Dynamical aspects of linear recurrence sequences: Final report

This short project enabled the investigators and Prof. van der Poorten to work on various dynamical realization problems; some of this work appears in [1]. The main results concern the dimension (as a $\mathbb{Z}$-module) of the space of sequences that are realizable (count periodic points for some map) and are linear recurrence sequences.

Theorem: *Let Δ denote the discriminant of the characteristic polynomial associated to a non-degenerate binary recurrence relation. Then the realizable subspace has*

(1) *dimension* 0 *if* $\Delta < 0$,
(2) *dimension* 1 *if* $\Delta = 0$ *or* $\Delta > 0$ *and non-square,*
(3) *dimension* 2 *if* $\Delta > 0$ *is a square.*

In the general case a less precise was found, and the nature of the Diophantine issues arising was clarified.

Theorem: *Let f denote the characteristic polynomial of a linear recurrence sequence with integer coefficients, assumed to be non-degenerate. If f is separable, with ℓ irreducible factors and a dominant root then the dimension of the realizable subspace cannot exceed ℓ. If $f(0) \neq 0$ then equality holds if either the dominant root is not less than the sum of the absolute values of the other roots or the dominant root is strictly greater than the sum of the absolute values of its conjugates.*

An indirect outcome of the research links established was the later collaboration on the monograph [2].

References

[1] G. Everest, A. J. van der Poorten, Y. Puri, and T. Ward, 'Integer sequences and periodic points', *J. Integer Seq.* **5** (2002), no. 2, Article 02.2.3, 10 pp. (electronic).

[2] G. Everest, A. van der Poorten, I. Shparlinski, and T. Ward, *Recurrence sequences*, in *Mathematical Surveys and Monographs* **104** (American Mathematical Society, Providence, RI, 2003).