Undernutrition and vulnerability to food insecurity:
a not so (log)normal distribution for caloric intake

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Abstract. *Accurate and precise measurement of hunger is crucial for policy makers to formulate effective strategies addressing food insecurity. This paper reviews the widely used FAO methodology for the estimation of the proportion of population below the minimum level of dietary energy consumption and Christiaensen and Boisvert's quantitative approach to measure vulnerability to food insecurity. Both of these methodologies use a parametric framework that requires specific assumption on the distribution of caloric consumption. Though this assumption can influence significantly the estimation of the number of hungry and/or vulnerable people, it has not been properly investigated in the literature. This paper addresses this methodological shortcoming and aims to identify a suitable parametric functional form for the distribution of caloric intake. Both gamma- and beta-type distributions are tested with numerical and graphical tools. By using household survey data on food consumption from Malawi, this paper finds that the easy and widely used lognormal distribution produces bias estimates, while the beta-type distributions, in particular Singh-Maddala and generalized beta of second kind, describe more accurately the distribution of caloric intake in Malawi allowing for more precise estimates of undernutrition.*

1. Introduction

At the World Food Summit in 1996, the international community set the goal of halving the number of people who suffer from hunger by 2015. Then, in 2000, they adopted 8 Millennium Development Goals (MDG) one of which is to halve the *proportion* of the world's hungry between 1990 and 2015 (MDG1, target 3). The Food and Agricultural Organization of

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the United Nations (FAO) estimated that 868 million people were chronically undernourished in 2010-2012 (FAO, 2012); however, with the proper set of policies and strategies, FAO believes that MDG1, target 3 can be met. In order to monitor progress, the proportion of population below the minimum level of dietary energy consumption (DEC) was chosen as an indicator. Accurate and precise measurement of this indicator is not only important for monitoring and formulating strategies to reduce world hunger, it is also has scientific value in understanding the phenomenon of world hunger and food security issues. This papers deals with methodological shortcomings that can undermine the validity of this and similar measurements.

Currently, the FAO global estimate of chronic hunger is not only used to monitor progress towards the achievement of MDG1, it is also the most cited source and widely used global hunger statistic by international institutions and donors to set policy and allocate foreign aid. Despite its wide use, FAO's methodology has come under scrutiny by international organizations and the academic community. Furthermore, as a static measure, it neglects the reality that at any given time there is risk and uncertainty and people move easily in and out of undernutrition. It follows that food security policy should be based not only on present conditions, but also utilize a dynamic approach. If policy makers can address factors that suddenly expose people to a lack of sufficient and nutritious food, they can design policy to build resilience. Vulnerability analysis tries to fill this gap. Although it is a relatively new field of research in the context of food security, both qualitative and quantitative approaches have already been applied. However, to date, there is a lack of scientific consensus on a specific methodology for measuring vulnerability to food insecurity. This paper will consider only quantitative approaches that analyze vulnerability from an economic perspective[2]. The primary quantitative approach to vulnerability analysis considered in this paper is the probabilistic approach prosed by Christiaensen and Boisert (2000) who define food vulnerability in terms of the current probability of caloric shortfall in the near future.

The aforementioned probabilistic approach to measure vulnerability and FAO's methodology share a major point of criticism: both utilize a parametric approach where a specific

[2] It is accepted that qualitative approaches such as the sustainable livelihoods approach are of unquestionable value; however, the quantitative analysis is more persuasive, and thus will be more effective in putting vulnerability analysis higher in the political agenda (Dercon, 2001).

statistical distribution is *assumed* for caloric consumption. This is a critical point which, to date, has received insufficient attention in the literature. In fact, the assumption of specific statistical distributions for food consumption may significantly influence the estimation of the number of hungry and the identification of vulnerable people. Instead of testing this assumption, easy assumptions have been preferred. In this context, this paper's objective is to expose biases in the assumption on the distribution of caloric intake, and identify a suitable parametric functional form.

The data used were obtained from a household survey conducted in Malawi during 2004-2005 by the World Bank. There are two fundamental reasons for the choice of Malawi. First, this paper aims to find a parametric distribution for caloric intake suitable for low-income and food-deficient countries. Malawi is a low-income country with 4 million people (23% of the population) suffering hunger (FAO, 2012). In 2002, a famine left 3.5 million people food insecure as a result of flooding, mismanagement of the country's grain reserve (almost depleted in 2001), and a chaotic response in terms of maize imports and food aid (Stevens, Devreux, & Kennan, 2002). Finally, with almost 2 million people at risk of food insecurity (Malawi Vulnerability Assesment Committee, 2012), Malawi represents a perfect candidate for this study. In fact the distribution of dietary intake in countries where many people do not meet their energy requirements is likely to be different (more skewed) than that of a population of adequately fed individuals. The second reason for the choice of Malawi was the availability of reliable data. The Living Standard Measurement Survey conducted in Malawi was one of the few surveys conducted in food insecure countries containing a detailed module on food consumption. Furthermore, the quality of the data was good and the National Statistical Office was helpful in providing conversion factors for unconventional units of measure and food composition table.

This paper is organized as follows: Section 2 provides the rationale of this study first defining the concepts of food security and undernutrition, then critically reviewing the current methodologies used to measure them. Section 3 provides a description of the data used in this study, and section 4 assesses the goodness of fit of several statistical distributions through graphical tools, summary measures and statistical tests. Finally in Section 5, this paper will draw specific conclusions and identify areas for further research.

2. Rationale

FAO's methodology and the vulnerability approach of Christiaensen and Boisvert address the problem of food insecurity by focusing on undernutrition that represents only one aspect of food insecurity.

In the past, food security referred only to the condition of having a sufficient supply of food. Sen (1981) introduced the idea that hunger and famine are not only caused by inadequate food supply, but also by a household's failure of food entitlement. People starve when their entitlement set does not provide them with adequate food. Thus, a famine and hunger can occur even if food supplies are adequate and markets are functioning well if people cannot access food. Accordingly, the most common definition of food security today includes the concept of food access as proposed at the World Food Summit in 1996: "food security exists when all people, at all times, have physical and economic access to sufficient, safe and nutritious food to meet their dietary needs and food preferences for an active and healthy life".[3] This definition captures the multidimensionality of food security. Undernutrition itself is a phenomenon resulting from many aspects (food intake, body size, metabolic rate, health, physical activity, etc.). Thus, a comprehensive measurement is not an easy task. Until now, two main approaches have been used: one considers the causes (food consumption) and the other considers the consequences (anthropometric measures) of undernutrition.[4] Aware that "there is no perfect way, and never will be, of defining and measuring undernutrition" (Svedberg, 2000), the challenge is to improve and/or introduce better indicators.

