MTH-ME75 : Financial Mathematics with Advanced Topics

1. Introduction: This unit is primarily concerned with the valuation of certain financial instruments known as derivatives. It has great application to finance and banking. The unit assumes some knowledge of differential equations and MTH-2C23 Differential Equations II, or equivalent, is a prerequisite. Some of the mathematical modelling is probabilistic but no prior knowledge of probability or statistics is assumed. Neither is any previous background in finance necessary.

2. Timetable Hours, Credits, Assessments: This unit is of 20 UCU and is taught in the Autumn Semester by 33 lectures in two parallel streams.. Assessment is by coursework (20%) and an examination (80%).

3. Overview: The Mathematical Modelling of Finance is a relatively new area of application of mathematics yet it is expanding rapidly and has great importance for world financial markets. The unit is concerned with the valuation of financial instruments known as derivatives. Introduction to options, futures and the no-arbitrage principle. Mathematical models for various types of options are discussed. We consider also Brownian motion, stochastic processes, stochastic calculus and Ito's lemma. The Black-Scholes partial differential equation is derived and its connection with diffusion brought out. It is applied and solved in various circumstances.

4. Recommended Reading:

Any of the following should prove useful:

1) Paul Wilmott introduces quantitative finance
P. Wilmott. Wiley, 2001.

2) The mathematics of financial derivatives
P. Wilmott, S. Howison, J. Dewynne. CUP, 1995.

3) An elementary introduction to mathematical finance, 2nd ed.
S.M. Ross. CUP, 2003.

4) A course in financial calculus
A. Etheridge. CUP, 2002.

5) The concepts and practice of mathematical finance
M. Joshi. CUP, 2003.

5. Lecture Contents:

Introduction to options, futures and the no-arbitrage principle -- using this to calculate fair delivery prices for futures. **(5 lectures)**

Models for the movement of stock prices, efficient markets, Brownian motion and geometric Brownian motion. Stochastic and deterministic processes. **(3 lectures)**

Basics of stochastic calculus and Ito's lemma. **(3 lectures)**

The Black-Scholes analysis. Derivation of the Black-Scholes partial differential equation and the assumptions behind it. Formulating the mathematical problem, determining boundary conditions for option pricing problems. **(6 lectures)**

Solving the Black-Scholes equation. Connection with heat conduction equation, solution of the heat

conduction equation - similarity solutions and the Dirac delta function. Derivation of the price of European options. **(7 lectures)**

Extension to options on assets paying dividends and American options, free boundary problems. **(6 lectures)**

Examples and revision. **(3 lectures)**

6. Advanced topic

The advanced topic will be chosen from: Exotic options; Interest rate derivative products.