Solving a differential equation

In a Year 13 class, students are asked to find the general solution of the differential equation \( \frac{dy}{dx} = y^2 \). One student proposes the following.

**Student:** This is a separable equation, so, I need to separate the variables:

\[
\frac{1}{y^2} \, dy = dx
\]

Now, I integrate both sides:

\[
\int y^{-2} \, dy = \int dx
\]

Which gives:

\[
y^{-1} = x
\]

\[
y = -\frac{1}{x}
\]

The problem asks for the general solution, I need to add the constant. So, the general solution should be:

\[
y = -\frac{1}{x} + C
\]

Let me check what the answer says at the back of the book … [checks]

Mmmm, it says:

\[
y = -\frac{1}{x+C}
\]

No, this cannot be correct, it must be a typo and it won’t be the first time!

**Questions:**

a. What is a solution to this mathematical problem?

b. What are the aims of using this problem in class?

c. What are the issues emerging from what the student does and says?

d. How would you respond to this student and to the whole class?