FAO as well as Christiaensen and Boisvert consider energy deficiency (dietary energy intake below certain energy requirement levels) disregarding the intake of protein or other specified nutrients. In fact, the revealed preference approach of Jensen and Miller (2010) suggests that individuals will diversify their diet and eat micronutrient-rich foods (such as meats, fish, eggs, dairy products) only after their basic calorie needs are satisfied. In fact, an individual needs a sufficient amount of calories to survive and if one balances the micronutrients intake without providing adequate calories she/he will not be able to

[3] For a broader review of food security concept see Maxwell S. (1996)

[4] Haddad et al. (1991; 1994), Headey et all (2012) and Carletto et al. (2012) provide an exhaustive review of indicators that have been used.

maintain her/his body-weight and working performance. On the other hand, if one increases DEC through normal staple foods, then a possible increase in micronutrients will occur[5]. Accordingly, often the increase of DEC may not be sufficient for nutritional improvement, but it is a necessary condition.

Another caveat of using data on micronutrient intake is limited availability. Household survey data usually includes data on the quantity of food consumed, but excludes information on the modality of consumption (raw or cooked food). Accordingly, it would be problematic to estimate the loss of micronutrients due to cooking processes. It is also true that food processing can increase calorie content, but some micronutrients such as vitamins are depleted also because of improper storage.

FAO's methodology

FAO derives the prevalence of undernourishment (PoU) from a probabilistic framework. It defines the PoU as the proportion of the population with food consumption (kcal/person/day) below the minimum energy requirement:

$$\boldsymbol{P(U) = P(x < r_L) = \int_{x<r_L} f(x)\,dx = F_x(r_L)} \quad , \qquad (1)$$

where P(U) is the proportion of undernourished, x is the dietary energy supply (DES) and r_L is the minimum energy requirement (MDER) (FAO, 2008).

DES is derived from the food and balance sheets using data on production, trade, stock changes, household waste and types of utilization (food, non-food use). Though DES measures simply food availability, it is used as a proxy for caloric intake. MDER is the mean energy requirement of the individuals of a country and is computed using a reference body-weight for attained-height and the recommended energy requirement per kilogram of body weight for each sex and age population group, as recommended by the Joint FAO/WHO/UNU Expert Consultation on human energy requirements.

The FAO's methodology is receiving criticism from the scientific community[6]. The primary criticism focuses on FAO's use of a univariate distribution framework rather than the bivariate distribution framework[7] originally proposed by Sukhatame (1961). Other criticisms

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[5] Ecket et al. (2010) found a highly significant and positive relationship between caloric intakes and some micronutrient intake (iron, zinc, vitamin A) using household survey data on Rwanda, Uganda and Tanzania.

[6] See a comprehensive overview of the debate by Naiken (2007).

[7] It involves the estimation of the joint density function of DEC and DER, both random variables.

are aimed at methodological assumptions on the coefficient of variation[8]. There have also been concerns about the data used (see for example (Svedberg, 2000)). Finally, other authors[9] have criticized the use of an analytical probability distribution framework and suggested the use of a “non-parametric approach”. They argue that the availability of data on food consumption from household surveys alleviates the need for assumptions on an analytic probability distribution common to all countries because the data itself determines the distribution. In other words, the non-parametric approach would be based on the direct comparison of the energy consumption of each sampled household with the energy requirements of the household itself.

A non-parametric approach may be preferred because it would not require assumptions underlying the probability distribution. Also, it would allow the disaggregation of food security information by geographic area or other socioeconomic variables. This additional information would be extremely useful for policy makers (de Haen, Klasen, & Qaim, 2011). However, household surveys are not conducted on regular basis largely because of time-constraint and implementation cost. For this reason, utilizing a non-parametric methodology at a global scale is impossible. Accordingly, the parametric approach remains a fundamental tool for measurement of global hunger and food security. The main problem of the parametric approach is that distribution of caloric intake (*f(x)* in equation (1)) is unknown; therefore researchers and scientist must rely on assumptions.

In the fourth World Food Survey (1977), FAO chose the beta distribution to estimate the prevalence of undernourishment in developing regions (excluding the Asian centrally-planned economies[10]) for 1969-71 and 1972-74. The distribution was chosen because it allowed the FAO to fix the lower and upper limits of the caloric intake distribution, thus simulating the physiological limits of intake. Later on, FAO compared the performance of the normal, lognormal and beta distributions using household survey data[11] and the lognormal outperformed both normal and beta distribution[12]. As a result, the FAO used a lognormal

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[8] For Svedberg (2000), FAO overestimates the CV, while Smith (1998) argues that the range fixed for the CV may be smaller than the actual range. Furthermore, Smith criticizes also FAO’s assumption of a constant CV.

[9] Smith L. C. (1998); De Haen, Klasen and Qaim (2011).

[10] China, Cambodia, North Corea, Mongolia and Vietnam

[11] Surveys from Brazil, Egypt, Indonesia, Republic of Corea, Sudan Thailand and Tunisia.

[12] Goodness of fit was measured by $G = \sum D^2$, where D is the percentage difference between the observed and the theoretical frequencies. On average, the lognormal G was about 20% of the beta G and 17% of the G for normal distribution.

distribution function in its DEC analysis for the Fifth World Food Survey (FAO, 1987). Furthermore, Naiken (FAO, 2003) advocates that the beta distribution would not be suitable to describe caloric intake when one is dealing with household survey data because surveys usually refer to food acquired (not consumed) by the household; on the other hand, the long upper tail of a lognormal distribution would reflect better the presence of wealthier households who are more likely to produce food wastages or use food to feed pets. Naiken's justification for the use of a lognormal distribution appears not to have any scientific foundation given that an opportune shift of the upper limit of the beta distribution would have been enough to "*reflect the fact that wastages, food fed to pets, etc. are likely to be confined to the upper tail"*. Furthermore, FAO does not use the actual food consumption, but approximates x in equation (1) by DES. In 2012, the FAO's Statistical Division revised again its methodology claiming that the distribution of food consumption may have become less skewed than implied by the lognormal model. FAO infers a zero or even a negative skewness, but it does not provide any empirical evidence. Rather, FAO justifies its claim from the fact that the mean of the distribution has increased and assuming that the level of food consumption below the mean would have increased proportionally more than do levels of consumption above the mean. Accordingly, the skew-normal distribution introduced by Azzalini (1985) has been adopted (FAO, 2012). However, it is characterized by only one shape parameter reflecting the skewness of the sample and it allows for negative values, which are incompatible with the phenomenon studied. Even if the assumption that people consuming less than the average person will increase their consumption more than people with an already high caloric intake is likely to be true[13], hinting at a zero or negative skewness seems excessive.

