the arms of the groma must be identified. They can be easily distinguished if one pair of angles is between similar bobs, and the other is not. However, this proposed method of improving the accuracy of the groma is undocumented, and must remain a speculative explanation for our observations of the apparent accuracy of the Saône plain survey.

It appears that at least one Roman survey of the late first century AD was executed to high precision, comparable to that achievable in recent times. No surveying errors can be detected at the level of accuracy of the modern 1:25,000 maps. This leads to some conclusions.

To a high degree of probability, there was a survey in centuries in the Saône plain. It seems highly unlikely that three features, the intersections at Beaune and in the Finage and the Chalon-Dijon road, would fit together so accurately by chance. Nor could Chouquer have pre-arranged such a fit. His methods were different, based on the use of transparent overlays, which would entail small inaccuracies. He also used geographic north as a reference for orientation. Despite the inherent inaccuracies of his approach he was forced, by what he saw on the ground, to the conclusion that the features are all part of the same grid. This perception is vindicated by calculation.

We thus have a measure of the very high level of precision which could be achieved by the *agrimensores*, and we have some support for Chouquer's proposals[59]. This weakens the current notion that centuriated surveys in France could not exist north of Lyon, and it could provide us with a model for other Roman systems of land division in the north-western parts of the Empire.

Even if we discount the clear evidence for the existence of the Limburg system *(described above)* and regard that of the Saône plain, with other similar systems in the same area (Chouquer 1983c), as the most northerly examples of centuriations, we can conclude, contra King (1989: 99), that they can no longer be represented as a purely Mediterranean phenomenon.

We may therefore consider the possibility that centuriations exist in other parts of the north-western Empire, and we may use computer-based models for them unless we have reason to think that they were inaccurately surveyed.[60]

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59 Note, however, that the existence of a single survey does not necessarily imply the existence of a single cadastre, in the administrative sense.

60 It must be confessed that the author started work in the faith that a computer-calculated set of coordinates could be an accurate model. It is naturally most gratifying to have this faith confirmed four years later.

3.1.5 Modelling a cadastral grid, example 4: - a supposed cadastre in south west France

As Paul Courbin (1988: 124) has pointed out, it is up to the archaeologist to identify fakes; this is the sum of his expertise in identification. So, assuming that the accuracy of surveying of purported examples of centuriations approaches that of the Saône plain system, we can use computer-based models to test them. An example of this arises in the case of a supposed cadastre in south west France.

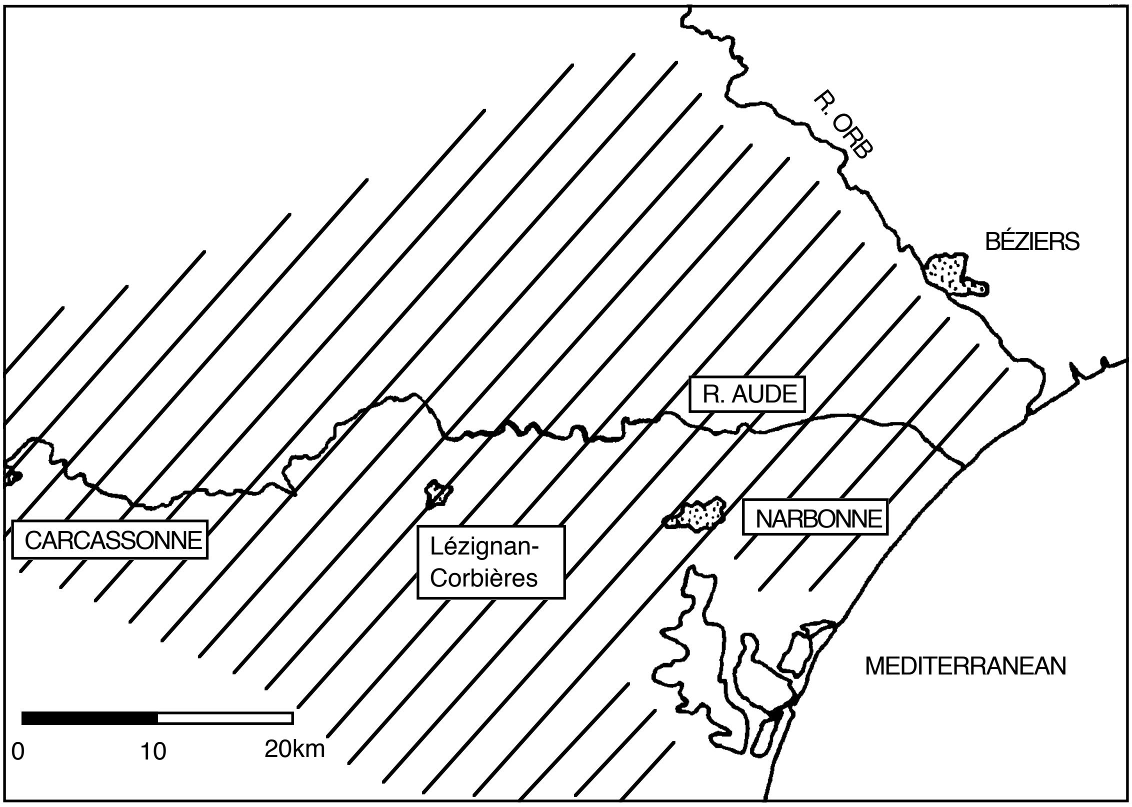

Figure 3.16 Eastern part of "Cadastre occidental de Narbonne", after Perez (1986, fig. 7).

Antoine Perez (1986) proposes that a single cadastre, with module 705 m, took the form of an irregular band stretching at least 220km from the Mediterranean near Narbonne, past Toulouse and north-westwards down the Garonne valley. He also shows (Perez 1986: fig. 7) a possible extension of the *pertica* nearly to Bordeaux; this would give a greatest extent of 330 km. This is very large by the standards of the Narbonnaise but, given the huge grids of Africa, it is possible.

Perez considers first the eastern extremity of this hypothetical cadastre *(figure 3.16)*. We can select some features *(figure 3.17)* which Perez shows coinciding with the *limites*. On the left are features from his figure 2 and, on the right, features which occur in his figure 3.

