Dynamical Systems Arising from Elliptic Curves

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ABSTRACT. We exhibit a family of dynamical systems arising from rational points on elliptic curves in an attempt to mimic the familiar toral automorphisms. At the non-archimedean primes, a continuous map is constructed on the local elliptic curve whose topological entropy is given by the local canonical height. Also, a precise formula for the periodic points is given. There follows a discussion of how these local results may be glued together to give a map on the adelic curve. We are able to give a map whose entropy is the global canonical height and whose periodic points are counted asymptotically by the real division polynomial (although the archimedean component of the map is artificial). Finally, we set out a precise conjecture about the existence of elliptic dynamical systems and discuss a possible connection with mathematical physics.