Christiaensen and Boivert's approach

Christiaensen and Boisvert (2000) build on a previous method of vulnerability to poverty developed by Chaudhuri (2000). In their analysis of household food vulnerability in northern Mali, they define food vulnerability as the probability now of being undernourished in the future. They obtain the vulnerability measure as:

[13] The underlying assumption of a pro-poor growth (the increase in calorie availability benefits the poor) is supported also by Blaydes and Kayser (2011)

$$V_{t,\alpha} = F(z)\int_{a}^{z}(z-x_{t+1})^{\alpha}\frac{f(x_{t+1})}{F(z)}dx_{t+1}$$

with $x \in [a,b]$ and $\alpha \geq 0$.[14]

Thus, vulnerability is given by the probability of consuming less than a threshold z times the conditional expected gap. Four components are needed for the analysis: (1) household food intake x, (2) ex-ante probability distribution of future caloric consumption *f(x)*, (3) minimum energy requirement *z*, and (4) vulnerability line.

There is a wide literature and some scientific consensus on the minimum energy requirement for having a healthy life, as well as on measurement of food intake and estimation of future consumption. Furthermore, different vulnerability lines can be easily applied and compared for sensitivity analysis. However, the most accurate ex-ante probability distribution of future caloric consumption is far from settled. The identification of the ex-ante probability distribution requires an assumption on the parametric functional form of food consumption distribution. Christiaensen and Boisvert *assume* lognormality[15], thus the estimation of ex-ante mean and variance of caloric consumption is sufficient to define the ex-ante probability distribution of future consumption in their study.

Distributions for caloric intake

The choice of a suitable parametric functional form for the distribution of food consumption is crucial in FAO's estimates of hunger and in approaches for the measurement of vulnerability to food insecurity, which draws from Chaudhuri's methodology (Chaudhuri, Jalan, & Suryahadi, 2002). Nevertheless, the choice of a suitable parametric function describing the distribution of food consumption has been disregarded in the literature. Scientists have preferred easy assumptions and neglected scientific inquiry on the subject.

As previously mentioned, FAO assumed lognormal distribution along with other skewed distributions (beta and skew-normal) in their measurement of undernourishment. The aforementioned vulnerability study assumed lognormality. In a broader study on secular

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[14] "If a=0, vulnerability is measured as the probability of shortfall (Vt,0=F(z)); the depth of shortfall is not reflected. If a=1, vulnerability (Vt,1) is measured as the product of the probability of shortfall and the conditional expected gap. Vt,1 accounts for the average size of shortfall, and given equal probabilities of shortfall (F(z)), people with higher conditional expected shortfall will be taken to be more vulnerable. By setting a>1, we can also reflect the spread of the distribution of the shortfalls such that those with a higher probability of large shortfalls are more vulnerable." (Christiaensen & Boisvert, 2000)

[15] The assumption is supported by both Kolmogorov-Smirnov and the Bera-Jarque test for normality.

trends in nutrition and mortality, Fogel (1997) assumed a lognormal distribution for caloric intake recalling that “studies covering a wide range of countries indicate that distributions of calories are well described by the lognormal distribution”[16]. Following Fogel’s study, Vecchi and Coppola (2006) investigated how the incidence of undernutrition changed in Italy between 1861 and 1911 by assuming that per capita calorie consumption across the population followed a lognormal distribution. Bekaert (1991) used the beta distribution to build the distribution of calories per adult equivalent in Belgium in 1812 and 1846. More recently, Vecchi (2011) assumed a beta distribution to describe calories available per capita for the Italian population between 1861 and 2011. However, to the knowledge of the author, there is a lack of studies testing alternative statistical distributions for caloric intake. In this context, this paper aims to contribute to this niche of study by testing and comparing statistical distributions in order to find a suitable parametric functional form for the distribution of caloric intake.

The idea of testing distributions for measuring caloric intake comes from the wide literature on income distribution. The literature shows that many parametric distributions outperform the lognormal distribution with regard to income. Kleiber and Kotz (2003) provide a comprehensive review. Shortly, among all the distributional forms proposed, the four-parameter generalized beta of the second kind is today widely acknowledged to provide an excellent goodness of fit for income distribution[17]. Furthermore, the flexibility of beta-type size distributions has led to many different applications ranging from income and wealth distribution to fire losses faced by universities, as well as healthcare costs in the actuarial literature (Kleiber & Kotz, 2003).

In this study, an initial assessment looked at the possible use of both beta- and gamma-type distributions; however, the comparison of suspend rootograms (Tukey, 1977) showed clearly the bad fit of the gamma-type distributions and a satisfactory fit for the beta-type ones. The generalized beta of first kind (GB1) and beta of first kind were excluded because they are supported on a bounded domain and there is no scientific evidence of a maximum level of calories a person can consume. Furthermore, data do not show any evidence of a possible mixture distribution. For these reasons, in order to find the best fit for the distribution of per adult equivalent daily caloric intake, the possible use of the generalized beta of the second

[16] Fogel does not mention which studies he is referring to.
[17] See also Bordley *et al.* (1996) and Jenkins (2009).

kind and its nested distributions is investigated and its performance compared to the lognormal distribution. In particular, the distributions tested are two-parameter (lognormal and Fisk), three-parameter (beta of second kind - B2 -, Singh-Maddala - SM - and Dagum[18]) and four-parameter distribution (generalized beta of second kind - GB2).

The GB2 is described by the following density function:

$$\boldsymbol{f(x)} = \frac{\boldsymbol{a x^{ap-1}}}{\boldsymbol{b^{ap} B(p,q)[1+(x/b)^{a}]^{p+q}}}, \qquad \boldsymbol{x > 0} \quad (2)$$

where B(p,q) is the beta function, all four parameters are positive, b is a scale parameter, and a, p, q are shape parameters controlling the tail behavior of the model. It has been shown to include as special or limiting case all the distributions mentioned above as pictured in Figure 1.

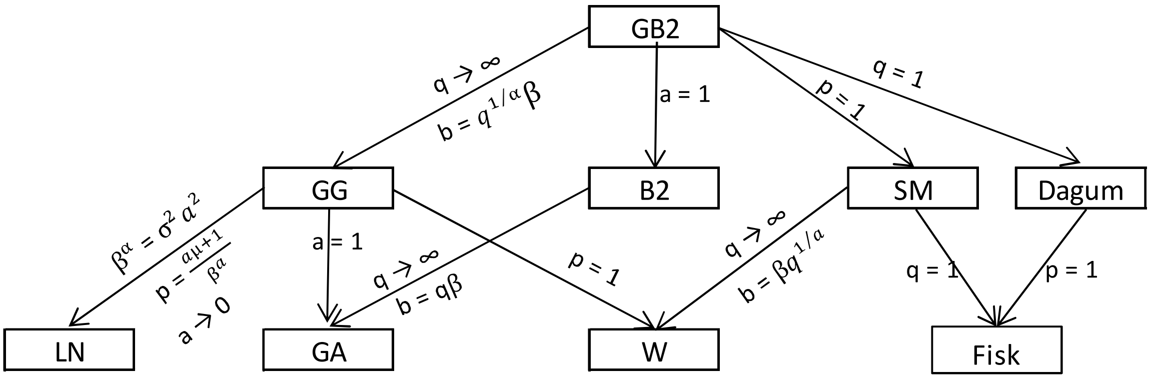

Figure 1. Beta-type distribution tree. Source: McDonald (1984), Mcdonald and Xu (1995)

The presence of three shape parameters describing the GB2 explains the flexibility of this distribution function and the higher fit that it should offer compared to less flexible distributions such as the lognormal (v is the only shape parameter): "the higher the distribution is on a branch in Figure 1, the better it will perform as measured by the same criterion" (McDonald, 1984). Thus, the GB2 should provide at least as good fit as any nested distribution. However, a special or limiting case might equal the GB2.