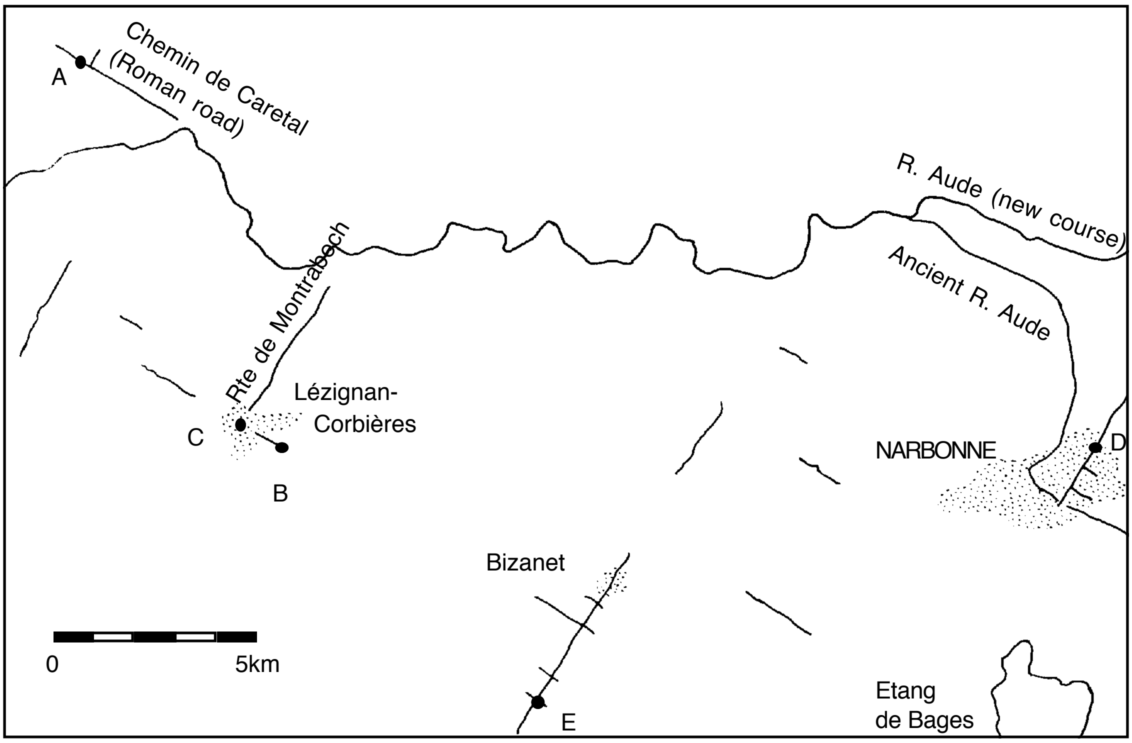

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Figure 3.17 Limites *and defining points in the eastern part of the "Cadastre occidental de Narbonne"*.

He describes how the Chemin de Caretal *(figure 3.17, top left)*, a Roman road, appears to be part of a present-day orthogonal pattern of boundaries, and how this best fits a square grid of 705m. He also perceives the road south west from Bizanet *(figure 3.17, bottom centre)* as "strictly orthogonal" to the Chemin de Caretal and observes that it is similarly related to the neighbouring parcelling. His search for further features extends to the rest of the Aude plain around Narbonne; and on the basis of the traces, three quarters of which are shown here, he claims that a single grid existed, with orientation "N 31,30° E".

Since orientations which are determined manually and with the use of transparent overlays (as in this case) are not normally given to an accuracy greater than half a degree, it seems reasonable to suppose that this orientation is N 31° 30' E, i.e. 31.5°. Perez does not state whether the orientation is specified with respect to true North or one of the Lambert kilometre grids.

The features shown here are to be found on three different IGN 1:25,000 topographic map sheets, from which the coordinates of points A-E could be most easily measured in terms of the Lambert zone III kilometre grid, since its intersection points are printed on the maps *(figure 3.18)*.

Point	**Description**	**Coordinates**
A	On Chemin de Caretal	63036,310885
B	Junction with D61	63519,309948
C	On line of route de Montrabech	63428,310001
D	On route de Rasimbaud	65530,309954
E	Intersection SW of Bizanet	64169,309333

Figure 3.18 Coordinates of points used to determine orientations for the "Cadastre occidental de Narbonne".

The ORIENT program on a Macintosh computer *(figure 3.3)* was used, as in the case of the Limburg system *(3.1.3)*, to determine ranges of orientations of grids defined by points A-E and to compare them with the orientation given by Perez for the cadastre as a whole.

For three pairs of points, the choice of grid module determines a variety of angles *(figure 3.19)*. The column for points A and B (headed A,B) gives possible angles for the layout shown in Perez' fig. 2. Similarly the column for points D and E shows possible orientations for the layout of Perez' figure 3. The position of a further point C was determined by fitting the first layout to the route de Montrabech north east of Lézignan-corbières.

MODULE (= 20 actus)		ANGLE (Degrees east of north)		
		A,B	D,E	C,E
	700	30.642	31.313	31240
	701	30.590	31.257	31.678
	702	30.539	31.202	32.145
	703	30.487	31.146	32.614
	704	30.436	31.090	33.119
	705	30.388	31.035	33.654
	706	30.333	30.979	34225
	707	30.282	30.923	-
	708	30.230	30.869	-
	709	30.178	30.811	-
	710	30.127	30.756	-

Figure 3.19 Calculations of possible orientations of "Narbonnaise Occidental".

The figure shows that, for a given grid module, there is a persistent difference between the angles obtained by using points A,B and D,E. This indicates that there is no grid module for which the orientation of these two fragments of the supposedly uniform cadastre would be the same. This conclusion is supported by studying the final column. It is noticeable that the grid orientation determined by points C and E is extremely sensitive to variations in the module. Only for a low value, 700 m, is the angle anywhere near that determined by the features shown in Perez' figures 2 and 3.

It is doubtful if such a small module was ever employed, and in any case it is very different from the figure (705m) given by Perez. A discrepancy of 5m per century would lead to a significant cumulative error in a cadastre as large as that postulated in this case. For example, Perez' (1986: fig. 5) illustration of the cadastre in the environs of Toulouse would be misplaced by about 750 m - more than a century.

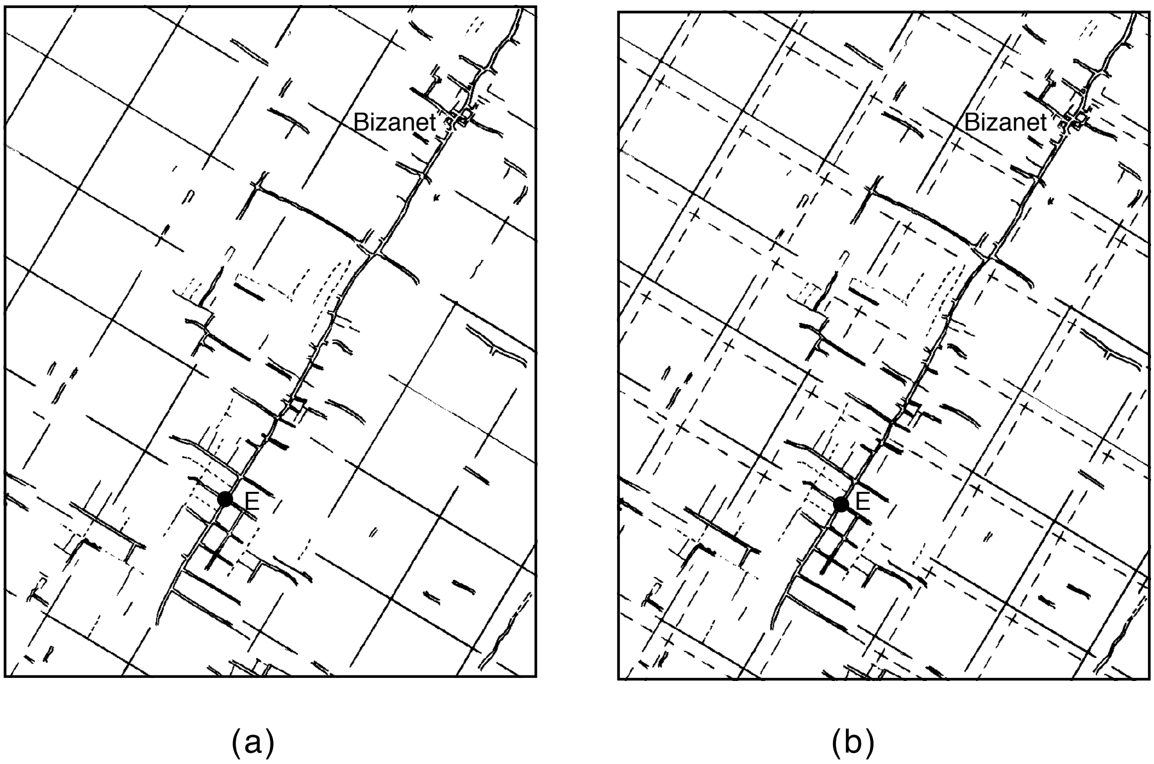

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Figure 3.20 *Comparison of positions of Narbonnaise occidental in the area of Bizanet, as defined by alternative pairs of points.*

