W. Kubiś and V. Uspenskij. *A compact group which is not Valdivia compact.* ***Proc. Amer. Math. Soc.***, 133 (2005), no. 8, pp. 2483-2487.

W. Kubiś and H. Michalewski. *Small Valdivia compact spaces.* ***Topology Appl.***, vol. 153 no. 14 (2006), pp. 2560-2573.

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An inverse system is a sequence S of mappings $p_\alpha^\beta : X_\beta \to X_\alpha, \alpha < \beta < \delta$ for some limit ordinal δ such that $\alpha < \beta < \gamma \Longrightarrow p_\alpha^\beta p_\beta^\gamma = p_\beta^\gamma$. The maps p_α^β are called the bonding mappings of S and the inverse limit of S is the subset of the product $\Pi_{\alpha<\delta} X_\alpha$ consisting of all x such that $p_\alpha^\beta(x(\beta)) = x(\alpha)$ for all $\alpha < \beta < \delta$. If each X_α is a compact space then the inverse limit is nonempty. Such systems have been studied extensively in set-theoretic topology, where most often $X_\beta \subseteq 2^\beta$ and the bonding maps are the natural projections. This concept has been around for a long time and it has been used by many people, and studied systematically by several authors. It is therefore a very pleasant surprise to have a new way of looking at this concept, allowing for the solution of a number of problems on Valdivia compacta, which is a class of compact spaces inspired by functional analysis, containing Corson compacta and dyadic spaces.

In *Small Valdivia compact spaces* it is shown that a compact space of weight $\leq \aleph_1$ is Valdivia compact iff it is the limit of an inverse sequence of metric compacta whose bonding maps are retractions. (A retraction is a continuous map which has a right inverse.) A corollary of this is that the class of Valdivia compacta of weight at most $\aleph_1$ is preserved both under retractions and under open 0-dimensional images, which answered quite a well known open question in the theory of Valdivia compacta. However, it was proved in *A compact group which is not Valdivia compact* that Valdivia compacta are generally not preserved by open continuous images — here a rather short elegant argument was based on the cohomology theory of compact connected abelian groups.

Every Valdivia compact space K has many retractions, which for instance implies that the Banach space $C(K)$ admits the so called projectional resolution of the identity (PRI, i.e. a comuting system of projections onto small subspaces). *Compact spaces generated by retractions* gives a counterexample to several questions on PRI (due to Montesinos and Kalenda) by considering a certain linear order which gives rise to a Valdivia compactum. In particular, it was shown that the property of Banach spaces having PRI is not hereditary.

A norm on a Banach space is called Kadec if the norm and the weak topology coincide on the unit sphere. It is a classical question in Banach space theory to study the class of Banach spaces that have a renorming which is Kadec. In *Kadec norms on spaces of continuous functions* the classical results about this class are greatly extended, as it is for example shown that if $C(K_1)$ has a pointwise Kadec renorming and K_2 belongs to the class of spaces obtained by closing the class of compact metrizable spaces under inverse limits of transfinite continuous sequences of retractions, then $C(K_1 \times K_2)$ has a pointwise Kadec renorming.

The new insight provided by these and other papers by Kubiś on this subject is the consideration of inverse systems in terms of category theory, on a quite abstract level. The above examples show how these abstract logical methods can then be applied to solve some very concrete problems asked outside of mathematical logic. In fact, the methods emerging in the recent close interactions of Banach space theory with mathematcal logic involve set theory, model theory and category theory, often a mixture of all. They provide an exciting new way to successfuly use foundations of mathematics

to attack problems in mathematical analysis, where the classical methods of logic, such as first order logic, fail.

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