3. Data description

In order to find a suitable parametric form for caloric consumption, this study uses household data from Malawi. Based on its low GNI ($870 in 2011 - values in PPP), Malawi is

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[18] The Singh-Maddala and the Dagum distributions are also known respectively as Burr type 12 and Burr type 3.

classified as a low-income country (WDI, 2012). In Malawi, the number of undernourished people has declined by 17% during the last decade, but it is still very high (23% of population) (FAO, 2012). Furthermore, 57% of households consider their own amount of food consumption to be inadequate (Malawi NSO, 2005) and the Vulnerability Assessment Committee (MVAC) indicated a sharp deterioration in food security conditions in 2012, compared to the previous year. This deterioration was the result of a drought in early 2012, higher maize prices, and currency devaluation (which worsens food access of affected households).

The data analyzed are from the Second Integrated Household Survey (IHS2), which is part of the Living Standard Measurement Surveys. It was designed by the National Statistics Office (NSO) of Malawi with technical assistance from the World Bank and was conducted from March 2004 through March 2005 by NSO. The sample was drawn using a two-stage stratified sampling procedure, and covered 11,280 households. It is nationally representative and stratifies the country into rural and urban strata[19]. The IHS2 includes a detailed module on food consumption, collecting information on 114 food items[20] consumed by sampled households in the last 7 days[21]. To estimate the per adult equivalent caloric intake, the Malawi NSO provided both a conversion factor table to convert food consumption data recorded with 19 different units of measure into grams, and a food composition table with the edible portions of each food item and their average energy content.

At the commodity level, the contamination of data from outliers was limited at 5% of total observations, and only for 12 households (0.11%) the computed DEC was detected as outlier. In this context, it is important to underline that this study will look at the head-count of undernutrition and it has been shown that additive separable indices are robust in the presence of 5% contaminated data (Cowell & Victoria-Feser, 1996).

[19] 9,120 are rural households, and 2,160 are urban households. Each of the twenty-six districts is considered as a separate sub-stratum of the main rural stratum. Thus, the total number of strata in the survey is thirty: twenty-six districts and four urban centers.

[20] In addition to the 114 food items, an invoice of “other” products for each of the 10 food groups was listed. Furthermore, the survey lists a group on “Cooked foods from vendor”. It is of particular interest, though it may not cover all food eaten outside the household, since one of the main shortcomings of studies on food security is the missing data on food consumption outside of the home, which has been increasing over time also in developing countries (Kennedy, Nantel, & Shetty, 2004).

[21] “Over the past one week (7 days), did you or others in your household consume any […]?”

4. Empirical results

The objective of this section is to investigate the possible use of beta-type distributions in describing the distribution of caloric intake. The Fisk, B2, SM, Dagum, GB2 and the lognormal (used as comparison term) distributions were fitted to the data on per adult equivalent daily caloric intake (DEC)[22]. Maximum likelihood estimates are used to estimate the unknown population parameters of each density function[23] and the results are reported in Table 1.

Table 1 Estimated parameters of selected distribution functions

	GB2	SM	Dagum	B2	Fisk	Lognormal
A	2.8	3.3	4.0	1.0	3.7	m = 8.0
B	3485.9	3396.2	3215.9	9470.9	2938.0	v = 0.5
P	1.3	1.0	0.8	5.9	1.0	
Q	1.8	1.4	1.0	18.1	1.0	
- ln Log-L *	-106823386	-106826931	-106841142	-106893968	-106866243	-107100000

* it is a log pseudolikelihood

The GB2 provides the largest log-likelihood (i.e. the best fit), as the distribution tree in Figure 1 would suggest. However, the computed log-likelihood is actually pseudo log-likelihood because of the use of sample weights in the estimation[24], thus likelihood-ratio test along with AIC test cannot be used. An alternative tool is the Wald test that gives directions in the choice among nested distributions. Unfortunately, the Wald test cannot be used for the lognormal model because this distribution is just a limiting case and not a linear restriction of the GB2. Thus, Kolmogorov-Smirnov (KS) test and the mean integrated squared error (MISE) are computed along with Wald test to assess the inadequacy of the lognormal model in describing the daily calorie consumption. The results of the tests on the restrictions required by each nested distribution of the GB2 are reported in Table 2.

For the Wald test, all the restrictions of nested models are rejected at 0.1% significance

[22] Statistical packages developed by Jenkins S. for Stata Software are used to fit distributions.

[23] “The MLE is rather sensitive to isolated observations located sufficiently far away from the majority of the data. There appears therefore to be some interest in more robust procedures. For a robust approach to the estimation of the Dagum model parameters using an optimal B-robust estimator (OBRE), see Victoria-Faser (1995)” (Kleiber & Kotz, 2003)

[24] “When there is clustering, individual observations are no longer independent, and the “likelihood” does not reflect this. Where there are pweights, the “likelihood” does not fully account for the “randomness” of the weighted sampling. The “likelihood” for pweighted or clustered MLEs is used only for the computation of the point estimates and should not be used for variance estimation using standard formulas. Thus the standard likelihood-ratio test should NOT be used after estimating pweighted or clustered MLEs.” (Sribney, 1997)

level; the SM restriction is the only exception. It implies that the more parsimonious three-parameter SM distribution already describes adequately the data. Nothing can be said about the lognormal. However, KS test, by checking for the equality of distributions, rejects with a 99.9% confidence the hypothesis that the dietary energy consumption may be distributed as a lognormal, a B2 or a Fisk distribution. On the other hand, KS test accepts the GB2, the SM and the Dagum distribution. Limitations on the use of the Kolmogorov-Smirnov test derives from the fact that the theoretical cumulative distribution must be fully specified, that is the location, scale and shape parameters need to be the real ones. Here, the parameters have been estimated through ML estimators. Similarly, the MISE criterion[25] suggests that either a GB2, or a SM or a Fisk distribution will give a better fit than the Dagum, B2 and lognormal distribution.