If, on the other hand, we accept the given module of 705 m, the discrepancy between the orientations shows up clearly in an examination of the modern-day boundaries at about N 30° E in the area of Bizanet *(figure 3.20)*. At (a) we see how they relate to a grid defined by points D and E (assuming that E is an intersection). This produces a rather impressive degree of correspondence. But at (b) we see that the same sized grid of dashed lines, as defined by the Chemin de Caretal and points B and C, does not correspond to this layout. Furthermore, a reduction of the grid module would have the effect of turning the *limites* of the this layout clockwise, and hence further away from the grid defined in (a).

It can thus be concluded that Perez' figures 2 and 3, if they are indeed a representation of parts of a grid of 705 m, cannot be made to fit together. This demonstrates the superiority of computational methods over manual techniques, particularly when the cadastre under investigation covers several 1:25,000 map sheets.

Perez asks (1986: 125) if we can talk of a "cadastre inter-provincial". This seems improbable, since, according to the calculations described here, even a relatively small part of his proposed system does not seem to fit together.

3.2 A simple statistical test of site distribution

We can make simple statistical tests of association of archaeological features with hypothetical cadastral grids.

There is no doubt that features of all periods, starting from the period when a cadastre is first established, tend to be influenced by and often located near its *limites*.[61] The reason for this could be symbolic, as perhaps in the case of the Roman temples near Valkenburg *(figure 3.8)* or Beaune *(figure 3.10)* and the modern crosses on the line of the *quintarius* at St-Gervais *(3.1.2)*. It could also be economic, particularly when the *limites* are materialised as means of communication, i.e. roads or canals. If so they provide a readily available means of access.

Whatever the reason, we expect that within the area of a centuriation the distribution of sites of many different types will show this non-random distribution pattern. Accordingly a cadastral hypothesis can be tested by examining the distribution of distances of sites from the grid lines, when compared to the distribution which would be expected on the null hypothesis that the points are scattered uniform randomly.

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[61] So, for example, sites in the northern *Ager Cosanus* dated to the 2nd century BC, which have been found "only on the major axes of the centuriation" (Attolini, et al. 1990: 145)). For the link with more modern sites see Caillemer and Chevallier (1954: 458). "Des routes, des voies ferrées, des pistes d'aérodrome, des limites de commune s'orientent de même pour éviter de couper les cultures dont les contours correspondent toujours à la répartition antique du sol; il arrivent souvent que de grandes fermes modernes soient situées à l'emplacement de ruines romaines, dans l'angle de centuries." This general picture, of the influence of the cadastre and its *limites* on a variety of modern features (even airport runways) is still valid.

In this sort of hypothesis test we follow Hodder and Orton (1976: 226-229) and Reilly (1988: 180-186)[62], at least in principle. Hodder and Orton counted the numbers of late Iron Age inscribed coins found in three bands covering arbitrary distances from Roman roads in central and southern England, and compared them with the relative areas within the bands. They used both the χ^2 and Kolmogorov-Smirnov tests to demonstrate a significant bias towards the roads. In the latter test, since their methodology divided the total area into areas at up to thee distances, they were obliged to interpolate the curve for expected distances from just three points. Given that GIS systems are now more widely available, this is now unnecessary. For example, the IDRISI distance function can be used to obtain an almost continuous distribution of cell distances from the roads. This can then be compared to the distances of the coins, with a more accurate (and more statistically significant) result (Kvamme 1992: 79).

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[62] Reilly studies the distribution of early Manx chapels (keeils) and burial grounds (rhullicks) in relation to the land unit called a treen (in effect a sub-parish) in order to see if there is support for two hypotheses. Firstly, were the keeils established deliberately on the basis of one keeil per treen? Secondly, were keeils and rhullicks deliberately established on treen boundaries?

Reilly (1988: 139-142) considers the lack of secure dating of the keeils and rhullicks to be a problem. He concludes that statistical tests of correlation between these sites and the treens can proceed if it is accepted that "even if the final distribution is the result of a palimpsest of activities, the end product might still be thought of as a contemporary network of sites of special sacred importance". By implication, and judging from the slightly defensive tone of his discussion, the statistical analysis cannot proceed unless we accept that all the sites were simultaneously in use at some point in time.

In his case the real problem is that the treen boundaries may have moved, thus invalidating a study of site location with respect to them. Fortunately this is not the case with a hypothetical centuriation. Local changes to the network may weaken the association, but this will only lead to a less statistically significant result, not an invalid one.

Reilly was facing a similar problem, but he chose to simulate the distribution of distances of points scattered at random, rather than measure bands of area. He did this in order to evaluate Goodier's (1983; 1984) semi analytical method for measuring boundary association.

Goodier had used empirical data which suggested that for irregular boundaries the boundary length would be less than one sixth of the square root of the area. She then used a formula to calculate the area of bands of constant width along the boundary. Reilly showed that this method is flawed. For irregular boundaries it is much more reliable to use the actual boundaries of the system being studied.

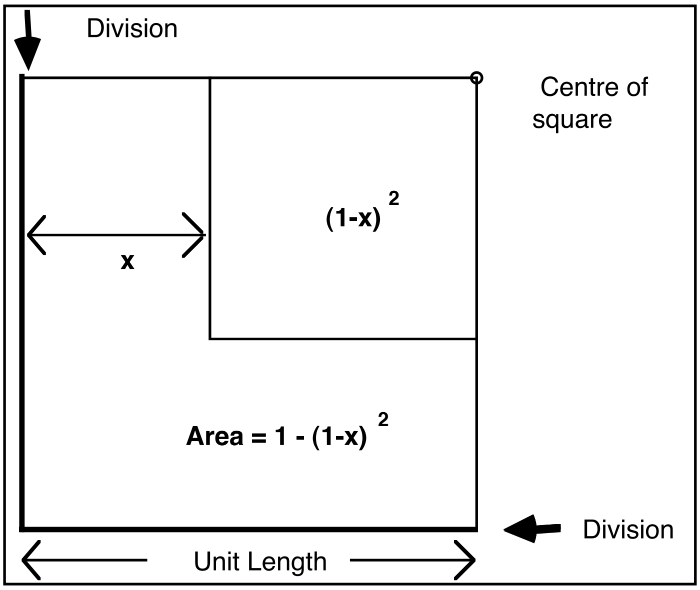


Figure 3.21 Expectation for distribution of distances of random points.

