Comments

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Lemma 3.16 can be stated slightly more generally with practically the same proof. Below we provide the generalization.

Lemma *Let* $p \geq 2$ *and* $n = p^m k \geq 4$ *where* k *is coprime to* p. *Let* $G \cong A_n$ *be the alternating group realized in* $GL(n-2, p)$. *Then* G *lifts to* $GL(n-2, H_m)$.

Proof. Set $R = \mathbf{Z}/p^m\mathbf{Z}$. Denote by M a free R-module with a basis $b_1, \ldots, b_n$. We turn M to an RG-module by forcing G to act on $b_1, \ldots, b_n$ by permutations. Set $K = R\langle b_1 + \cdots + b_n \rangle$, $L = R\langle b_2 - b_1, b_3 - b_1, \ldots, b_n - b_1 \rangle$ and $M_1 = R\langle b_2 - b_1, b_3 - b_1, \ldots, b_{n-1} - b_1 \rangle$. Clearly, K and L are RG-submodules of M. Observe first that $K \subset L$. Indeed, $b_1 + \ldots + b_n = nb_1 + \sum_{i=2}^n (b_i - b_1) \in L$ as $nb_1 = 0$ in M. Observe that M_1 is a free R-module (if $\sum_{i=2}^{n-1} r_i(b_i - b_1) = 0$ for some $r_i \in R$ then $\sum_{i=2}^{n-1} r_i b_i - \sum_{i=2}^{n-1} r_i b_1 = 0$ which implies that $r_2 = \cdots = r_{n-1} = 0$ as $b_1, \ldots, b_n$ is a free basis in M). Besides, $M_1 \cap K = 0$ by the same reason (if $\sum_{i=2}^{n-1} r_i(b_i - b_1) + r(b_1 + \ldots + b_n) = 0$ for some $r, r_i \in R$ then $\sum_{i=2}^{n-1}(r_i + r)b_i - (\sum_{i=2}^{n-1} r_i)b_1 + rb_1 = 0$; therefore, $r_i = -r$ for $i = 2, \ldots, n-1$ so $-(\sum_{i=2}^{n-1} r_i)b_1 + rb_1 = r(n-1)b_1 = -rb_1 = 0$ which is not true). Therefore, the R-module L/K is isomorphic to M_1 so L/K is a free R-module. Clearly, L/K is an RG-module, so we are done.