Table 2 - Goodness of fit of GB2's nested distributions

	Lognormal	**GB2**	**SM**	**Dagum**	**B2**	**Fisk**
Adj. Wald			2 (0.1579)	8.74 (0.0002)	48.24 (0.0000)	8.91 (0.0002)
KS	1 (0.000)	0.0090 (0.314)	0.0107 (0.148)	0.0108 (0.140)	0.3020 (0.000)	0.0943 (0.000)
MISE	5.47E-10	2.48E-10	2.79E-10	3.04E-10	4.03E-10	2.16E-10

note: p-value in parentheses; KS test user-written command allowing for weights used (Nikolas Mittag http://home.uchicago.edu/~mittag/programs.html)

The statistical tests allowed summarizing differences between the shape of a variable and a theoretical distribution in one number. However, it inevitably hides a lot of information. Accordingly, it seems more appropriate to complement these results using graphical tools.

The graph of observed and predicted probabilities for calorie consumption (Figure 2) offers the first chance for visual comparison. At first glance, the lognormal distribution seems to offer a poor fit and a tendency to accentuate the skewness.

[25] MISE is equal to: $E\int\left(f_n(x)-f(x)\right)^2 dx$, where, f_n is one of the estimated distributions of dietary energy consumption, while f is the real distribution. Here, f was approximated using the non-parametric adaptive kernel estimation.

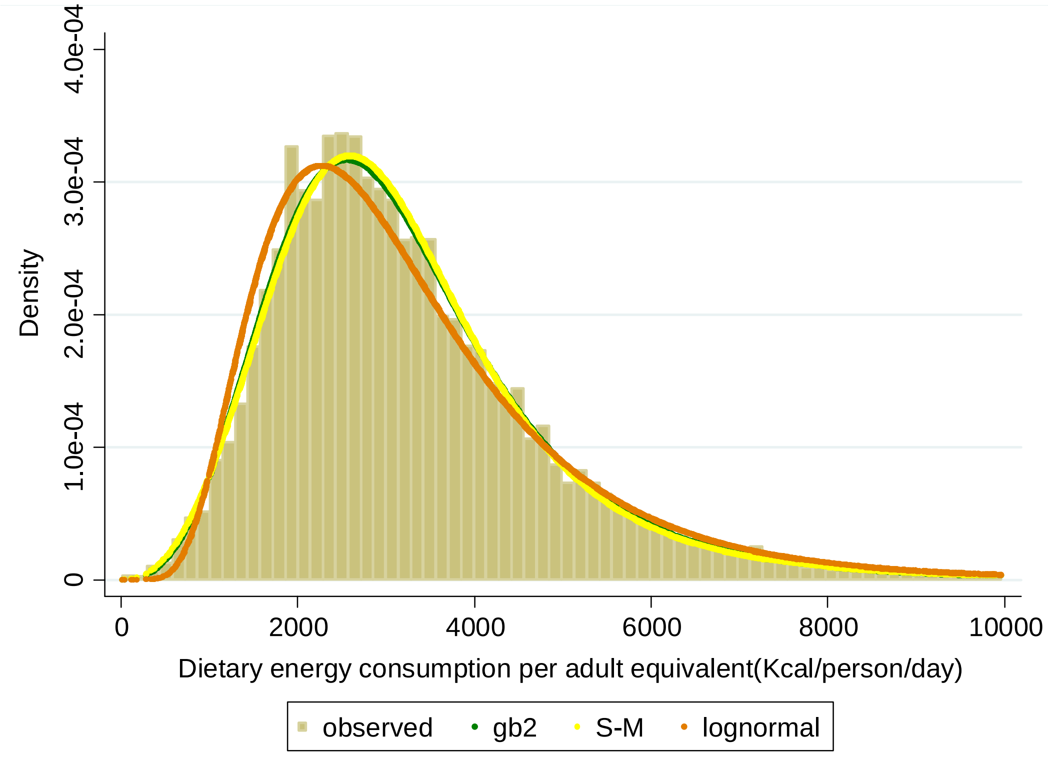

Figure 2 Distribution of dietary energy consumption

The quantile-quantile plots (q-q plot) in Figure 3 allow making comparisons between each statistical distribution and the sample. In fact, in a q-q plot, the quantile of the sample data are plotted against the quantile of the hypothesized distribution; if the data do not depart substantially in shape from the distribution, then the plot of the quantiles will be a straight line with slope 1, pointed towards the origin. In

Figure 3, all the plots show an important departure from the sample data distribution in the right tail, however it is important to highlight two details. First, the departure of the statistical distribution from the real data starts beyond the 99th percentile and there is not any systematic lack of fit in other parts of the data. Second, the extreme values do not show any systematic pattern across districts. As a result of this as well as because the aim of the research is to look at the problem of undernourishment, the discrepancy for the last percentile does not have a significant impact on the results of this study. In addition, Wilk and Gnanadesikan (1968) advise that when the sample data have long tails this type of graphical representation tends "to emphasize the comparative structure in the tails and to blur the distinctions in the 'middle'"[26]. Thus, a complementary tool is used. It is the percent

[26] The reason for this is that the quantile change rapidly when the density is sparse (e.g. in the tails) and a slowly when the density is high (in the middle).

versus percent plots (p-p plot), which has a behavior opposite than the q-q plot, being more sensitive to discrepancies in the middle of a distribution. At a first global look, p-p plots reveal some discrepancies only for the lognormal distribution.