However, in the case of formal cadastres, particularly centuriations, analytical methods can be used. Thanks to the foresight of the Roman land surveyors the distribution of features within their systems can be studied by the simplest of computational means.

For a uniform random scatter within a square grid the continuous cumulative distribution is $1-(1-x)^2$, where x is a fraction of half the grid distance *(figure 3.21)*. For example, for x = 0.5 we have the expectation that $1-(1-0.5)^2$ or 75% of points scattered at random in any grid square will fall within the band so defined, i.e. at distances up to a quarter (0.5 x 1/2) grid distance from a grid line. Again, it is certain that any point will fall within half the grid distance, since the formula gives the expectation for this distance as 1 (or 100%).

In this case where, on the null hypothesis, a continuous distribution is expected, and where sample sizes are less than the minimum of 5 required for the χ^2 test, we can use the Kolmogorov-Smirnov single sample test. This approaches the problem by comparing the observed cumulative frequency distribution of the sample to that expected from the population specified by the null hypothesis. The test statistic, D, is the maximum deviation between the observed and the expected distributions (Lapin 1973: 422). At an early stage of the research two programs, KS1 and KS2, were written in VAX Basic to calculate the D values for the observations.

KS1 calculates the distance to the nearest *limes* from each point in the set of observations. It uses a routine which calculates the shortest perpendicular distance between a point and an infinite array of parallel lines which represent one set of *limites*. These are at a fixed distance m apart and pass through a an 'origin' at a,b at an angle ß. For a point x,y this distance d is the lesser of

$$\left|(y-b)\mathrm{Sin}ß - (x-a)\mathrm{Cos}ß\right|_m \text{ and}$$

$$m - \left|(y-b)\mathrm{Sin}ß - (x-a)\mathrm{Cos}ß\right|_m$$

This distance is calculated for ß and ß $+\frac{\pi}{2}$ (i.e. for the two sets of *limites*) and the shorter distance is added to a list of distances expressed as fractions of 0.5 x m. This is done for all n observations and the list of distances sorted.

KS2 calculates the a D value for each observation, value d, by comparing $\frac{r}{n}$, the fraction of the observations which this observation represents, against the number of observations expected for this d on the null hypothesis, which is $1 - (1-d)^2$. It prints the list of sorted distances, D values, the total number of observations and the maximum value of D.

The probability with which the null hypothesis can be rejected can be read from a table of critical values of D, which are commonly available for sample sizes up to 100[63]. For larger samples the critical value, D_{α}, for a given probability, α, of rejection can be calculated using the formula (Rohlf and Sokal 1969: 249):

$$D_{\alpha} = \frac{\sqrt[2]{\dfrac{-\log_e(\frac{1}{2}\alpha)}{2}}}{\sqrt[2]{n}}$$

For commonly used probabilities the values of the numerator of this expression are:

Probability	.8	.9	.95	.98	.99	.995	.998	.999
Value	1.07	1.22	1.36	1.52	1.63	1.73	1.86	1.95

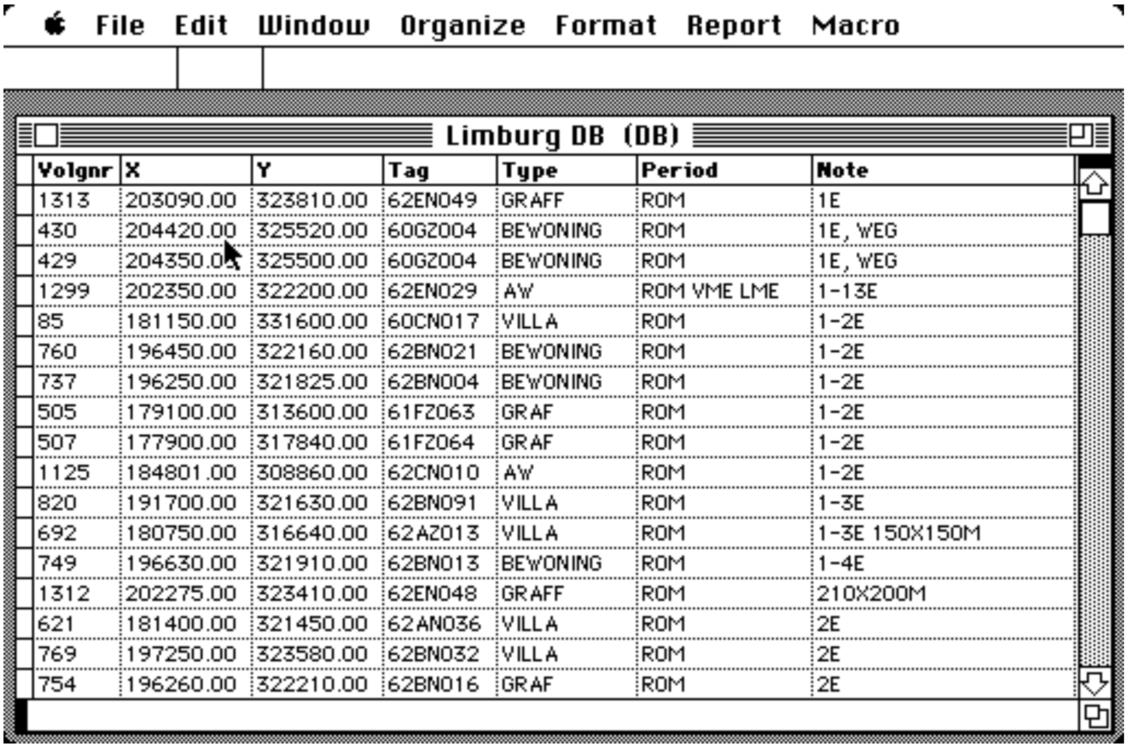
Figure 3. 22 Numerator values in the asymptotic formula for Kolmogorov-Smirnov critical probability levels.

These numerator values can then be divided by the square root of the sample size to obtain the critical values of D. So, for example, if we have 400 observations (square root = 20) the critical value for a .99 probability of rejection is .082. If the D value for the observations achieves this level then we can say that the observed distribution would have occurred with less than 1% probability on the basis of the null hypothesis.

3.2.1 Example of Kolmogorov-Smirnov tests on the Limburg data set

As mentioned above *(3.1.3)* the large amount of data available on the Limburg area gives us the opportunity to demonstrate the use of Kolmogorov-Smirnov tests, and perhaps to say something about the probability that the supposed centuriation exists.

