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Topics in orbit equivalence. (English)

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These notes provide a rapid but thorough description of orbit equivalence with both ergodic theory and the perspective of set theory in mind. Orbit equivalence is concerned with the equivalence relation induced on a measure space X by a measure-preserving or measure-class-preserving action of a group Γ (points are equivalent if one is in the orbit of the other). These notes deal with the classical setting, in which Γ is countable and discrete, and X is a standard Borel space. The authors introduce two useful thematic notions. The first is 'elasticity', exemplified by the extension by *D. S. Ornstein* and *B. Weiss* [Bull. Am. Math. Soc., New Ser. 2, 161-164 (1980; Zbl 0427.28018)] of *H. A. Dye*'s theorem [Am. J. Math. 81, 119-159 (1959; Zbl 0087.11501) and Am. J. Math. 85, 551-576 (1963; Zbl 0191.42803)]: any two Borel nonatomic probability preserving actions of amenable groups are orbit equivalent. Thus the equivalence relation induced by such an action does not remember anything about the group that created it apart from the vague recollection that it was amenable. The second is 'rigidity', in which the equivalance relation induced by a group action carries a great deal of information about the group. For example, Furman has shown that if a free, nonatomic, probability-preserving action of some countable group Γ is orbit equivalent to the natural action of $SL(3,\mathbb{Z})$ on the 3-torus, then Γ is isomorphic to $SL(3,\mathbb{Z})$ and the actions are Borel isomorphic. A more recent instance of 'rigidity' is a result of Gaboriau, which gives the statement (under similar hypotheses) that orbit equivalence for actions of free groups remembers the rank of the free group.

These notes are in three parts. The first gives a quick treatment of ergodic theory. The second deals with orbit equivalence in the presence of amenability – the 'elastic' world: Dye's theorem, the Ornstein-Weiss theorem on orbit equivalence of measure-preserving amenable group actions, the Theorem by *A. Connes, J. Feldman*, and *B. Weiss* [Ergodic Theory Dyn. Syst. 1, 431-450 (1981; Zbl 0491.28018)] on hyperfiniteness of amenable equivalance relations. More recent work is included, for example the theorem by *S. Jackson, A. S. Kechris* and *A. Louveau* [J. Math. Log. 2, No. 1, 1-80 (2002; Zbl 1008.03031)]: any Borel action of a finitely generated group of polynomial growth on a standard Borel space induces a hyperfinite equivalence relation. The third part describes the theory of costs for equivalence relations, including a development of the work of Gaboriau, Hjorth and others. This part ends with a list of 15 open problems taken from the text.

The brevity of these notes may have squeezed out some of the detailed history of these developments, but the authors have nonetheless managed to make the mathematics self-contained and complete. With little in the way of prerequisites, these notes carefully cover a great deal of mathematics not always easily garnered from the literature. They will be of great interest to ergodic theorists as well as set theorists.

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Keywords : orbit equivalence; amenability; costs

Classification :

∗37A15 General groups of measure-preserving transformation
03E15 Descriptive set theory (logic)
28D05 Measure-preserving transformations
28D15 General groups of measure-preserving transformations
37A20 Orbit equivalence, cocycles, ergodic equivalence relations

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