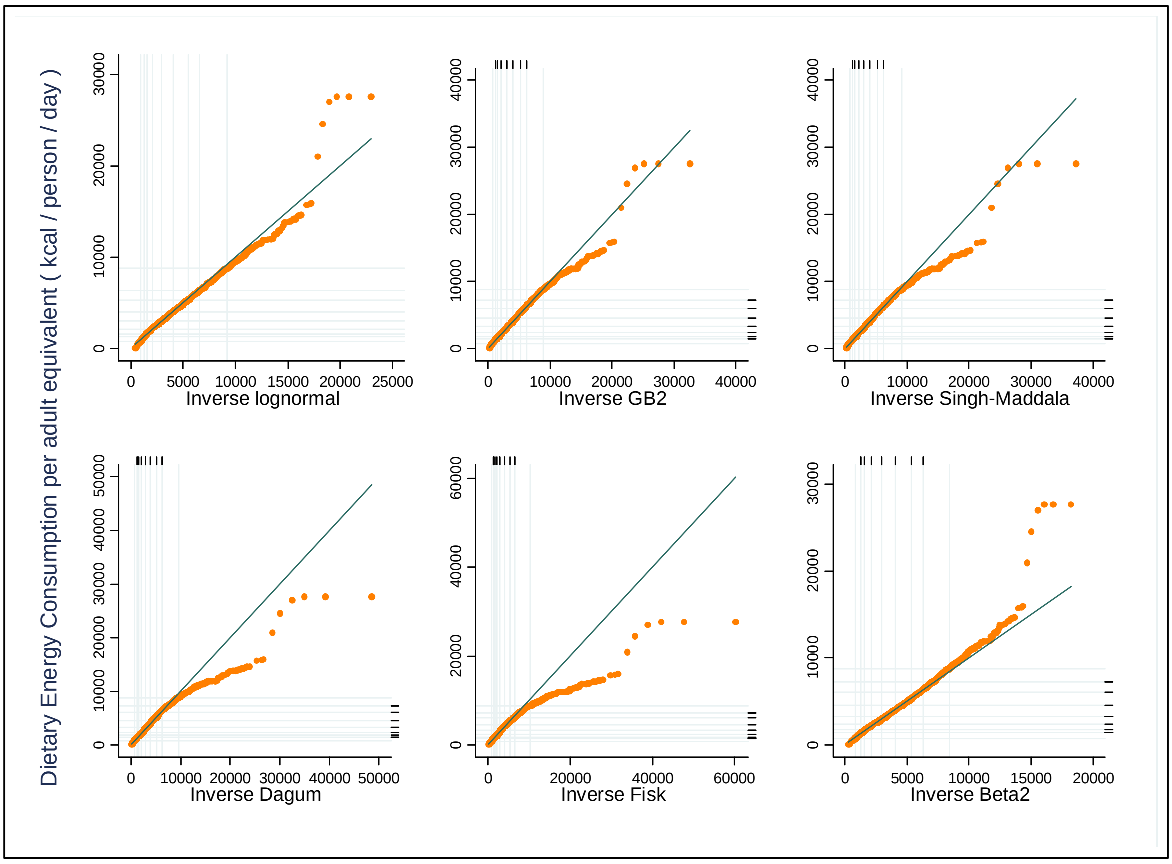

Figure 3 Quantile-Quantile plots of dietary energy consumption and selected statistical distributions. Grid lines are 1, 5, 10, 25, 50, 75, 90, 95, 99 percentiles

The large range of data and the big discrepancy in the right tail do not allow seeing the different quality of fit offered by each distribution in the left tail. Then, by focusing on the left side of the distribution, all the q-q plots show the extreme left end (fifth percentile) curving downward. It indicates that the sample data have a heavier tail than the statistical distributions therefore the model will underestimate the undernourished. However, the discrepancy looks negligible (only less the 10 data points) for the GB2, SM and Dagum, while the lognormal distribution shows the worst fit. The lognormal underestimates the number of undernourished up to the tenth percentile and slightly overestimates it thereafter.

Finally, the percentiles of the observed energy intake and the corresponding values predicted using the statistical distributions, along with key distributional summary measures, are reported in Table 3. They help to assess the overall goodness of fit.

Table 3 Distributional summary statistics of dietary energy consumption - kcal/person/day

	Sample	lognormal	GB2	SM	Dagum	Fisk	B2
p01	765	915 *(19.6%)*	785 *(2.6%)*	761 *(-0.5%)*	754 *(-1.4%)*	835 *(9.2%)*	835 *(9.2%)*
p05	1293	1280 *(-1.0%)*	1260 *(-2.6%)*	1250 *(-3.4%)*	1260 *(-2.6%)*	1310 *(1.3%)*	1250 *(-3.4%)*
p25	2154	2080 *(-3.4%)*	2160 *(0.3%)*	2170 *(0.7%)*	2180 *(1.2%)*	2170 *(0.7%)*	2120 *(-1.6%)*
p50	2944	2910 *(-1.1%)*	2970 *(0.9%)*	2970 *(0.9%)*	2960 *(0.6%)*	2940 *(-0.1%)*	2970 *(0.9%)*
p75	4013	4070 *(1.4%)*	4000 *(-0.3%)*	3990 *(-0.6%)*	3960 *(-1.3%)*	3970 *(-1.1%)*	4080 *(1.7%)*
p95	6329	6590 *(4.1%)*	6260 *(-1.1%)*	6260 *(-1.1%)*	6350 *(0.3%)*	6580 *(4.0%)*	6290 *(-0.6%)*
p99	8793	9240 *(5.1%)*	8960 *(1.9%)*	9120 *(3.7%)*	9620 *(9.4%)*	10300 *(17.1%)*	8440 *(-4.0%)*
<ADER	27.63%	29.77% *(7.8%)*	27.14% *(-1.8%)*	26.90% *(-2.6%)*	26.59% *(-3.7%)*	26.86% *(-2.8%)*	28.32% *(2.5%)*
<MDER	11.11%	11.71% *(5.5%)*	10.84% *(-2.4%)*	10.83% *(-2.5%)*	10.60% *(-4.6%)*	10.00% *(-10.0%)*	11.58% *(4.2%)*
Mean	3268	3290 *(0.7%)*	3270 *(0.1%)*	3270 *(0.1%)*	3290 *(0.7%)*	3330 *(1.9%)*	3270 *(0.1%)*
Std. Dev.	1645	1740 *(5.8%)*	1680 *(2.1%)*	1710 *(3.9%)*	1820 *(10.6%)*	1970 *(19.7%)*	1610 *(-2.1%)*

* Goodness-of-fit; **ADER=2233 kcal/person/day; ***MDER=1610 kcal/person/day
Worst fit; best fit ; number inside the parenthesis is the percentage difference between the statistical distribution and the sample.

Taking into consideration the mean of energy consumption, the lognormal performs only better that the Fisk distribution. By choosing a lognormal density function one would overestimate the mean of energy intake only by 1%, but would predict a distribution with a higher variance (standard deviation 6% higher than in the actual distribution). Thus, implying a higher inequality among the population in having access to food. On the other side, the GB2, along with SM and B2 predict almost perfectly (+0.1%) the mean and they give a better representation of the variance (+2% for GB2 and B2, +4% for the SM). In summary, the numerical comparison of percentiles confirms the results of the graphical assessment of goodness-of-fit, which is the inadequacy of the lognormal distribution and the superiority of GB2 and SM in describing the sample caloric intake distribution.

Least but not last, Table 3 gives the percentage of undernourished people in Malawi. In order to compute the head-count of the undernourished people (first column), the dietary energy requirement is used as cut-off point. Dietary energy requirement (DER) is defined by the Joint FAO/WHO/UNU Expert Consultation as the “amount of food energy needed to balance energy expenditure in order to maintain body size, body composition and a level of necessary and desirable physical activity consistent with long-term good health”. The Joint FAO/WHO/UNU Expert Consultation gives clear guidelines on how to compute the energy requirement for each age distinguishing between male and female, and three different levels of physical activity (light, moderate and heavy)[27]. The methodology is based on the use of a “reference” man and women; in other words, the study uses a normative value for body-weight consistent with good health. FAO’s methodology derives weights from the weight-for-height reference table using the actual height. Here the reference-weights are obtained from the WHO reference BMI-for-age. This is considered as a strength in this research because the use of the actual body-weight could imply bias (e.g. a person who is underweight would be assigned an energy requirement to keep his body underweight). Two values of DER are used as thresholds. One is the average dietary energy requirement (ADER), which is the energy necessary for an individual of average physical stature to perform moderate activities. The other is the minimum dietary energy requirement (MDER), which is the energy necessary to a sedentary individual of a physical stature compatible with the lower limit of the range of variation of body-weight[28].