63 For example (Beyer 1966: 322).



Volgnr	X	Y	Tag	Type	Period	Note
1313	203090.00	323810.00	62EN049	GRAFF	ROM	1E
430	204420.00	325520.00	60GZ004	BEWONING	ROM	1E, WEG
429	204350.0[illegible]	325500.00	60GZ004	BEWONING	ROM	1E, WEG
1299	202350.00	322200.00	62EN029	AW	ROM VME LME	1-13E
85	181150.00	331600.00	60CN017	VILLA	ROM	1-2E
760	196450.00	322160.00	62BN021	BEWONING	ROM	1-2E
737	196250.00	321825.00	62BN004	BEWONING	ROM	1-2E
505	179100.00	313600.00	61FZ063	GRAF	ROM	1-2E
507	177900.00	317840.00	61FZ064	GRAF	ROM	1-2E
1125	184801.00	308860.00	62CN010	AW	ROM	1-2E
820	191700.00	321630.00	62BN091	VILLA	ROM	1-3E
692	180750.00	316640.00	62AZ013	VILLA	ROM	1-3E 150X150M
749	196630.00	321910.00	62BN013	BEWONING	ROM	1-4E
1312	202275.00	323410.00	62EN048	GRAFF	ROM	210X200M
621	181400.00	321450.00	62AN036	VILLA	ROM	2E
769	197250.00	323580.00	62BN032	VILLA	ROM	2E
754	196260.00	322210.00	62BN016	GRAF	ROM	2E

Figure 3.23 The 'van Leusen' data for Limburg, as received.

491 pieces of data were made available, covering sites of the Roman period[64]. A Microsoft Works database was set up *(figure 3.23)* and an extra column subsequently included, containing the grid references in a modified form suitable for input to the KS procedure on the VAX multiaccess system. It was thus possible, while communicating with the VAX, to select sites of different types from the Works database, copy the list of coordinates and paste them into a VAX data file for immediate processing.

Prior to performing the tests no attempt was made to modify the data in any way. It was clear that some coordinates referred to the same site, which might for example have both signs of habitation (bewoning) and graves (graff). It was also clear that the most objective way of treating the data would be to ignore these cases,

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[64] The data was most kindly supplied by Martijn van Leusen, University of Amsterdam. Prior to sending it he was not aware of the author's reconstruction described above. The data was transmitted as a text email message and read directly into the database.

since they were not likely to bias the result of the tests in any particular direction.

Several sets of data were tested *(figure 3.24)*.

	Type	No.	D	Near %	P not <	α
1	All sites	491	.08247	56.4	0.995	.0025
2	All definite sites (not IA)	419	.08463	56.8	0.995	.0049
3	Definite habitations (not IA)	85	.17927	62.4	0.995	
4	Temples	2	.80067	100	0.95	
5	Villas (all)	153	.11977	56.9	0.95	.025
6	Habitations (all)	107	.12234	57.0	0.9	.08
7	Villas (definite)	135	.10452	54.8	0.8	.104
8	Roads (definite)	15	.29019	66.7	0.8	
9	Roads (all)	18	.22353	55.6	not signif't	
10	Definite IA	17	.20327	41.2	not signif't	
11	Graves (all)	89	.10237	58.4	not signif't	
12	Graves, def Roman (not IA)	74	.09672	56.8	not signif't	
13	Other sites	142	.07134	54.2	not signif't	
and						
14	Questionable villas	18	.27526	72.2	0.9	

Figure 3.24 Kolmogorov-Smirnov test results for Limburg data.

In the table, the column headed "Near %" gives the percentage of the sites in each category which lie in the half of the area nearest to the *limites*. For this category the value of distance is less than $1-\sqrt{0.5}$ which is .29289. The use of this measure of degree of bias will be discussed below.

"P not <" gives the minimum probability (confidence) of rejection of the null hypothesis. There is clearly some correspondence between this and the measure of bias, but the correspondence is not perfect. The column headed "α" gives the value of the probability of D being exceeded, according to the formula for sample sizes over 100, viz: $\alpha = 2e^{-2nD^2}$.

Looking at the table one is struck by an exceedingly clear result. If we take the complete data set *(line 1)*, making no attempt to alter or analyse it in any way, we can say that there is not more than a 0.5% chance that the 491 values are drawn from a set of points distributed at random with respect to the cadastral grid. In other words the odds are more than 200:1 against the hypothesis of random distribution. Although the degree of bias of sites towards the *limites* is lower than might have been expected *(see below)*, the number of observations is so high that a significant result is obtained.

Having considered the population as a whole we may consider the statistics of properly defined subset populations. As David Clarke said:

> "One important corollary of the aggregate or composite nature of archæological entities is that such populations exhibit their own specific 'behavioural' characteristics which are more complex than the simple sum of the characteristics of the components and more predictable than that of the individual components. One of the main tasks therefore, is to detect and trace these persistent regularity patterns in archæological data and to use these predictable regularities as tests for real data. If the real data displays the regularity predicted then it should fulfil some already established conditions. If the real data departs from the predicted pattern then some conditions are not fulfilled and the nature of the discrepancy may suggest the divergent conditions responsible for the anomaly." (Clarke 1978: 150).

While we express reservations below *(7.2)* concerning the general applicability of this approach, there is no doubt about the expected regularity of distribution of populations of sites in Roman cadastres. Thus, in the spirit of Clarke, we can observe the discrepancies to see if they suggest divergent conditions. Of course Clarke's method cannot be applied in an unmodified form because only one variable is being measured, i.e. the distance of sites from *limites*; hence the need to consider subset populations which have already been defined by the attribute values in the database. This does nothing to invalidate the result obtained from the population as a whole, and may provide us with additional useful information.

If sites with previous Iron Age use, and sites not certainly identified or not certainly Roman, are excluded *(line 2)*, the bias towards the grid lines increases slightly, but the value of α is nearly halved. This is uninformative; the lowering of the α value may be due to the reduction in the sample size.

Another more interesting result is the difference between those definite Roman habitation sites which did not have Iron Age occupation on the same site *(line 3)*, and the habitation sites in general *(line 6)*. The former have a very definite bias towards the *limites*, and their distribution is approximately 20 times more unlikely. This seems to be a confirmation of the expectation that, in general, sites with signs of Iron Age habitation will not be significantly associated with the grid, and that their inclusion in the set of Roman habitation sites will reduce its apparent degree of association.

For all villas *(line 5)*, certain and questionable, the departure from a random distribution can be asserted with odds of 20 to one, but when doubtful examples are excluded a result of much less significance is obtained. This apparently paradoxical result may not be totally due to a reduction in the sample size. If so, the following hypothesis can be advanced: that some of genuine villas were deliberately placed away from *limites*[65]. If so the exclusion of the questionable examples, which are not all villas in reality, but other habitations, would make the distribution of the set of villas with respect to the grid more random.

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[65] We know that Columella I. v. 7 (1977: 63), while admitting the value of access roads, advised gentlemen not to site their dwelling near a main road, for fear of having to offer accommodation to passers-by. Some of the *limites* may have been too busy to be attractive. There is also the fact that more important villas occupy large curtilages, which would imply that even if the curtilage abutted a *limes* the actual dwelling house would be at some distance. Furthermore there is the possibility that a large villa could be the centre of a *fundus exceptus*, and hence possibly unrelated to the cadastre.

This idea can be tested by looking at the set of questionable villas *(line 14)*. These figures show that there is a chance of less than 10% that their distribution is random and that they are highly biased towards the grid lines *(figure 3.25)*.[66] They thus seem to have the characteristics of habitation sites in general, rather than definite villas.

