"JUST THE MATHS"

SLIDES NUMBER

1.11

ALGEBRA 11
Inequalities 2

by

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UNIT 1.11 - ALGEBRA 11
INEQUALITIES 2.

1.11.1 RECAP ON MODULUS, ABSOLUTE VALUE OR NUMERICAL VALUE

$$| x | = x \quad \text{if} \quad x \geq 0;$$

$$| x | = -x \quad \text{if} \quad x \leq 0.$$

Notes:
(i) Alternatively $| x | = +\sqrt{x^2}$;

(ii) It can be shown that, $| a + b | \leq | a | + | b |$; the **"Triangle Inequality"**.

1.11.2 INTERVAL INEQUALITIES

(a) Using the Modulus notation

We investigate the inequality

$$| x - a | < k,$$

where a is any number and k is a positive number.

Case 1. $x - a > 0$**.**

$$x - a < k, \quad \text{that is,} \quad x < a + k.$$

Case 2. $x - a < 0$**.**

$$-(x-a) < k, \quad \text{that is,} \quad a - x < k \quad \text{or} \quad x > a - k.$$

Conclusion

$$| \, x - a \, | < k \quad \text{means} \quad a - k < x < a + k.$$

Similarly for $| \, x - a \, | \leq k$.

EXAMPLE

Obtain the closed interval represented by the statement

$$| \, x + 3 \, | \leq 10$$

Solution

Using $a = -3$ and $k = 10$, we have

$$-3 - 10 \leq x \leq -3 + 10.$$

That is,

$$-13 \leq x \leq 7.$$

(b) Using Factorised Polynomials

EXAMPLE

Find the range of values of x for which

$$(x+3)(x-1)(x-2) > 0$$

Solution

Critical values are $x = -3, 1, 2$ dividing the line into
$x < -3, \quad -3 < x < 1, \quad 1 < x < 2, \quad x > 2;$

$x < -3$ gives (neg)(neg)(neg) and therefore < 0;

$-3 < x < 1$ gives (pos)(neg)(neg) and therefore > 0;

$1 < x < 2$ gives (pos)(pos)(neg) and therefore < 0;

$x > 2$ gives (pos)(pos)(pos) and therefore > 0.

$$\text{Ans}: \quad -3 < x < 1 \quad \text{and} \quad x > 2.$$

Note:
Alternatively, sketch the graph of the polynomial.