# “An introduction to number theory” by Everest and Ward

errata file (items marked * are corrected in June 2006 reprint)

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p. 12, equation (1.4): In fact the proof shows the slightly stronger inequality $\sum_{p\le N}\frac{1}{p} > \log\log N - 1$.

*p. 19, just after equation (1.12): “We now prove the inequality (1.11) by induction”

p. 20, line 10: We need to have $n \geq 5$ here because a few lines later the inequality $\frac{4}{9}n^2 > 2n$ is used.

p. 20, line 11: The inequality should be $p \leq n$ rather than $p < n$.

p. 21, equation (1.17): The term $\frac{4}{3}\log 2$ should be $\frac{4}{3}n\log 2$.

p. 26, Example 1.14: $23n + 1$ is also prime for $n = 6$, which should therefore be added to the displayed list of numbers.

*p. 48, line -4: Displayed equation should read

$$(-1)(-2)\cdots(-2n)(2n)(2n-1)\cdots 3{\cdot}2{\cdot}1 = (2n!)^2(-1)^{2n} \equiv -1 \pmod{p}.$$

p. 51, before line -10: The possibility that $w = x = y = z = 0$ must be excluded in this argument. Note that the $a$ and $b$ found in Lemma 2.9 may be chosen with $|a|, |b| < p/2$. Then $0 < a^2+b^2+1 < 2(p/2)^2+1 < p^2$, so $0 < m < p$. If $w, x, y, z$ are all zero then

$$m^2|a^2 + b^2 + c^2 + d^2 = mp,$$

giving a contradiction if $m > 1$.

p. 73, Exercise 3.16: The third Legendre symbol should be $\left(\frac{1003}{113}\right)$ (the point is 111 is not prime).

p. 76, line -12: This should read $1 \leq |e| < 1 + 2\sqrt{d}$.

p. 79, proof of Theorem 3.22: This is garbled and should read as follows.

Proof. Assume that $(\alpha, \gamma)$ is a primitive integral solution to Equation (3.18). By Theorem 1.23, there are integers $\beta, \delta$ with

$$\alpha\delta - \beta\gamma = 1.$$

Let

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}.$$

Notice that $\det \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} = 1$, so this matrix is an invertible transformation on $\mathbb{Z}^2$. Express Equation (3.18) in the variables $X$ and $Y$ to obtain

$$a(\alpha X + \beta Y)^2 + b(\alpha X + \beta Y)(\gamma X + \delta Y) + c(\gamma X + \delta Y)^2$$
$$= nX^2 + (2r + \rho)XY + sY^2 = n,$$

where $s = a\beta^2 + b\beta\delta + c\delta^2$ and

$$2r + \rho = 2a\alpha\beta + 2c\gamma\delta + b(\alpha\delta + \beta\gamma).$$

Notice that $s$ is an integer and $\alpha\delta+\beta\gamma = 1+2\beta\gamma$ is odd, so $b(\alpha\delta+\beta\gamma)-\rho$ is even, and

$$r = a\alpha\beta + c\gamma\delta + \frac{1}{2}\left(b(\alpha\delta + \beta\gamma) - \rho\right)$$

is an integer also. The equation

$$nX^2 + (2r + \rho)XY + sY^2 = n$$

has the solution $X = 1, Y = 0$ corresponding to

$$\begin{bmatrix} \alpha \\ \gamma \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

The discriminant is unchanged, so

$$(2r + \rho)^2 - 4sn = \Delta,$$

so

$$r^2 + \rho r - \left(\frac{\Delta - \rho}{4}\right) = sn,$$

showing that $r$ is an integer solution of the congruence (3.19).

The end of the proof should have the displayed equation:

$$nx^2 + (2r + \rho)xy + sy^2 = n.$$

p. 87, line 11,12: This should read $\alpha = c + g\delta$ and $\beta = h$.

p. 96, Exercise 5.2: $(1, 0)$ is not on the curve – this should be $(0, 0)$.

p. 101, line 12: This should read "Let $T^2$ denote the denominator of $v$;"

p. 115, line 5: "By the strong form of Siegel's Theorem (Theorem 7.13)..."

p. 171, Example 8.19: Summations should start at $n = 0$ throughout.

p. 178, Exercise 8.23: The equation in (c) should read

$$B(s) = 1 + \frac{1}{2^s} - 2 \cdot \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} - 2 \cdot \frac{1}{6^s} + \cdots .$$

p. 191, line -9: range of integration should be from $y$ to $y + 1$.

p. 192, equation (9.8): This should be

$$-c_k + c_{-k} = 2i \int_0^1 g(x) \sin(2\pi kx) dx \to 0 \text{ as } k \to \infty.$$

Equation (9.9): $\sin(2K + 1)x$ should be $\sin((2K + 1)\pi x)$.

p. 193, line -10: The right-hand side should be just $G_N(x)$.

p. 195, line -4: This should read "uniformly and absolutely".

p. 204, Exercise 9.11: The reference should be to Exercise 8.24 not equation (8.24).

*p.206, line 6: "The disproof of Mertens' conjecture"

p.209, equation (10.6): The right-hand side should be $\sum_{n=0}^{\infty}(-t^2)^n$.

p. 221, line -3: Lower limit of summation should be $\nu = 0$.

p. 249, line -1: A factor of $C_1(k)$ should appear at the end of the expression.