Questionable Villas

No.	Volgnr	XY	Distance	Observed	Expected	D	
							Max D
1	1325	2004432390	0.005382	0.055556	0.010735	0.044821	0.275263
2	636	1882431960	0.050649	0.111111	0.098733	0.012378	
3	836	1941232095	0.060834	0.166667	0.117967	0.048699	**Total No.**
4	463	1790032220	0.077231	0.222222	0.148497	0.073725	18
5	2	1866634378	0.088480	0.277778	0.169131	0.108646	
6	496	1773031310	0.111168	0.333333	0.209978	0.123356	
7	1181	1971630873	0.165945	0.388889	0.304352	0.084537	
8	452	1749031925	0.201910	0.444444	0.363052	0.081392	
9	560	1749031925	0.201910	0.500000	0.363052	0.136948	
10	1310	2027532215	0.211787	0.555556	0.378720	0.176835	
11	315	1898232693	0.232526	0.611111	0.410984	0.200127	
12	1284	2003032295	0.251196	0.666667	0.439293	0.227374	
13	482	1789531380	0.275000	0.722222	0.474375	0.247847	
14	802	1945232460	0.308166	0.777778	0.521366	0.256412	
15	582	1781431004	0.335222	0.833333	0.558070	0.275263	
16	322	1809532733	0.452854	0.888889	0.700631	0.188258	
17	1301	2037531950	0.711881	0.944444	0.916987	0.027457	
18	581	1794531068	0.806953	1.000000	0.962733	0.037267	

Figure 3.25 Tabulation of D values for questionable villas.

Temples, despite their low numbers, also have a statistically significant distribution. They are both close to *limites*[67], and one is at a corner. This is to be expected[68].

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[66] These figures were obtained directly from the MS Works Database and processed as an MS Works spreadsheet. This way of working could replace the Kolmogorov-Smornov procedure previously used on the VAX. Its advantage is that it offers flexibility in the display of data and the ability to work almost anywhere. Its disadvantage is the degree of manual intervention necessary to execute the steps of the procedure.

[67] They may in fact be coincident, given the precision with which most of the site coordinates are given.

[68] Indeed, it was expected. The author said to van Leusen, when the possibility of using this data was discussed, that he anticipated that temples would tend to

Roman roads are not numerous, but again, as in the case of habitation sites, we see an increase in significance when the questionable examples are removed. Ten are in the nearer half of the area, of which five have coordinates within 40m of the grid. Given the imprecision of these coordinates, the relationship of these roads to the grid may repay further investigation.

The sites with definite Iron Age use were also investigated. This produced another apparently paradoxical result. One would expect them to be randomly distributed with respect to the grid, and the figures *(line 10)* do not, on the face of it, dispute this. But we observe that the D value is positive, whereas on the other hand the sites are generally biased away from the grid lines.

Sites with Definite Iron Age use

No.	Volgnr	XY	Distance	Observed	Expected	D	Max D
1	316	1853132736	0.012621	0.058824	0.025083	0.033741	0.203272
2	343	1825033050	0.014290	0.117647	0.028376	0.089271	
3	910	1939531923	0.030234	0.176471	0.059554	0.116917	**Total No.**
4	338	1889032567	0.041385	0.235294	0.081057	0.154237	17
5	1099	1940631818	0.046504	0.294118	0.090845	0.203272	
6	634	1858032165	0.213379	0.352941	0.381227	-0.028286	
7	1017	1946231418	0.255372	0.411765	0.445529	-0.033764	
8	205	1878133705	0.349110	0.470588	0.576342	-0.105754	
9	215	1886033250	0.355285	0.529412	0.584343	-0.054931	
10	1027	1980031721	0.397474	0.588235	0.636962	-0.048727	
11	1037	1969531439	0.419729	0.647059	0.663286	-0.016227	
12	76	1876333183	0.420286	0.705882	0.663932	0.041951	
13	764	1964032240	0.491608	0.764706	0.741538	0.023168	
14	203	1878033695	0.576592	0.823529	0.820726	0.002804	
15	987	1956031716	0.689001	0.882353	0.903280	-0.020927	
16	201	1877033690	0.869212	0.941176	0.982894	-0.041718	
17	195	1856433683	0.869710	1.000000	0.983025	0.016975	

Figure 3.26 Tabulation of D values for definite Iron Age sites.

The reason for this result lies in the detailed values *(figure 3.26)*. There are 5 sites lying within 17m of the grid, and hence possibly coincident. This may be chance, but other explanations could be explored. Perhaps the attribution of some of the Iron Age material

coincide with the grid, more than any other class of site. This provides a rare example of prediction in archaeology.

is faulty, or perhaps the cadastre was constructed to incorporate some pre-existing sites.

The conclusion is that the Limburg data indicates, with a probability of at least 200:1, that the cadastre proposed in 1959 does exist. Detailed investigation shows that there is probably a hierarchy of association with the grid: first temples, then habitations of unspecified type, then villas. Since results from other cadastres indicate much the same distinction, the probability is subjectively strengthened.

Thus the statistical approach gives some clear support to an old theory and it also suggests that there are further questions to answer. What is the relationship of the Roman road fragments? Is there a reason, other than chance, for the close association of a some Iron Age sites?

3.3 A Bayesian approach to examining site distributions

Despite the rather positive results which we have obtained from the use of Kolmogorov-Smirnov tests on the Limburg data, there are normally several factors which make conventional statistical tests unsuitable for use with the sort of data that we encounter in archaeological records. These are

i) the rather small data sets available in some cases,

ii) the "closed box" nature of the tests themselves, which make their results subject to scepticism, and

iii) the difficulty of combining the results with other evidence.

Sample size can be a problem; for example, out of the 491 entries in the Limburg database only two are temples. Recommendations are that the χ^2 test should not be used when any of the expected or actual frequencies is less than 5 (Moroney 1956: 258), so it could be wrong to use it even if we thought that in general it was an appropriate test. The Kolmogorov-Smirnov test can be correctly used with very small data sets, but we have seen that in a situation such as Limburg, where the sites as a whole have a bias towards the grid lines in the range 55:45 to 60:40, a very large amount of data is required before a statistically significant result appears.

The scepticism which may greet conclusions based on conventional significance tests is not surprising. As Clive Ruggles (1986) has suggested, methods of testing hypotheses using classical statistical techniques have generally become a closed box for archaeologists. In the Kolmogorov-Smirnov tests described above the method and the table of significance levels for D were taken on trust and it seems unlikely that many users of the statistical test would study its theoretical basis. As Ruggles says (1986: 9), "Archaeologists are largely separated from what is being done to their data, seeing only the input and end result. They are also likely to lose sight of the implicit methodological assumptions underlying the techniques being used". This is the sort of "mechanical ritual" which Gigerenzer et al. (1989: 106-109) describe as a degenerate form (frequently encountered) of what has come to be regarded as "statistical method" or, as they term it, "the monolithic logic of inductive inference".

Ruggles suggests the possible merits of a Bayesian approach. This is an idea which will be explored here because it offers two advantages. It may, by a very straightforward approach, reconnect the archaeologist with the data. It will also allow results obtained from one sort of data to be combined with indications of likelihood obtained by other means. This, which is the very nature of the Bayesian view of probability, conforms well to the realist eclectic approach advocated below *(7.2.2)*.

Bayesian statistics differs from the traditional approach in maintaining that probability statements are expressions of subjective belief. This does not, in itself, make the methods imprecise or "unscientific".

Bayes' theorem itself is based upon the generally accepted notion that the probability of two independent events occurring jointly is the product of the probabilities that each will occur.

