

## MTH-2B72 : Mathematics for Geophysical Science II

**1. Introduction:** This is a sequel to **Mathematics for Geophysical Science I** and provides an introduction to Mathematical Modelling using the mathematical techniques developed in **Mathematics for Geophysical Science I**. It covers the same material as the course **Mathematics II** for Environmental Science students. The unit provides mathematical solutions that should be useful in ENV courses on Oceanography, Meteorology, Geophysics and Geophysical Modelling. It may be followed by advanced level 3 applied mathematics courses. A discussion of Fourier series arises from the solution of the partial differential equations of mathematical physics.

**2. Timetable Hours, Credits, Assessments:** This unit is of 20 UCU and is taught in the Spring Semester by 39 lectures in two parallel streams. It is supported by 5 example classes. Assessment is by set regular homework (20%) and an examination (80%).

**3. Overview:** This unit has four topics which all aim to develop skills in mathematical modelling. The topics are Fluid Dynamics, Fourier Series, Partial Differential Equations and Solid Mechanics. This course shows how mathematics can be applied to the environmental and geophysical sciences.

Fourier series are useful for analysing time series and solving partial differential equations. The concept of representing a function by a sum of appropriate trigonometric functions is developed, leading to examples of whole range and half range Fourier series. The range of validity, and the behaviour of Fourier series are explored in a laboratory class, using the computer algebra package MAPLE.

The solution of partial differential equations is one of the cornerstones of geophysical modelling. Analytic techniques based on the separation of variables, allied to Fourier series, are used to solve prototype problems in wave motion, unsteady heat conduction, and steady heat flow.

Fluid dynamics is the study of fluid motion and is a foundation of meteorology, oceanography and many branches of engineering.

**4. Recommended Reading:** The following books are recommend for different parts of the course.

E Kreyszig "Advanced Engineering Mathematics" (Wiley)  
(This is available in the library)

A R Paterson "A first course in fluid dynamics" (CUP 1983)

R O Davis & A P S Selvadurai "Elasticity and Geomechanics", (CUP 1996)

### 5. Lecture Contents:

**Fluid Dynamics:** Introduction. Continuum hypothesis. Pressure. Material derivative. Buoyancy. Two-dimensional flows. Conservation of mass. Bernoulli's equation. Flow over weir. Hydraulic jumps. Lift. **(5 lectures)**

Three-dimensional flows. Euler's equations. Boundary conditions. Stream function. Vorticity. Vortex tubes. Circulation. Irrotational flow. Kelvin's circulation theorem. **(4 lectures)**

Velocity potential. Uniform stream. Line source. Dipole. Line vortex. Combined examples. Lift. **(3 lectures)**

Water waves. Linear surface gravity waves. Phase and Group speed. Deep water and shallow water approximation. Particle paths. **(4 lectures)**

**Fourier Series :** Taylor's series, convergence of series, Fourier's theorem for piece-wise smooth functions on  $(-\pi, \pi)$ , comments, examples, periodic extension outside  $(-\pi, \pi)$ , summing series, extension to the interval  $(-\ell, \ell)$ . Even and odd functions, half-range Fourier sine and cosine series on the interval  $(0, \ell)$ , examples. **(5 lectures)**

**Second-order partial differential equations :** Basic concepts and definitions, wave equation, heat conduction equation, Laplace's equation. The method of separation of variables. Application of this method for solving initial/boundary value problems associated with the aforementioned P.D.E.'s. D'Alembert's solution for the wave equation. Application for solving initial value problems for the wave equation on infinite and semi-infinite domains. **(6 lectures)**

**Solid Mechanics :** Theory of stress. Traction on a surface, resultant force and torque, in terms of components of stress. Normal and shear stresses. Principle stresses and axes of stress. Equations of equilibrium and symmetry of stress matrix. Displacement, strain and small deformations. **(6 lectures)**

Elasticity. Linear stress-strain relations. Navier equations of equilibrium and motion. Elementary examples. Propagation of plane P and S waves. **(4 lectures)**