

MTH-2G30 : Computability

1. Introduction: This unit provides an introduction to Computability: a topic in Mathematical Logic which is at the foundations of theoretical computer science. It is largely self-contained and is suitable for non-Mathematics students with an appropriate background.

2. Timetable Hours, Credits, Assessments: The course is a 10 UCU unit of 15 lectures with 3 seminars. Assessment is by coursework (20%), consisting of homework sheets, and an examination (80%).

3. Overview: The course is concerned with **computability**. It addresses issues such as which functions can be computed by a mechanical process or computer program. This is idealised in the very precise notions of an **unlimited register machine (URM)** or a **Turing machine**. Once we have shown that some familiar mathematical functions can be computed using URM-programs, we introduce the Goedel numbering of programs and show how this leads to the **undecidability** of various problems: the **non**-existence of algorithms to solve certain classes of mathematical problems.

4. Recommended Reading:

- 1) N Cutland "Computability", CUP
- 2) A G Hamilton "Logic for Mathematicians", CUP

The lectures will follow (1) quite closely with some material from (2).

5. Lecture Contents:

URMs and some examples of functions computable by them. Subroutines, recursion, minimalisation. **(8 lectures)**

Goedel numbers, universal programs and some undecidable problems. **(5 lectures)**

Other approaches: Turing machines, recursive functions. The Church-Turing thesis. **(2 lectures)**