Suppose that an event **D** (the datum) may occur only under a finite number of associated events or conditions $\mathbf{H_1}, \mathbf{H_2}, \mathbf{H_3}$ ... (the hypotheses). The probability of **D** and a particular $\mathbf{H_i}$ is

$$p(\mathbf{D}|\mathbf{H_i})p(\mathbf{H_i}) \quad \text{----------} \ (1)$$

where, in general, the notation $p(\mathbf{A}|\mathbf{B})$ stands for the conditional probability of **A**, given **B**.

Thus (1) expresses the fact that $\mathbf{H_i}$ has occurred with probability $p(\mathbf{H_i})$, and, given that this has happened, **D** has occurred with probability $p(\mathbf{D}|\mathbf{H_i})$.

The total probability that **D** will occur, $p(\mathbf{D})$, is then the sum of these probabilities for all **H**.

$$p(\mathbf{D}) = \sum p(\mathbf{H_j})p(\mathbf{D}|\mathbf{H_j}) \quad \text{----------} \ (2)$$

Now, for some particular $\mathbf{H_k}$, the probability of $\mathbf{H_k}$ and $\mathbf{D}$ both occurring can also be expressed as

$$p(\mathbf{H_k}|\mathbf{D})p(\mathbf{D})$$

This is the same probability as expressed in (1), so

$$p(\mathbf{H_k}|\mathbf{D})p(\mathbf{D}) = p(\mathbf{D}|\mathbf{H_k})p(\mathbf{H_k}).$$

Using (2) and dividing, we have Bayes' theorem in its general form

$$p(\mathbf{H_k}|\mathbf{D}) = \frac{p(\mathbf{H_k})p(\mathbf{D}|\mathbf{H_k})}{\sum p(\mathbf{H_j})p(\mathbf{D}|\mathbf{H_j})}$$

where

$p(\mathbf{H_k})$ is the prior probability of hypothesis $\mathbf{H_k}$, that is our subjective belief in its likelihood,

$p(\mathbf{D}|\mathbf{H_k})$ is the likelihood of event $\mathbf{D}$ occurring if $\mathbf{H_k}$ is true,

$\sum p(\mathbf{H_j})p(\mathbf{D}|\mathbf{H_j})$ is the total likelihood of $\mathbf{D}$ occurring under any of the hypotheses under consideration, and

$p(\mathbf{H_k}|\mathbf{D})$ is the new, posterior, probability of $\mathbf{H_k}$, given $\mathbf{D}$.

Thus, if an event $\mathbf{D}$ is observed, Bayes' theorem may be used to revise the subjectively expressed prior probabilities of a number of alternative hypotheses which would be associated with it.

If only two hypotheses, $\mathbf{H_1}$ and $\mathbf{H_2}$, could explain the event then Bayes' theorem can be put in odds-likelihood form

$$\frac{p(\mathbf{H_1}|\mathbf{D})}{p(\mathbf{H_2}|\mathbf{D})} = \frac{p(\mathbf{H_1})}{p(\mathbf{H_2})} \times \frac{p(\mathbf{D}|\mathbf{H_1})}{p(\mathbf{D}|\mathbf{H_2})}$$

or

$$\mathbf{\Omega'' = \Omega' L}$$

where $\mathbf{\Omega''}$ and $\mathbf{\Omega'}$ are posterior and prior odds (the ratio of probabilities of the two hypotheses), and $\mathbf{L}$ is the ratio of the likelihoods of the event according to each hypothesis.

Phillips (1973: 79-81) gives an example of the application of this formula to the problem of determining the odds that a coin is biased. It is assumed that a biased coin would show heads 60% of the time. Suppose that $\mathbf{H_1}$ is the hypothesis that the coin is biased, $\mathbf{H_2}$ is the alternative hypothesis that it is fair, and that the event $\mathbf{D}$ is heads. Then

$p(\mathbf{D}|\mathbf{H_1}) = 60\%$ and

$p(\mathbf{D}|\mathbf{H_2}) = 50\%$

so the likelihood ratio is 60/50 (1.2) for heads and, by a similar calculation, 40/50 (0.8) for tails.

Thus, for one trial in which heads shows, the odds that the coin is biased increase by a factor of 1.2. Conversely, if tails shows it decreases. In this way the evidence leads to a revision of prior belief.

Each new posterior value of the odds can be used as the prior odds for another trial. After several independent trials, with actual values of $\mathbf{L} = \mathbf{L_1}, \mathbf{L_2}, \mathbf{L_3}$ ..., the posterior odds can be expressed as

$$\mathbf{\Omega}'' = \mathbf{\Omega}' \prod \mathbf{L_i}$$

which is the odds-likelihood ratio form of Bayes theorem.

So in the case of the suspected biased coin, where only two likelihoods can appear, and $\mathbf{n}$ heads and $\mathbf{m}$ tails have occurred,

$$\mathbf{\Omega}'' = \mathbf{\Omega}' \, 1.2^{\mathbf{n}} 0.8^{\mathbf{m}}$$

Given that we have a record of a series of throws, the posterior odds depend on two other values, our prior belief that the coin was likely to be biased (the prior odds), and our knowledge that a biased coin would show heads 60% of the time, since this allows the likelihood ratios to be determined.

This example is simplified by Phillips in order to make it easy to understand. In practice we have to know that a biased coin will show heads. If you went to the bank, took a new coin, tossed it and it came up heads, you would not know if the odds should be increased or decreased.

Bayesian statistics can still tackle this problem, but now we have to adopt three hypotheses: that the coin is biased to heads, that it is fair, or that it is biased to tails. Suppose that the new (and very probably unbiased) coin is tossed and it comes up six times in a row. What are our posterior beliefs about it?

Hypothesis	Likelihood of a Head	Likelihood of 6 Heads	Prior probs.	Priors x likelihoods	Posterior probs.
Biased Heads	0.6	0.046656	0.001	.0000467	0.00299
Fair	0.5	0.015625	0.998	.0155937	0.99675
Biased Tails	0.4	0.004096	0.001	.0000041	0.00026
Sum			1	.0156445	1

Figure 3.27 Bayesian calculation of posterior probabilities after 6 tosses of a supposedly fair coin turns up 6 heads.

We can see *(figure 3.27)* that these data have very little effect upon our belief that the coin is fair. We are only a little more than one part in one thousand less sure than we were. Because of our prior conviction that new coins from banks should be fair, we would dismiss such a run of heads as a chance event.

However, this is a slight digression. In the investigation of cadastres we can employ Bayes theorem in its odds-likelihood ratio form because there are only two hypotheses: that the theoretical cadastre has influenced site location or that it has not.

This claim needs some justification because the sceptic might see the possibility that other factors could produce a spacing of 355m or 710m between sites. He would perhaps argue in favour of a natural environmental influence, such as the regular spacing of ridges of higher ground, or a cultural influence, such as the spacing of boundaries determined by field size. This may appear possible, but it would depend upon the frequency of these features being close the supposed frequency of the cadastral boundaries, otherwise, even if the two started in step, they would soon be out of phase. If, on the other hand, the frequency determined by some natural or cultural agent was close to that of the cadastre then the two would have to start in step in order to remain so. The chance of this is reduced if the definition of the position of the cadastre is independent of the area being considered.