The ADER and MDER are computed for each household using its sex-age composition and the energy requirement is compared with the actual energy intake. If the household’s intake is less than its energy requirement, all the components of the household are counted as undernourished. For the parametric functions, the percentage of people undernourished in Table 3 refers to the probability of being undernourished (energy intake less than energy requirement):

$$P(x < r) = \int_{x<r} f(x)dx$$

where *x* is per adult equivalent DEC, *r* is the energy requirement and *f(x)* is one of the functional form tested[29].

[27] To compute ADER, the PAL level was set at 1.8, midpoint of the physical activity level value range for moderate activity, while it was set and 1.55 for MDER, midpoint of the physical activity level value range consistent with a sedentary life

[28] FAO’s threshold is the minimum energy requirement (MER), which imply that undernourishment is defined as “an extreme form of food insecurity, arising when food energy availability is inadequate to cover even minimum needs for a sedentary life” (FAO, 2012).

[29] r is the weighted mean of ADER (MDER)

The non-parametric analysis conducted with the household survey data suggests that 27.63% of the population in Malawi (3.4 million) do not meet the energy requirement necessary to perform moderate physical activities, and 11.11% of the population consume less than their minimum energy requirement. The use of parametric distributions for the estimation of the proportion of undernourished gave similar results predicting between 3.3 and 3.7 million people with calorie deficiency[30]. However, important differences among these estimates exist. Figure 4 highlights them. As one can see, the GB2 is the model whose estimates gets closer to the real numbers slightly underestimating them (-1.8% and -1.4% the number of people with calorie consumption less than ADER and MDER, respectively). The assumption of a SM distribution for caloric intake also underestimates calorie deficient people by 2.6 and 2.5 per cent. By contrast, the widely used lognormal distribution overestimates the probability of being undernourished by 8% and severely undernourished by 5%, thus giving the worst estimation.

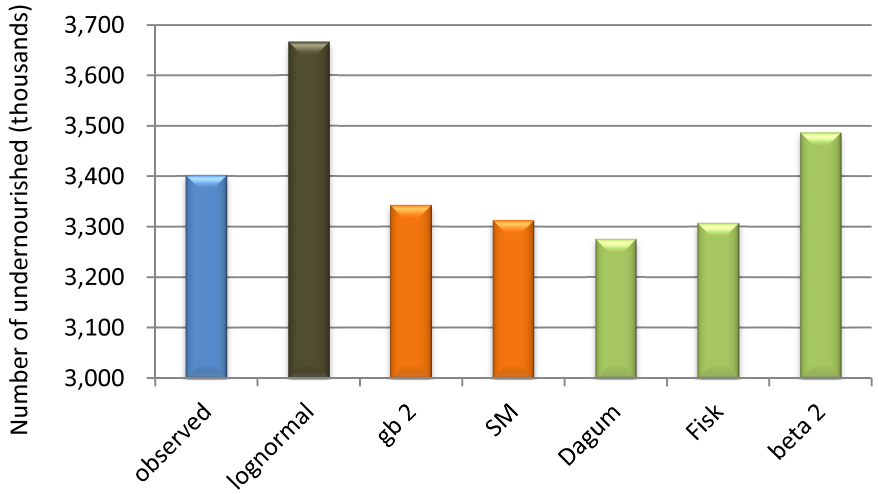

Figure 4 Observed and estimated number of calorie deficient people (DEC<ADER) (thousands)

In summary, this comparison reveals how the choice of a parametric distribution can have a major impact on the measurements of undernutrition, and consequently it may effect policy decisions. The analysis on caloric intake in Malawi highlights the limits of the lognormal distribution. Its lack of flexibility negatively affects its ability to fit the dietary energy consumption, and suggest the use of a different distribution. The GB2 model offers overall the best fit followed by Sigh-Mandala. However, as the Wald tests suggest, there are not significant gains from the use of a four-parameter GB2 rather than the three-parameter SM.

[30] Calorie consumption less than average dietary requirement (ADER)

5. Conclusion

This paper addressed a major shortcoming in the current methodologies of undernutrition measurements deriving from the assumptions on the distribution of caloric intake. The current methodologies used by both international organizations, such as the FAO, and scientific community assume that the distribution of caloric intake is lognormal.

This study tested the potential bias deriving from the use of lognormal distribution using household food consumption data from Malawi. Both graphical and numerical assessments showed the superiority of the beta-type distributions over the lognormal. In particular, the lognormality assumption results in a significant overestimation of the undernourished in Malawi. Models applying the GB2 or the SM distribution for caloric intake distribution obtained better estimates. However for the Wald test, the fourth parameter of the GB2 does not add any significant information suggesting that the more parsimonious three-parameter SM distribution already describes adequately the data and it is a suitable candidate for representing the caloric intake distribution.

This result does not pretend to have any universal value; however, it sheds light on an important issue that has been neglected in the literature. The assumption of SM distribution needs to be tested using survey data from different years and country, and the tradeoff between accuracy and complexity should be opportunely taken into account. For Salem and Mount (1974), "parameters should be simple to estimate and also to interpret in an economically meaningful way". The assumption of lognormality used in different studies may have been a convenient choice. The lognormal distribution is simply characterized by the mean and standard deviation that are easy to compute or estimate. However, also the SM distribution has attractive characteristics: it has a simple closed form for the density, cumulative distribution, and quantile function. It is characterized by a scale parameters (b), a shape parameter (q) determining the right tail of the distribution such that the right tail becomes lighter as q increase, and a further shape parameter (a) characterizing the general shape of the distribution[31]. However, the parameters of the SM are more difficult to interpret in an economically meaningful way.

The last highlight of this paper concerns the relevance of this topic for policy formation. The analysis conducted on household food consumption data from Malawi showed that the

[31] For a>1 the function is unimodal

different assumptions on the statistical distribution of food consumption results in different estimations of hunger, within a range of 400,000 people. Clearly, it is easy to see how the misspecification of the model may undermine the monitoring process for the achievement of the MDG1. Furthermore, it may introduce misleading information preventing policy makers from formulating effective policies addressing food insecurity. The analysis of vulnerability to food insecurity offers a significant example. If the parametric model chosen overestimates the number of vulnerable people, it may result in policies that neglect those that are hungry now. Thus, in an environment of limited resources, policy makers may divert resources to unneeded ex-ante interventions for the prevention of food insecurity rather than allocate them to ex-post interventions aimed at alleviating hunger. The two types of interventions are clearly different, and policies aimed at reducing the risk of vulnerable people or enhancing their resilience are less likely to be effective in the fight against hunger.

