Then

$$0 \longrightarrow E_1(K) \longrightarrow E_0(K) \xrightarrow[\text{reduction}]{} \tilde{E}_{ns}(k) \longrightarrow 0$$

is an exact sequence.

Summary:– (for $\mathbb{Q}$; $y^2 = x^3 + ax + b$, $\Delta = 4a^3 + 27b^2 \neq 0$)

Reduction type	Δ mod p	$-2ab$ mod p	$\tilde{E}_{ns}$	$N = \lvert \tilde{E}(\mathbb{F}_p) \rvert$
good	$\neq 0$		$\tilde{E}$	$\lvert N-p-1\rvert \leq 2\sqrt{p}$
cusp	0	0	$\mathbb{G}_a(k)$	p
split multiplicative	0	$\square$	$\mathbb{G}_m(k)$	$p-1$
non-split multiplicative	0	$\neq \square$	$\mathbb{G}_m[-2ab]$	$p+1$

$\square$ = a square

$\mathbb{G}_a$ = affine line $\mathbb{A}^1$ (additive group).

$\mathbb{G}_m$ = multiplicative group $\mathbb{A}^1(k) \setminus \{0\} = k^\times$

$\mathbb{G}_m[a]$ = twisted multiplicative group defined as follows:–
if $a \in k \setminus k^2$, $L = k[\sqrt{a}]$,
G = plane affine curve $X^2 - aY^2 = 1$,
operation

$$(x,y) \times (x',y') = (xx' + ayy', xy' + x'y)$$

e.g. $k = \mathbb{R}$, $a = -1$, $\mathbb{G}_m[a]$ = circle $X^2 + Y^2 = 1$.

Conductor:– a refinement of Δ (has the same prime factors as Δ, powers adjusted according to reduction data modulo each prime and ramification data).