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Restricted orbit equivalence for actions of discrete amenable groups. (English)

Cambridge Tracts in Mathematics. 146. Cambridge: Cambridge University Press. vi, 201 p. £35.00; $ 50.00 (2002). [ISBN 0-521-80795-6/hbk]

One of the most striking early results in ergodic theory is that of *H. A. Dye* [Am. J. Math. 81, 119-159 (1959; Zbl 0087.11501)], showing that any two ergodic measure-preserving transformations on non-atomic standard probability spaces are orbit equivalent. This result is so definitive that it seemed to close off orbit equivalence as an interesting notion for measure-preserving transformations (though it initiated the very important study of the rich orbit equivalence structure of non-singular transformations). At the opposite extreme of strength of equivalences, *D. Ornstein* [Adv. Math. 4, 337-352 (1970; Zbl 0197.33502)] showed that Bernoulli transformations of the same entropy are measurably isomorphic, and characterized the property of being a Bernoulli shift in terms of being finitely determined. The notion of 'restricted' orbit equivalence interpolates between these two results in the following way. A notion of 'size' is developed, and for each size m there is an equivalence relation (m-equivalence), an associated entropy (m-entropy), and a distinguished class of transformations (m-finitely determined). The equivalence theorem then shows that any two m-finitely determined transformations with the same m-entropy are m-equivalent. With different choices of m this recovers the Dye and Ornstein theory, and much else besides, including the notion of even Kakutani equivalence. The theory in this form was developed by *D. J. Rudolph* [Mem. Am. Math. Soc. 323, 150 p. (1985; Zbl 0601.28013)] for single transformations and then extended to actions of higher-rank abelian groups by *J. W. Kammeyer* and *D. J. Rudolph* [Ergodic Theory Dyn. Syst. 17, No. 5, 1083-1129 (1997; Zbl 0897.58028)].

In this book several projects are carried out. The restricted orbit equivalence theory is extended to actions of discrete amenable groups, using in part the basic tools developed for amenable ergodic theory by *D. S. Ornstein* and *B. Weiss* [J. Anal. Math. 48, 1-141 (1987; Zbl 0637.28015)]. The theory of restricted orbit equivalence is also cleaned up: those basic results and definitions that with hindsight might have been done in a way that extends and applies better have been recast, and the connections between the definitions explained in an appendix. The copying lemmas are 'modern' in the sense that they avoid using the marriage lemma. Those parts of the equivalence theory for amenable group actions that are of wider interest (that is, of interest for amenable ergodic theory in general) are isolated and presented in a clear fashion. Finally, a short but careful introduction explains how restricted orbit equivalence arose both mathematically and historically. This makes very interesting reading, and it explains the influential work of *Vershik* on (what could now be called) restricted orbit equivalences for actions of infinite sums of finite groups, and the work of *A. B. Katok* [Math. USSR, Izv. 11, 99-146 (1977; Zbl 0379.28008)] and others in the former Soviet Union on restricted equivalences.

As suggested above, this profound book deserves a wider readership than those interested only in the topic of the title. It has much to say about the ergodic theory of discrete amenable group actions, and gives an attractive overview and backward look at the restricted orbit equivalence theory for single transformations and other 'small' groups.

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Keywords : measure-preserving transformation; amenable; entropy; restricted orbit equivalence

Classification :

∗43-02 Research monographs (abstract harmonic analysis)
37-02 Research exposition (Dynamical systems and ergodic theory)
22-02 Research monographs (topological groups)
28D15 General groups of measure-preserving transformations
43A07 Means on groups, etc.
28D20 Entropy and other measure-theoretic invariants

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