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★**Multidimensional continued fractions.**

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The digits in the classical continued fraction expansion of a real number in $(0, 1)$ are generated by an algorithm satisfying several desirable properties: The algorithm terminates at 0 if and only if the real number is rational; it is eventually periodic if and only if the real number is a quadratic irrational; it generates a sequence of rational approximants that are not only good but optimally so; the algorithm viewed as a map on the interval preserves an absolutely continuous measure on the interval, and the resulting measure-preserving system has many strong ergodic properties.

C. G. J. Jacobi [J. Reine Angew. Math. **69** (1868), 29–64; JFM 01.0062.01] introduced a higher-dimensional continued fraction for a pair of reals. His hope was to emulate the classical quadratic result of Lagrange by showing that the two-dimensional algorithm is eventually periodic if and only if the two reals belong to a cubic number field. This is still not known to hold.

Since then there has been a great proliferation of continued fraction algorithms; no single algorithm in higher dimensions has been found that shares all the appropriate analogs of the desirable properties, so different algorithms are studied for different reasons. Some are better adapted to producing simultaneous Diophantine approximation results, some are more convenient algorithmically, some have better dynamical or stochastic properties. A. J. Brentjes [*Multidimensional continued fraction algorithms*, Math. Centrum, Amsterdam, 1981; MR0638474 (83b:10038)] gave an extensive survey of the classical literature and overview of the state of higher-dimensional algorithms, with an emphasis on practical implementations of them.

This book is a welcome addition to the literature, with particular emphasis placed on the dynamical and ergodic-theoretic aspects of those higher-dimensional continued fraction algorithms that can be described using fractional linear maps. The machinery of fibred systems described in the author's monograph [*Ergodic theory of fibred systems and metric number theory*, Oxford Univ. Press, New York, 1995; MR1419320 (97h:11083)] is used extensively. The book has an extensive bibliography and provides a good introduction to a fascinating field, which on the one hand dates back to Jacobi and Lagrange, while on the other points forward with interesting open problems and deep connections to geometry, ergodic theory, and number theory. *Thomas Ward* (4-EANG)