As has already been said, in Roman cadastres there is often a noticeable bias of Roman and later sites towards the major divisions. A simple way of measuring this bias, in a 20x20 *actus* centuriation, is to divide each square of 20x20 or 10x10 *actus* into two portions equal in area *(figure 3.28)*. Then, given a set of observations, **n** in one area and **m** in the other, the odds-likelihood form of Bayes' theorem can be used.

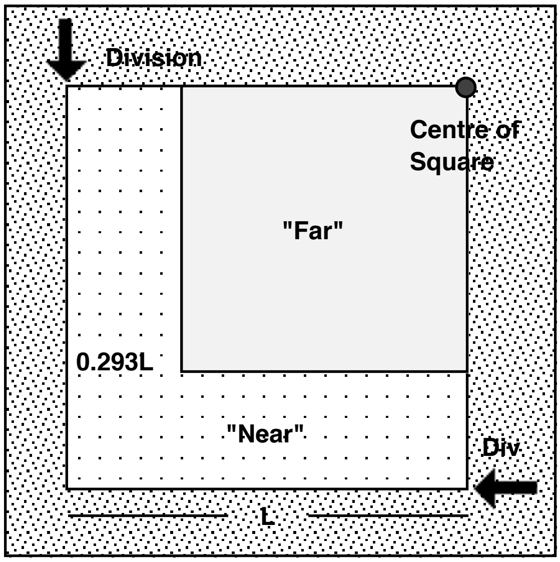

Figure 3.28 Equal division of a quarter square.

We have two hypotheses:
H_1 - a centuriated cadastre existed in a particular area and more than 50% of the sites are likely to lie in the area near the divisions,
H_2 - the cadastre did not exist; thus the distribution with respect to the hypothetical grid is random and there is a 50% likelihood that a site appears in either area.

The next step is to obtain some agreement on the likelihood of finding a site in the "nearer" half of a genuine cadastre. Although, as we shall see, this value need not (and cannot) be determined precisely, this agreement should be <u>public</u> (Phillips 1973: 78), otherwise it may be difficult to accept that a trustworthy value can be calculated for posterior odds.

Experience determines a plausible value for the proportion of the sites which are likely to lie "near" the divisions. From the observations made of the Limburg cadastre *(figure 3.24)* we can see that we might expect a figure of about 55-60%. This can be compared to the central part of the *ager collatinus*, where, according to Chouquer et al (1987: 286-287), the individual Roman settlements whose orientation conforms to the cadastre are clearly related to the divisions at 15 *actus*.

In order to estimate what proportion of them are near the *limites*, a 150 x 150 *actus* square in the centre of the cadastre was chosen and these sites counted by hand. This gave 27 sites "near" the divisions and 18 "away from" them according to the above definition. This gives a "near" proportion of 60%.

For the time being, let us take a likelihood if 60% in the "near" half as being a reasonable figure. This gives likelihood ratios of 1.2 and 0.8 for the two events, "near" and "far" from the divisions.

We can now consider the case of Romney Marsh.

From the evidence of topography and the association of possible cadastral traces with a significant division of soil type *(presented below in section 4.2.2)* , there is a non-zero prior probability (a tentative belief by the author) that there was a Roman cadastre in Romney Marsh. This seems to have a module of 355m.

There are very few Roman finds in this area, but there are a large number of medieval church sites and courts (manorial halls). This latter class of sites was chosen for investigation, not because these points lay near the hypothetical grid lines, but because churches and halls are normally regarded as being the most important indicators of the location of medieval settlement. In order to obtain as accurate a set of data as possible, the author was aided by Eleanor Vollans, a medieval historian and specialist on the Marsh, who verified, located and if necessary added sites to a draft map prepared by the author in April 1989. It was important for the objectivity of the hypothesis test that, in deciding which sites to include, she had no precise knowledge of the location of the grid. She also defined an arbitrary boundary to the landward side of the Marsh.

There were 54 points defined, 33 churches or church sites and 21 courts. Using a cadastral grid size of 355 m, the OS grid coordinates were processed by the routine for calculating Kolmogorov-Smirnov D values, which automatically sorts the data in order of ascending distance from the grid lines. There were 37 points in the half near the grid lines and 17 in the other half.

Using the odds-likelihood ratio form of Bayes' theorem

$\mathbf{\Omega}'' = \mathbf{\Omega}' \times 1.2^{37} \times 0.8^{17} = \mathbf{\Omega}' \times 19.15.$

Proportion in "near" half	$\prod \mathbf{L_i}$
60	19.15
61	22.95
62	26.94
63	30.95
64	34.79
65	38.24
66	41.10
67	43.16
68	44.26
69	44.27
70	43.18
71	41.02
72	37.90
73	34.02
74	29.63
75	25.00

Figure 3.29 $\prod L_i$ *values.*

So, if the author's prior probability for the existence of the cadastre was, say, 0.1 (that is, it had a 1 in 10 chance of existing), we would have prior odds of 1:9 and posterior odds of 2.13:1. Thus the posterior probability of existence is $\frac{2.13}{3.13} = 0.68$. In other words, the examination of these data makes it more than likely that the cadastre exists. This result is of course dependent upon the prior probability. Had someone more sceptical set it at only a 1 in 100 chance, a recalculation gives posterior odds of 0.193:1 and a posterior probability of 0.16. Thus even someone initially sceptical should now have a rather different view[69].

Two further point may be considered.

Firstly, for these data, how sensitive is

[69] We note that the complete dogmatist, who is so convinced that such cadastres could not exist in Britain that he has a prior probability of zero, would not be swayed by any amount of information. Such a position is, however, difficult to defend as scientific.

the product of the likelihood ratios, $\prod \mathbf{L_i}$, and hence any change in the probability, to changes in the likelihood values?

Given the data observed in this case, this value may be computed for the range of likelihoods of 60% - 75% in the "near" half *(figure 3.29)*. We can see that a better estimate of the prior likelihood ratio for this sample, which is about 2:1 (in fact 69% to 31%) rather than 3:2 (60% to 40%), would have produced a $\prod \mathbf{L_i}$ more than twice as large. This would have given posterior odds of 4.92:1 and increased the probability from 0.1 to 0.83. This is not fundamentally different from the previous result. The publicly agreed likelihood can have a considerable range of values and the data will still lead to a major revision of prior odds. In this example the revision of probability is affected by the choice of likelihood but it is not very sensitive to it.

Secondly, how can we now proceed in practice?

The essence of the Bayesian approach is that we can now use any additional information to further revise our probability of 0.68. We now anticipate, from the data that we already have, that in the defined area the likelihood ratio is about 2:1. If further research reveals another church or court site we can see if it lies within $0.293 \times \frac{355}{2} = 52$m of a theoretical *limes*. If so we multiply the odds by 4/3, if not, by 2/3.

This approach has two advantages. It is very simple. In general, once we have decided on a likelihood ratio, **r**, which will probably be in the range 57-67%, we measure the distance from the site to the nearest *limes* in terms of half the grid distance. If it is less than 0.293, we multiply the odds of the cadastres existence by 2**r**. Otherwise we multiply it by 2(1-**r**).

The other advantage is that we can use data as it arises naturally, item by item, in the course of research.