Definite integral as area under a curve

In a Year 12 class in the UK, students are given the following problem:

“Find the area bounded by the curve \( f(x) = x^3 \) and the x-axis between -1 and 1”

Here is what one student says:

**Student:** That’s easy. You just need to integrate the definite integral \( \int_{-1}^{1} (x^3 - x) \, dx \).

So,

\[
\int_{-1}^{1} (x^3 - x) \, dx = \left[ \frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^{1} \\
= \left[ \frac{1^4}{4} - \frac{1^2}{2} \right] - \left[ \frac{(-1)^4}{4} - \frac{(-1)^2}{2} \right] \\
= \left[ \frac{1}{4} - \frac{1}{2} \right] - \left[ \frac{1}{4} - \frac{1}{2} \right] \\
= 0
\]

So, the area is 0! Erm….what?!

You are the teacher and you just heard the student.

**Questions:**

a. What is a solution to this problem?
b. What are the aims of using this problem in class?
c. What do you think are the issues emerging from the student’s response?
d. How would you respond to this student and to the whole class?

This is a Task developed by the MathTASK 2016-17 team. Let us know whether it is useful and how we can improve it at @mathtask or email Irene Biza at i.biza@uea.ac.uk. For more tasks, visit MathTASK.