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A nonlinear dynamics perspective of Wolfram's new kind of science. Vol. 2. (English)

World Scientific Series on Nonlinear Science. Series A 57. Hackensack, NJ: World Scientific. i–xviii, 369–947. $ 108.00, £ 58.00/v.2; $ 245.00, £ 132.00/set (2007).

This is the second volume of A nonlinear dynamics perspective of Wolfram's new kind of science; some of the constituent chapters appear in the journal *International Journal of Bifurcation and Chaos*, with additional co-authors. The pre-publication book review with which this volume starts ends by saying "The volumes are somewhat special and exciting because they [...] show gradual development of ideas and concepts in an educational and entertaining *hence* mathematically rigorous manner" (my emphasis).

The book should be viewed in the context of the earlier work of *S. Wolfram* [A new kind of science, Wolfram Media. xiv, 1197 p. (2002; Zbl 1022.68084)] and one of its many substantial reviews – for example, that of *L. Gray* [Notices Am. Math. Soc. 50, No. 2, 200–211 (2003; Zbl pre02115042)].

The bulk of the book is taken up with various graphical representations of cellular automata, including multi-coloured portraits on grids, less familiar representations of them and their orbits via cobweb diagrams made from "characteristic functions" on intervals (obtained by associating binary strings to digits in binary expansions of real numbers), and various finite-state automata associated to cellular automata.

The result physically is a sort of coffee-table book of striking illustrations. The result mathematically is rather less straightforward to characterize. From a mathematical point of view, cellular automata are on the one hand fascinating (because they are both simple to define and manifestly capable of exhibiting great orbit complexity) while on the other quite difficult to deal with (apart from a small subset, their basic dynamical properties are not fully understood). Despite the finite definition of any given cellular automata, the natural questions about them (What are the invariant measures? Are there distinguished invariant measures? What are the resulting statistical properties of the orbits? What is the topological entropy?) all involve genuinely infinite phenomena (via limiting processes or the need to select something from an infinite set of possibilities). Thus there is a genuine unavoidable gap between what is observed in any finite presentation of the orbits of a cellular automata and what may be the genuine statistical or dynamical properties of the system. Given the ease with which orbit portraits of cellular automata may be produced, and the great diversity of patterns then seen, it is all too easy to allow the healthy dialogue between finite examples and rigorous theory to change until the tail wags the dog.

While the pictures and examples in this book are both striking and interesting, it is difficult to develop much confidence in the mathematical development. For example, on p. 493 (footnote) there is the strange suggestion that an irrational real number has the property that its binary expansion contains every possible finite block of 0s and 1s infinitely often. This anxiety about quite how the statistical properties of cellular automata are being defined and tested is reinforced by the index, which does not contain

measure, *mixing* nor *entropy*; *ergodic* refers to the rather confused discussion on p. 493. The absence, in both index and text, of any real development of invariant measures for cellular automata again makes it difficult to understand what is being asserted (for example) on p. 499: "We have completely characterized the [...] asymptotic behaviors of 206 one-dimensional CA with three inputs [...] A single randomly chosen initial state [...] is used as a probe...".

The contents of the book comprise the following chapters: From Bernoulli shift to $1/f$ spectrum; Fractals everywhere; From time-reversible attractors to the arrow of time; Mathematical foundation of Bernoulli σ_f-shift maps; The arrow of time; Concluding remarks. The epilogue to the book begins "This book heralds the *end of the beginning* of our *analytic* approach to cellular automata and complexity theory."

While there is much of interest here, and in particular many interesting examples presented in novel ways, I can only suggest that anyone interested in the asymptotic behaviour of cellular automata consult this book in conjunction with Gray's review [op. cit.] and a conventional source on ergodic theory.

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Keywords : cellular automata; time reversal; nonlinear dynamics

Classification :

∗37B10 Symbolic dynamics
37N20 Dynamical systems in other branches of physics
37B15 Cellular automata
68Q60 Specification and verification of programs