References

Ecker, O., Weinberger , K., & Qaim, M. (2010). Patterns and determinants of dietary micronutrient deficiencies in rural areas of East Africa. *African Journal of Agricultural and Resource Economics, 4*(2), 175-194.

Azzalini, A. (1985). A Class of Distributions Which Includes the Normal Ones. *Scandinavian Journal of Statistics, 12*(2), 171-178.

Bekaert, G. (1991). Caloric consumption in Industrializing Belgium. *The Journal of Economc History, 51*(3), 633-655.

Blaydes , L., & Kayser, M. (2011). Counting Calories: Democracy and distribution in the developing world. *International Studies Quarterly, 55*, 887-908.

Bordley, R. F., McDonald, A., & Mantrala, A. (1996). Something new, something old: parametric models for the size distribution of income. *Journal of Income Distribution, 6*, 91-103.

Carletto, C., Zezza, A., & Banerjee, R. (2012). Towards better measurement of household food security: harmonizing indicators and the role of household surveys. *Global Food Security*, http://dx.doi.org/10.1016/j.gfs.2012.11.006.

Chaudhuri, S., Jalan, J., & Suryahadi, A. (2002). *Assessing Household Vulnerability to Poverty from Cross-sectional Data: a Methodology and Estimates from Indonesia.* Department of Economics . New York: Columbia University .

Christiaensen , L. J., & Boisvert, R. N. (2000). *On Measuring Household Food Vulnerability: Case Evidence from Northern Mali.* Working papers 127676. Cornell University, Department of Applied Economics and Management.

Cowell, F. A., & Victoria-Feser, M. P. (1996). Poverty measurement with contaminated data: A robust approach. *European Economic Review, 40*, 1761-1771.

de Haen, H., Klasen, S., & Qaim, M. (2011). What do we really know? Metrics for food insecurity and undernutrition. *Food Policy, 36*, 760-769.

Dercon, S. (2001, August). Assessing Vulnerability to Poverty. *mimeo*. Oxford, UK: Department of Economics, Oxford University.

FAO. (1977). *The Fourth World Food Survey.* FAO Food and Nutrition Series, 10. Rome: FAO.

FAO. (1987). *1985. The fifth World Food Survey.* Rome: FAO.

FAO. (1996). *Report of the World Food Summit.* Roma: FAO.

FAO. (2003). Measurement and Assessment of Food Deprivation and Undernutrition. (p. Proceedings of an International symposium). Rome: FAO.

FAO. (2008). *FAO methodology for the measurement of food deprivation.* Statistic Division. Rome: FAO.

FAO. (2012). *The State of Food Insecurity in the World 2012.* Rome: FAO.

Fogel, R. (1997). New findings on secular trends in nutrition and mortality: some implication for population theory. In N. Rosenzweig, & O. Stark, *Handbook of population and family economics* (pp. 433-481). Amsterdam: Elsevier Science.

Haddad, L., Sullivan, J., & Kennedy, E. (1991). *Identification and evaluation of alternative indicators of food and nutrition security: some conceptual issues and an analysis of extant data.* Washington, DC: US Agency for International Development.

Haddad, L., Kennedy, E., & Sullivan , J. (1994). Choice of indicators for food security and nutrition monitoring. *Food Policy, 19*(3), 329-343.

Headey, D., Ecker, o., & Trinh Tan, J. (2012, January 17-19). Measuring food security in a volatile world. *draft version for the FAO International Scientific Symposium on Food and Nutrition Security Information: from valid measurement to effective decision making*. Rome.

Jenkins, S. (2009). Distributionally-sensitive inequality indices and the GB2 income distribution. *Review of Income and Wealth, 55*(2), 392-398.

Jensen, R., & Miller, N. (2010). *A Revealed Preference Approach To Measuring Hunger and Undernutrition.* on-line working paper series PWP-CCPR-2010-022. Los Angeles: California Center for Population Research, University of California.

Kennedy, G., Nantel, G., & Shetty, P. (2004). *Globalization of food systems in developing countries: a synthesis of country case studies.* FAO Food and Nutrition Paper n.83. Rome: FAO.

Kleiber, C., & Kotz, S. (2003). *Statistical Size Distributions in Economics and Actuarial Sciences.* New York: Wiley.

Malawi NSO. (2005). *Integrated Household Survey 2004-2005. Household socio-economic characteristics. Report.* Zomba, Malawi: National Statistical Office.

Malawi Vulnerability Assesment Committee. (2012). National food Security Assessment: June 2012 and October 2012. Lilongwe, Malawi: Ministry of Economic Planning & Development.

Maxwell, S. (1996). Food security: a post-modern perspective. *Food Policy, 21*(2), 155-170.

McDonald, J. (1984). Some Generalized Functions for the Size Distribution of Income. *Econometrica, 52*(3), 647-665.

McDonald, J., & Xu, Y. (1995). A generalization of the beta distribution with applications. *Journal of Econometrics, 66*, 133-152.

Naiken, L. (2007). *The Probability Distribution Framework for Estimating the Prevalence of Undernourishment: Exploding the Myth of the Bivariate distribution.* Statistics Division Working Paper, Series ESS/ESSG/009e. Rome: FAO.

Salem, A., & Mount, T. (1974). A convenient descriptive model of income distribution: the gamma density. *Econometrica, 42*(6), 1115-1127.

Sen, A. (1981). *Poverty and Famines: An Essay on Entitlement and Deprivation.* Oxford: Clarendon Press.

Smith, L. (1998). Can FAO's measure of chronic undernoiurishment be strengthned? *Food Policy, 23*(5), 425-445.

Stevens, C., Devreux, S., & Kennan, J. (2002). The Malawi Famine of 2002: More Questions than Answers. Brighton, Sussex: IDS.

Sukhatame, P. (1961). The World's Hunger and the Future Needs in Food Supplies. *Journal of the Royal Statistics Society, 124*, 463-585.

Svedberg, P. (2000). *Poverty and Undernutrition. Theory, Measurement, and Policy.* Oxford: Oxford University Press.

Tukey, J. W. (1977). *Exploratory Data Analysis .* MA: Addison-Wesley.

Vecchi, G. (2011). *In richezza e povertà. Il benessere degli italiani dall'unità a oggi .* Bologna: Il Mulino.

Vecchi, G., & Coppola, M. (2006). Nutrition and growth in Italy, 1861–1911: What macroeconomic data hide. *Explorations in Economic History, 43*(3), 438-464.

Wilk, M., & Gnanadesikan, R. (1968). Probability Plotting Methods for the Analysis of Data. *Biometrika, 55*(1), 1-17.